EFFECTS OF JOINT STIFFENING ON THE DYNAMIC RESPONSE OF FRAMES

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ABSTRACT

This paper examined the effects of joint stiffening on the dynamic response of frames subjected to lateral loads using the stiffness method. Modified Stiffness coefficients and fixed-end reactions were developed as functions of two parameters α and β representing the ratio of the length of the stiffened portion to that of the flexible portion of the left and right ends of a member respectively. In the absence of joint stiffeners (i.e. α = β = 0) the obtained modified coefficients revert to conventional expressions. The set of equations governing the motion of the frame with stiffened joints was derived using the lumped-mass procedure. Solution of the system gave the Natural Frequencies, Joint Displacement, Bending Moment, Shear Force, and Axial Force for various values of the parameters α and β ranging from 0.00 to 0.20 respectively. An earlier work showed that stiffening of joints enhances stability. This work established that natural frequency increases with increase in joint stiffening while joint displacement decreases with increase in joint stiffening and that joint stiffening allows substantial reduction in moments which leads to economic design of framed structures.

1.0 INTRODUCTION

Frames are subjected not only to static loads (i.e. gravity loads) but also to dynamic loads (i.e. lateral loads) which set up vibration in the structure [1]. Lateral loads include wind, earthquake, vibrating machinery, etc. [2]. The characterization of each of these dynamic loads is accomplished completely by a definite function of time with controlling parameters such as amplitude, frequency, period and phase [3]. Dynamic loads are generally divided into two categories, a transient or shock load of short duration and a steady-state load of relatively long duration. The present work is concerned with steady-state loads

Traditionally, the shear frame model was the ideal structure used for the determination of dynamic response. In shear frame model, rotation of joints is assumed not to occur and the structure is assumed to sway only in its plane [4-6]. The shear frame model is relatively easy to handle but is known to give results which may differ greatly from the actual [7] and it does not include the effects of axial deformations on the bending stiffness of columns.

Based on the ideas developed by Osadebe [8] the present work would introduce on improvement model by using frames with stiffened joints [9]. In this model, modified stiffness coefficients and fixed-end reactions were developed as functions of two parameters α and β representing the ratio of the length of the left and right ends of a member respectively. In the absence of joint stiffeners (i.e. $\alpha = \beta = 0$) the obtained modified coefficients revert to conventional expressions showing the problem formulation to be accurate. Given a frame with stiffened joints subjected to steady state dynamic loads the problem is to formulate equations of motion whose solution yield response information. The principle involved was applied to a frame with many degree of freedom (MDOF) for various values of the parameters α and β ranging from 0.00 to 0.20 from the results obtained relevant graphs were plotted for various parametric studies.

Karamanski [10] agreed that the overall performance of a framed structure can be improved by increasing the rigidity of its connecting joints. To achieve this in practice requires gradual widening of the sizes of individual members in the neighbourhood of the connecting joints. Another way this can occur is when a uniform beam intersects a column whose size is much greater than that of the beam [11]. Consequently the joints are assumed to be infinitely rigid or stiffened.

Previous work by Ekere [11] showed that joint stiffening enhances stability of frames. Also, design criteria for reinforced concrete box culvert with infinitely rigid joints by moment distribution method were developed by Diaz de Cossio [12]. The performance was satisfactory.

2.0 ASSUMPTIONS OF THE FRAME-WITH-STIFFENED-JOINT-MODEL

In this model the assumptions include,

I. Some lengths, a and b, at each end of a member are infinitely rigid while the remaining portion, L, is flexible and, therefore, has a finite rigidity.

II. Sway is permitted in the plane of the structure and joints rotate as rigid bodies without being deformed. Rotation of a stiffened joint as a rigid body is due to flexure of the flexible portion of adjoining beams and columns.

III. The masses are lumped at the floor levels.

IV. The effect of vertical inertia is negligible.

V. The structure is idealized into a conjugate system with horizontal translational restrictions. The dynamic structure stiffness coefficients K_{ij} is obtained by imposing unit translations, in turn, at each floor and determining the resulting reactions at the point of restrictions.

3.0 STIFFNESS COEFFICIENTS FOR A BEAM ELEMENT WITH INFINITELY RIGID ENDS

Making adoption from Ezeokpube [9], the stiffness coefficients for a beam element with infinitely rigid ends are as follows:

3.1 Rotational Stiffness for Beam on Fixed Supports

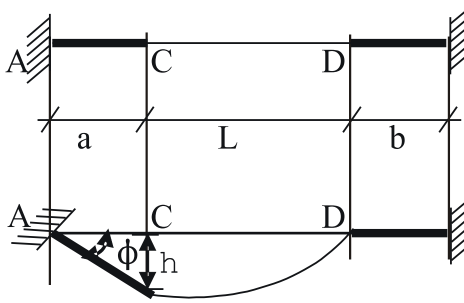


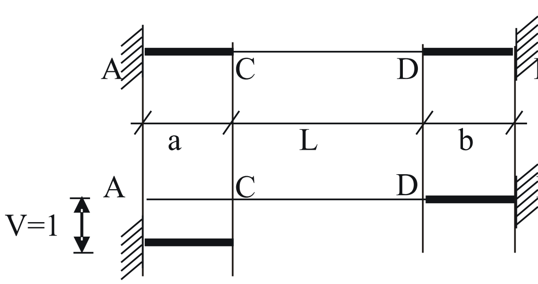
Fig. 1.
Application of unit rotation (φ=1) at joint A

Consider the beam AB with infinitely stiff portions AC and DB of lengths a and b respectively as shown in figure 1. The flexible portion CD has length L and constant flexural Rigidity EI. Then for a unit rotation (φ = 1) of Joint A and by considering the equilibrium of sections A-C and D-B we have that the modified stiffness coefficients, for the moments and shears at both ends of the beam, are

$M_A = (4EI/L)(1 + 3\alpha + 3\alpha^2)$
$M_B = (2EI/L)[1 + 3(\alpha + \beta) + 6\alpha\beta]$ …(1)
$Q_A = (6EI/L^2)\ (1 + 2\alpha)$
$Q_B = (-6EI/L^2)\ (1 + 2\alpha)$
where, $\alpha = a/L$ and $\beta = b/L$

3.2 Translational stiffness for Beam on Fixed Supports

Fig. 2. Application of unit translation



(V=1) at joint A

The joint A of the beam AB with infinitely rigid ends as shown in Fig. 2. is now subjected to a unit translation (V=1). By considering the equilibrium of sections A-C and D-B we have that the moments and shears, at both ends of the beams, are

$M_A = (6EI/L^2)(1 + 2\alpha)$
$M_B = (6EI/L^2)(1 + 2\beta)$ …(2)
$Q_A = 12EI/L^3$, $Q_B = -12EI/L^3$

4.0 FIXED-END REACTIONS DUE TO EXTERNAL LOADS FOR A BEAM WITH INFINITELY RIGID ENDS ON FIXED SUPPORTS

Making adoption from Ezeokpube [9] the fixed-end reactions for a beam element with infinitely rigid ends due to external loads are as follows:

4.1 Beam AB subjected to the action of a Uniformly Distributed Load (UDL) q

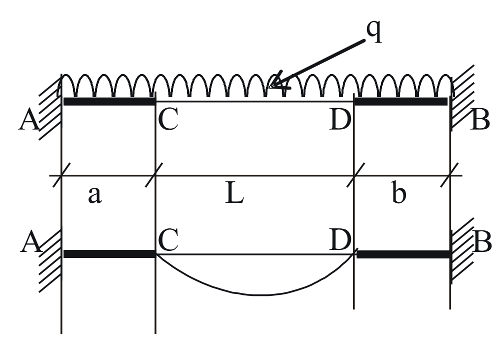


Fig. 3.
Application of uniformly distributed load

Figure 3. shows the beam AB subjected to the action of a uniform distributed load. The fixed-end moments and shears are

$M_A = (-ql^2/12)\ (1 + 6\alpha + 6\alpha^2)$
$Q_A = -ql(1 + 2\alpha)/2$
$M_B = (ql^2/12)\ (1 + 6\beta + 6\beta^2)$ …(3)
$Q_B = -ql(1 + 2\beta)/2$

4.2 Beam AB subjected to the action of a Concentrated Load P

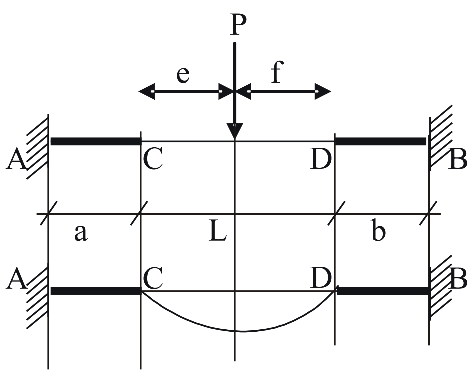


Fig. 4. Application of concentrated load

Considering the beam AB in Fig 4 in which the concentrated load P is acting as shown the fixed-end moments and shears are

$M_A = -Pf^2\{e(1 + 2\alpha) + \alpha L\}/L^2$
$Q_A = -Pf^2(L+2e)/L^3$
$M_B = Pe^2\{f(1 + 2\beta) + \beta L\}/L^2$ …(4)
$Q_B = -Pe^2(L + 2f)/L^3$

5.0 FREE VIBRATION

At any instant of time in the course of a free undamped vibration of an MDOF frame the equation of motion is obtained by adding the force of inertia due to the masses in motion and the restoring forces due to the stiffness of members. Thus,

$$m_i \frac{d^2 x_j}{dt^2}(t) + K_{ij} x_i(t) = 0 \qquad \ldots(5)$$

where, $x_i(t)$ = displacement function

m_i = mass at the i^{th} floor

K_{ij} = the reaction at the i^{th} floor obtained from the bending moment diagram due to the application of unit displacement at the j^{th} floor of the conjugate frame.

$\frac{d^2 x_j}{dt^2}(t)$ = acceleration of the mass m_i

It is assumed that the motion of the frame is simple harmonic and so the displacement function is further defined by

$x_i(t) = X_i \sin \omega t$ …(6)

where X_i = amplitude of displacement of the mass m_i

ω = natural frequency

Performing the differentiation in Equation (5) we have

$$m_i \frac{d^2}{dt^2}(X_i \sin \omega t) + K_{ij} x_i(t) = 0$$

$-m_i \omega^2 x_i(t) + K_{ij} x_i(t) = 0$ …(7)

Using the amplitude, Equation (7) becomes

$K_{ij} X_i - m_i \omega^2 X_i = 0$

or $[K_{ij} - m_i \omega^2][X_i] = 0$ …(8)

Cramer's rule which is used to solve Equation (8) requires, for a non-trival solution, that the determinant of the coefficients of X equals zero i.e.

$|K_{ij} - m_i \omega^2| = 0$ …(9)

Thus, Equation (9) is an eigenvalue problem whose solution yields the natural frequencies $\omega_1, \omega_2, \ldots, \omega_n$.

where, $\omega_1 < \omega_2 < \ldots < \omega_n$ …(10)

6.0 FORCED VIBRATION

The equation of motion for forced vibration is also time dependent and is obtained by adding the forcing function to Equation (7) and replacing the natural frequency with the forcing frequency. Thus,

$m_i \theta^2 x_i + K_{ij} x_i(t) + R_{ip}(t) = 0$ …(11)

Using the amplitudes, equation (11) becomes

$[K_{ij} - m_i \theta^2][X_i] + [R_{ip}] = 0$ …(12)

where, X_i = Amplitude of joint displacement due to forced vibration

θ = forcing frequency

R_{ip} = the reaction at the i^{th} floor obtained from the bending moment diagram due to the application of the external load to the conjugate frame.

After obtaining the amplitude of joints displacement from the solution of equation (12) bending moment, shear force, and axial force due to forced vibration are then determined using the following relations

$$\text{Bending moment, } M = \sum_{i=1}^{n} M_i X_i + M_p \quad (13)$$

$$\text{Shear force, } Q = \sum_{i=1}^{n} Q_i X_i + Q_p \quad (14)$$

$$\text{Axial force, } N = \sum_{i=1}^{n} N_i X_i + N_p \quad (15)$$

7.0 CASE STUDY

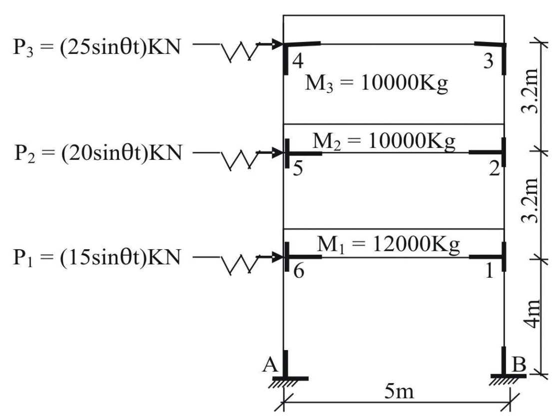
The three story building frame with stiffened joints loaded as shown in the figure 5 has three degrees of freedom with masses m_1, m_2, and m_3 lumped at the 1st, 2nd, and 3rd floors respectively. The following stiffening factors are applied in turn to the frame

case 1: α = β = 0.00,
case 2: α = β = 0.05,
case 3: α = β = 0.10
case 4: α = β = 0.15,
case 5: α = β = 0.20

Using the given stiffening factors and given that the flexural rigidity EI = 3 × 10^4KNm^2, forcing frequency, θ = 2.5 √EI × 10^{-3} rad/sec., and acceleration due to gravity g = 9.81ms^{-2}

i. Determine the natural frequencies ω_1, ω_2, and ω_3.
ii. Determine the amplitudes of joint displacement
iii. Determine the bending moment and shear force due to forced vibration.
iv. Plot the graph of
a. joint displacement Vs stiffening factors
b. joint moment Vs stiffening factors
c. natural frequency Vs stiffening factors

Fig. 5. MDOF Frame with stiffened joints



8.0 RESULTS OF ANALYSIS

Table 1: Amplitude of Joint Displacements due to Forced Vibration

n	Stiffening factors for case n(X)	Joint displacement (mm)		
		1st floor (Y_1)	2nd floor (Y_2)	3rd floor (Y_3)
1	α = β = 0.00	16.3	29.1	37.5
2	α = β = 0.05	6.9	14.1	19.2
3	α = β = 0.10	4.6	8.1	10.4
4	α = β = 0.15	3.1	5.6	7.1
5	α = β = 0.20	2.3	4.1	5.2

Table 2: Dynamic Bending Moment at Joint 2

n	Stiffening factors for case n(X)	Bending Moment (KNm)		
		(M_{24})	(M_{2B})	(M_{21})
1	α = β = 0.00	144	-190	-334
2	α = β = 0.05	102	-139	-240
3	α = β = 0.10	86	-108	-195
4	α = β = 0.15	80	-100	-180
5	α = β = 0.20	76	-98	-172

Table 3: Natural Frequencies

n	Stiffening factors for case n(X)	Natural Frequencies($\sqrt{EI}x10^{-3}$ rads/sec)		
		(ω_1)	(ω_2)	(ω_3)
1	α = β = 0.00	2.9	9.2	17.7
2	α = β = 0.05	3.4	10.6	20.4
3	α = β = 0.10	3.8	11.9	23.4
4	α = β = 0.15	4.3	13.4	25.9
5	α = β = 0.20	4.8	14.9	28.9

FIG. 6 GRAPH OF JOINT DISPLACEMENT Vs STIFFENING FACTOR

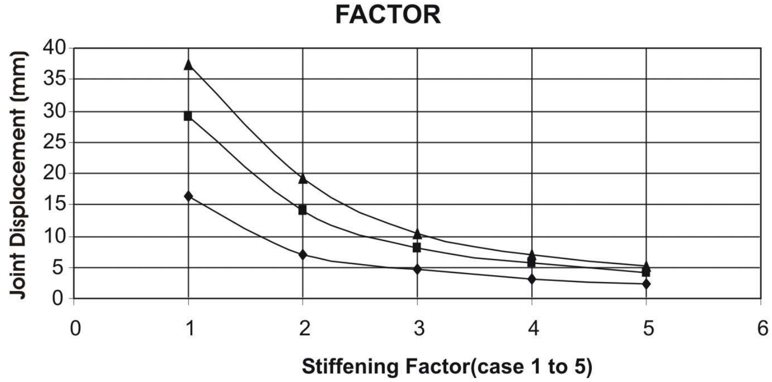


FIG. 7 GRAPH OF JOINT MOMENT Vs STIFFENING FACTOR

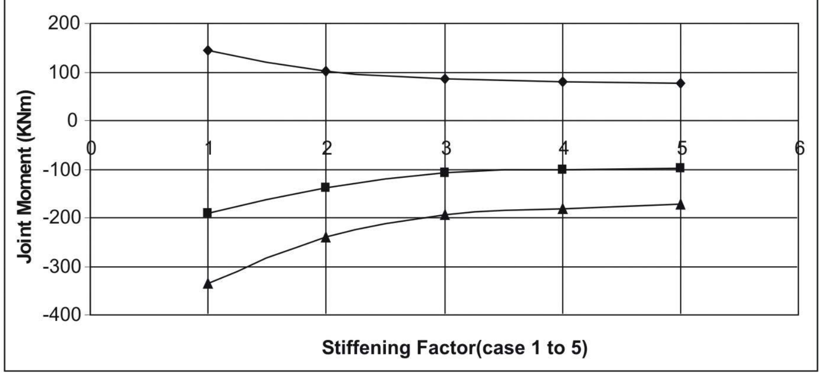
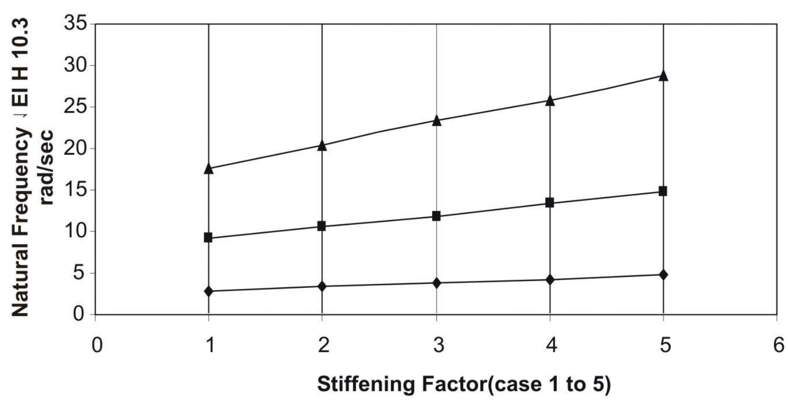


FIG. 8 NATURAL FREQUENCY Vs STIFFENING FACTOR



9.0 DISCUSSION OF RESULTS AND CONCLUSION

The graph of joint Displacement Vs Stiffening Factor (Fig 6), using Table 1, shows that Joint Displacement decreases with increase in Joint Stiffening. Using the results of Table 2 and the accompanying graph, Joint Moment Vs Stiffening Factor, in Fig 7, it can be inferred that joint stiffening allows substantial reduction in moments which leads to economic design of framed structures. The result in Table 3 and the graph of Natural Frequency Vs Stiffening Factor, in Fig 8 show that joint stiffening increases with natural frequency of framed structures.

In conclusion, joint stiffening substantially reduces dynamic joint displacements and moment values. This leads to greater stability and economic design of framed structures as a result of saving in material consumption. Additionally, joint stiffening increases the magnitude of natural frequencies. This can be used to advantage to keep the magnitude of any natural frequency away from that of the forcing system so as to avoid resonance occurrence.

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