EFFECTS OF FLEXURAL RIGIDITY OF REINFORCEMENT BARS ON THE FUNDAMENTAL NATURAL FREQUENCY OF REINFORCED CONCRETE SLABS

F. O. OKAFOR and O. OGUAGHAMBA
Department of Civil Engineering,
University of Nigeria, Nsukka.

ABSTRACT

An understanding of the orthotropic plates' behavior in their dynamic regime is essential because the loading can cause severe damage in the plates, such as: - cracking, loss of aesthetic, fear to the occupants, etc. To this end, a new set of stress - strain relations for. Orthotropic plates were derived. The principle of force of inertia was introduced, yielding the corresponding dynamic governing equation of orthotropic plate. The solution of the equation was obtained by numerical method, and the results show that the flexural rigidity of the bars has significant effect on the fundamental natural frequency of heavily reinforced concrete sections

KEYWORDS: *Fundamental Natural Frequency, Reinforced Concrete Slab, Flexural Rigidity, Reinforcement Bars*

INTRODUCTION

The competitive trends of the world market have long been forcing structural engineers to develop minimum weight and labour cost solutions [1-4]. A direct consequence of this new design trend is a considerable increase in problems related to unwanted floor vibrations. This phenomenon is very frequent in a wide range of structures subjected to rhythmic dynamical load actions. This development makes related work in this respect justifying. Man, equipment and facilities including various types of machines (Technological loads) are the source of impact and dynamic loads. Wind and earthquake loads are not to be mentioned here, but it should be noted that even weak wind pressures could be the excitor to facilities and indirectly make unpleasant noises and vibrations [5-6].

Euler [7] performed a free vibration analysis of plate problems, probably gave the first impetus to a mathematical statement of plate problems. Weaver et al [8] gave the formula for the frequencies of the various modes of vibration of plate isotropic. Malaikah et al [9] investigated the effect of the embedded steel bars in the concrete cylinders on the dynamic modulus of concrete. Their work revealed that the presence of the single bar made the specimen less susceptible to micro cracking. Much later, AI Wardany et al [10] used the Frequency-Wave Number (FK) method to demonstrate the effects of reinforcing bars in the spring back energy. Their result supports those obtained }n the numerical study of Wu et al [11]; which demonstrated that the existence of steel reinforcement bars in concrete causes a certain amount of the elastic energy generated to bounce back and forth between the concrete surface and the steel bars. This energy is primarily dependent on the steel bars diameter, the cover thickness and the spacing between the reinforcement bars. The effect of various parameters like the width-to-thickness ratio, the material anisotropy, the fibre orientation, the aspect ratio, the edge conditions and the number of layers on the fundamental frequency of vibration is studied by Latheswary et al [I2].

Cantieni [13] experimentally identified and measured the dynamic characteristics of a reinforced concrete structure, using Ambient Vibration Testing (A-VT). He excited the concrete slab by throwing a 5 kg medical ball from a height of roughly 1 m in irregular intervals of one to four seconds.

Vellasco et al [1] investigated the structural behaviour of Commonly used composite floors subjected to rhythmic dynamical load actions, identified the occurrence of unwanted vibrations that could cause human discomfort, or in extreme cases, structural failure. Mello et al [2] presented an analytical methodology for the evaluation of structural behaviour of composite floors against human comfort. This procedure takes into account a more realistic loading model developed to incorporate the dynamic effects induced by human walking.

EI-Dardiry et al [14] developed an isotropic and an orthotropic flat plate models for predicting simple and reasonably accurate dynamic behaviour of composite floors. Hsu [15] modelled numerically the vibration response of isotropic and orthotropic plates with mixed boundary conditions using a solution that is based on the differential quadrature method (DQM). The results demonstrated the efficiency of the numerical method in treating this class of engineering problem. Ventsel and Krauthammer [16] exploited the advantages Galerkin's numerical method to determine the frequencies of isotropic plates of different boundary conditions.

An indirect method, based on principle of orthogonality is used in this study for the free vibration analysis of orthotropic plates. This study has been motivated by the lack of open literature on dynamic analysis of reinforced concrete orthotropic plates, based on the constituent materials, using Galerkin’s method. Moreover, the effects of various constituent materials of the plates have rather been suggested by literature to have effect on the dynamic behaviour of the reinforced concrete slabs. With these facts in view, the present study is carried out.

FORMULATION OF DYNAMIC EQUATION FOR ORTHOTROPIC PLATES USING GALERKlN'S METHOD

It can be recalled that the assumption of isotropy implies that material properties at a point are the same in all directions [16, 17]. Orthotropic materials are materials that display orthogonal dependent properties. Thus, the governing differential equation for orthotropic plates is as given in Equation (l.0).

$$D_x \frac{\partial^4 w}{\partial x^4} + 2H \frac{\partial^4 w}{\partial x^2 \partial y^2} + D_y \frac{\partial^4 w}{\partial y^4} = p(x\,,y) \tag{1.0}$$

The expressions for flexural rigidities of such an orthotropic plate (two-way reinforced concrete slab) arc as follows [16, 17]

$$D_x = \frac{E_C}{(1-\mu_C^2)}\left[\frac{h^3}{12} - I_{xs} + \frac{E_s}{E_c}I_{xs}\right];$$
$$D_y = \frac{E_C}{(1-\mu_C^2)}\left[\frac{h^3}{12} - I_{ys} + \frac{E_s}{E_c}I_{ys}\right] \quad (2.0)$$
$$D_s = \frac{1-\mu_c}{2}\sqrt{D_x D_y}; H = \sqrt{D_x D_y};$$
$$D_{xy} = \mu_c\sqrt{D_x D_y} \quad (3.0)$$

Where

E_c and μ_c are modulus of elasticity and Poisson's ratio for concrete, respectively.

E_s is the modulus of elasticity for steel; and I_{xs}, I_{ys} are the moments of inertia of steel bars about the *x* and *y* axes, respectively.

Let a differential equation of a given 2D boundary value problem be of the form:-

$$L[w(x,y)] = P(x,y) in\ some\ 2D\ domain \Omega \quad (4.0)$$

Where, $w = w(x, y)$ is an unknown, P = a given load term defined also in the domain Ω, L = a symbol indicating either a linear or non - linear differential operator.

The function *w* must satisfy the prescribed boundary conditions on the boundary $\Gamma$of that domain.

However, requiring that the magnitude of the function [L (w) - P] be minimum, leads to the following Galerkin equation:

$$\iint_A^0 [L(W_N) - P] f_i(x,y) dxdy = 0 \ldots (5.0)$$

In the dynamic regime, the forcing function appearing on the right - hand side of the governing differential equation becomes [18-20]:

$$P = P(x,y,t) - ph\frac{\partial^2 w}{\partial t}(x,y,t) \ldots (7.0)$$

Thus, the differential Equation 1.0 becomes,

$$D^{**}\nabla^2\nabla^2 w(x,y,t) = P(x,y,t)$$
$$-ph\frac{\partial^2 w}{\partial t^2}(x,y,t) \ldots \quad (8.0)$$

Where, D^{**} = Flexural rigidity of Orthotropic Plates (Equation 1.0)

Equation (8.0) is the differential equation of forced, undamped motion of plate. Thus, for natural or free vibrations, P(x, y, t) is set equal to zero, and equation (9.0) becomes:

$$D^{**}\nabla^2\nabla^2 w(x,y,t) + Ph\frac{\partial^2 w}{\partial t^2}(x,y,t) = 0 \quad (9.0)$$

Note: Deflection w must satisfy the boundary conditions at the plate edge (these conditions practically do not differ from those in the case of static equilibrium) and the following initial conditions:

$$w = w_0(x,y,); \frac{\partial w}{\partial t} = \vartheta_0(x,y); at\ t = 0 \quad (10.0)$$

Where, W_o = initial deflection for point (x, y), ϑ_0 = Initial velocity for point *(x. y)*

A complete solution of the problem of a freely vibrating plate is reduced to determining the deflections at any point at any moment of time. To solve Equation (9.0) and obtain w(x, y, t) in general, one can assume the following solution:

$$w(x,y,t) = (A\cos\omega t + B\sin\omega T)W(X,Y) \quad (11.0)$$

Which is a separable solution of the shape function *W(x, y)* describing the modes of the vibration and some harmonic function of a time; co is the natural frequency of the plate vibration which is related to vibration period *T* by the relation,

$$\omega = \frac{2\pi}{T} \ldots \quad (12.0)$$

Introducing Equation (11.0) into Equation (9.0)

$$\Rightarrow D^{**}\nabla^2\nabla^2 W(x,y) - ph\omega^2[W(x,y)] = 0 \quad (13.0)$$

Assume that a shape function for the plate is approximated by series (14.0), which satisfies, term by term, all boundary conditions

$W(x,y) = \sum_{i=1}^{n} C_i W_i\,(x,y)\ldots.$ (14.0)

Substituting equation (14.0) into equation (13.0) and then, using the general procedure for the Galerkin's method given in equation (5.0), the expression given by equation (15.0) is obtained.

$\iint_A^S [D^{**} \sum_{i=1}^{n} C_i \nabla^2\nabla^2 W_i -$
$ph\omega^2 \sum_{i=1}^{n} C_i W_i(x,y)] W_k dxdy = 0;$
$K = 1,2,3, \ldots n.\ \ldots$ (15.0)

The numerical implementation of the above conditions leads to the Galerkin system of linear algebraic homogenous equations for orthotropic plates of the form.

$a_{11}C_1 + a_{12}C_2 + \ \ldots\ldots\ldots + a_{1n}\, C_n = 0$

$\vdots\ \vdots\ \vdots\ \vdots$

$a_{n1}C_1\ + a_{n2}\, C_2 + \ \cdots\ \cdots\cdots\ \cdots + \ a_{nn}\, C_n\ =$ 0… (16.0)

Where,

$a_{ik} = a_{ki} = \iint_A^S [D^{**}\nabla^2\nabla^2 W_i -$
$ph\omega^2 W_i] W_k dxdy\ldots$ (17.0)

This system of homogenous equations has non-trivial solution, if its determinant made up of the coefficients $\mathbf{a_{ik}}$ is equal to zero. The latter (Equation 17.0) results in the nth order characteristic Equation of the form

$\nabla(\omega) = 0\ldots$ (18.0)

Which yields therefore, the frequency or characteristic root, ω

ANAL YSIS AND RESULTS

Case **1: Clamped Rectangular Plate**

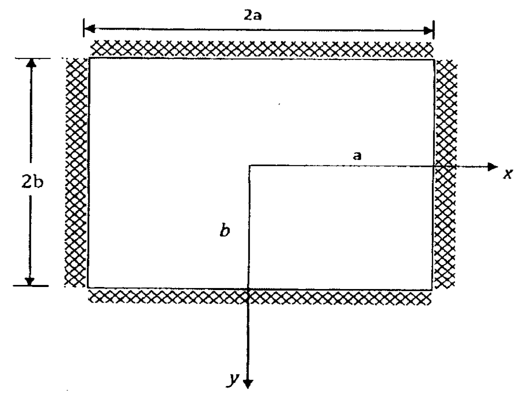

Figure 1: Clamped Rectangular Plate

In the clamped rectangular plate shown in figure 1,considering only the first term of the series, i.e.

$W(x,y) = C_{11}(X^2 - a^2)^2(y^2 - b^2)^2$ (19.0)

Note that, the amplitude C_{ik} cannot be determined from the linear eigenvalue problem.

Using equation (17.0),

$a_{11} = \iint_A^S \Big[\Big[D_x \frac{\partial^4}{\partial x^4}[C_{11}(x^2 - a^2)^2(y^2 -$
$b^2)^2]$
$+2H \frac{\partial^4}{\partial x^2 \partial y^2}[C_{11}(x^2 - a^2)^2(y^2 - b^2)^2]$
$+D_y \frac{\partial^4}{\partial x^4}[C_{11}(x^2 - a^2)^2(y^2 - b^2)^2]]$
$-ph\boldsymbol{\omega^2}[C_{11}(x^2 - a^2)^2(y^2 - b^2)^2]]$
$[C_{11}(x^2 - a^2)^2(y^2 - b^2)^2]dxdy = 0$

$\Rightarrow \omega = \frac{1}{a^2b^2}\sqrt{\frac{(31.5b^4D_x + 18a^2b^2H + 31.5a^4D_y)}{ph}}$

From the Isotropic solution of Ventsel et al [16] ,

$\omega = \frac{1}{a^2b^2}\sqrt{\frac{D(31.5b^4 + 18a^2b^2 + 31.5a^4)}{ph}}\ldots$ (21.0)

Case 2: Simply Supported Plate

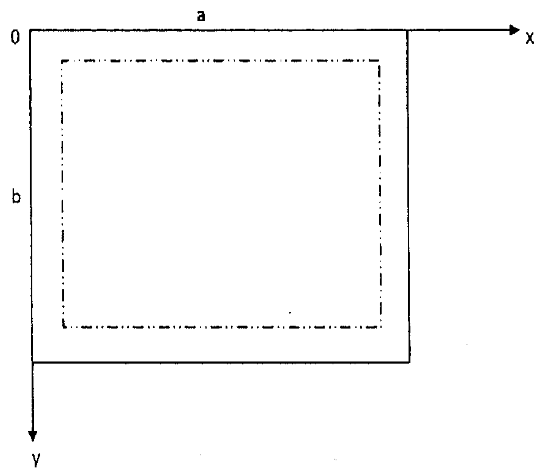

Figure 2: Simply Supported Plate (All round)

For the simply supported plate of figure 2, considering the shape function as in Equation (22.0)

$W(x,y) =$

$\sum_{i=1}^{\infty}\sum_{k=1}^{\infty} C_{ik} \sin\left(\frac{i\pi x}{a}\right) \sin\left(\frac{k\pi y}{b}\right) \ldots$ (22.0)

Retaining only the first term of the series of the shape function, i.e.

$W(x,y) = C_{ik} \sin\frac{\pi x}{a} \sin\frac{\pi y}{b}$

U sing Equation (17.0),

$a_{11} = \int_0^a \int_0^b \left[\!\left[D_x \frac{\partial^4 W(x,y)}{\partial x^4} + 2H \frac{\partial^4 W(x,y)}{\partial x^2 \partial y^2}\right.\right.$

$\left. + D_y \frac{\partial^4 W(x,y)}{\partial y^4} \right] - ph\omega^2 \sin\left(\frac{\pi x}{a}\right) \sin\left(\frac{\pi y}{b}\right) \Big]$

$\sin\left(\frac{\pi x}{a}\right) \sin\left(\frac{\pi y}{b}\right) dxdy = 0$

$\Rightarrow \left[\pi^4 \left[\frac{D_x}{a^4} + \frac{2H}{a^2 b^2} + \frac{D_y}{b^4}\right] - ph\omega^2\right] = 0,$

$\Rightarrow \omega = \pi^2 \sqrt{\frac{1}{ph}\left[\frac{D_x}{a^2} + \frac{2H}{a^2 b^2} + \frac{D_y}{b^4}\right]} \ldots$ (23.0)

From the Isotropic solution of Ventsel et al [16],

$\searrow\Rightarrow \omega = \pi^2 \left(\frac{1}{a^2} + \frac{1}{b^2}\right) \sqrt{\frac{D}{ph}} \ldots$ (24.0)

Case 3: Plate With Mixed Supports

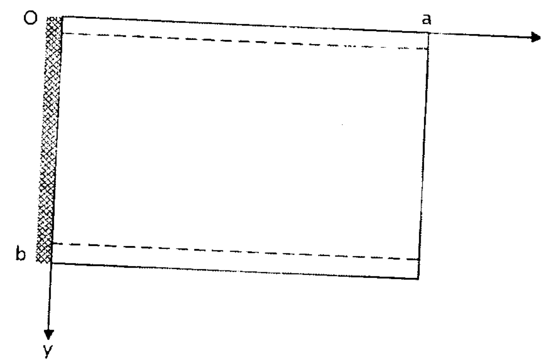

Figure 3: Mixed Support Condition of Plate s

For the plate with mixed support condition of figure 3, considering the shape functions as in equation (25.0),

$W(x,y) = \sum_{i=1}^{n}\sum_{k=1}^{n} C_{ik} \left(\frac{ix}{a}\right)^2 \sin\frac{K\pi y}{b} \quad \ldots$ (25.0)

Considering the first term of the series, then

$W(x,y) = C_{11} \left(\frac{x}{a}\right)^2 \sin\frac{\pi y}{b}$

From Equation (17.0)

$a_{11} = \iint_A^A \left[\!\left[D_x \frac{\partial^4}{\partial x^4} \left(\left(\frac{x}{a}\right)^2 \sin\frac{\pi y}{b}\right)\right.\right.$

$+2H \frac{\partial^4}{\partial x^2 \partial y^2} \left(\left(\frac{x}{a}\right)^2 \sin\frac{\pi y}{b}\right)$

$+D_y \frac{\partial^4}{\partial x^4} \left(\left(\frac{x}{a}\right)^2 \sin\frac{\pi y}{b}\right)$ -

$ph\omega^2 \left(\frac{x}{a}\right)^2 \sin\frac{\pi y}{b} \Big] \left(\frac{x}{a}\right)^2 \sin\frac{\pi y}{b} = 0$

$\therefore \omega = \frac{\pi}{b} \sqrt{\frac{1}{ph}\left(\frac{\pi^2 D_y}{b^2} - \frac{10H}{3a^2}\right)} \ldots$ (26.0)

From the Isotropic solution of Ventsel et al

$\omega = \frac{\pi}{b} \sqrt{\frac{D}{ph}\left(\frac{\pi^2}{b^2} - \frac{20}{3a^2}\right)} \ldots$(27.0)

NUMERICAL STUDY

(a) Singly Reinforced Sections:

In the preliminary design of a singly reinforced concrete slab of arbitrary support conditions, the following physical and geometric properties were adopted: f_{cu}= 32.5 MPa, μ_c = 0.20, 12 mm diameter mild steel spaced 150 mm both ways, Modulus of elasticity of steel, E_s = 205,000 N/mm^2Thickness of slab, h = 150 mm, Density of reinforced concrete, p = 2563 Kg $/m^3$= 0.000002563 Kg/mm^3 a = 5000 mm, for b/a = 0.5,0.6,0.7,0.8,0.9, 1.0, 1.1 and 1.2. The fundamental frequency of the slab was evaluated based on the above data.

(b) Doubly Reinforced Sections:

In the preliminary design of a doubly reinforced concrete slab of arbitrary support conditions, the following physical and geometric properties were considered: f_{cu}= 35MPa,μ_c = 0.20, 12 mm diameter mild steel spaced 150 nun both ways top and bottom. Modulus of elasticity of steel, E_s = *205,000* N/mm^2 Thickness of slab, h = 175 mm, Density of reinforced concrete, p = 2563 Kg/m^3 = 0.000002563 Kg/mm^3 a = 5000 mm, for b/a = 0.5, 0.6, 0.7, 0.8, 0.9, 1.0, 1.1 and 1.2; cover to all reinforcement, c = 25 mm.

The fundamental frequency of the slab was evaluated using the above data.

Solution

The results for the two cases were computed using the Q-Basic computer program, and are tabulated in Table 1 and Table 2 respectively.

From Nevile [21], $E_c = 4.73(f_c')^{0.5}$ (28.0)

Where, f_c' and E_c are the characteristic strength and modulus of elasticity of the concrete respectively.

From Spiegel et al [22] and Rajput [23],

$$I_{sx} = \frac{\pi d_{sy}^4}{S_{vy}64} \, and \, I_{sy} = \frac{\pi d_{sx}^4}{S_{xy}64}$$

$$I_{LM} = I_{XX}(or\, I_G) + Ah^2 \ldots \text{(29.0)}$$

Where, S_{vy} and S_{vx} are the spacing of the reinforcements perpendicular to *x* and *y* directions respectively d_{sy} and d_{sx} are diameters in those directions respectively. I_{LM} is the Moment of Inertia of an Area, for Lamina of infinite elemental components. I_{xx} = Moment of Inertia of an Area, for Lamina about an axis, xx, A = Area of section, h = centroid of a lamina about xx-axis. I_{sx} and I_{sy} are moment of inertia of steels about *x* and *y* directions respectively.

Tables 1 and 2 show the results of the fundamental frequencies obtained for the case of singly reinforced sections and doubly reinforced sections respectively.

Table 1: fundamental natural frequencies,ω, for single reinforced plate of varying conditions

	fixed supported all round ω, (rad/sec)			Simply supported ω, (rad/sec)			Mixed support condition ω, (rad/sec)		
b/a	[7]	Present study	% Difference	[7]	Present study	% Difference	[7]	Present study	% Difference
0.5	4.472041	4.472396	+ 0.0355	8.953696	8.954409	+ 0.0713	6.53021	6.53073	+ 0.0520
0.6	3.268715	3.268975	+ 0.0260	6.765015	6.765553	+ 0.0538	4.327417	4.327762	+ 0.0345
0.7	2.562288	2.562491	+ 0.0203	5.445309	5.445742	+ 0.0433	2.989200	2.989438	+ 0.0238
0.8	2.120047	2.120216	+ 0.0169	4.588759	4.589134	+ 0.0375	2.108193	2.108361	+ 0.0168
0.9	1.830054	1.830200	+ 0.0146	4.001529	4.001847	+ 0.0318	1.487757	1.487875	+ 0.0118
1.0	1.632958	1.633088	+ 0.0119	3.581476	3.581763	+ 0.0287	1.020133	1.020214	+ 0.0081
1.1	1.494991	1.495110	+ 0.0119	3.270689	3.270949	+ 0.0260	0.6325393	0.6325896	+ 0.00503
1.2	1.395919	1.396030	+ 0.0111	3.034308	3.034549	+ 0.0241	0.2055341	0.2055505	+ 0.00164
r	-0.92069	-0.92069		-0.93308	-0.93308		-0.94658	-0.94658	

Table 2: Fundamental Natural Frequencies, ω of Doubly Reinforced Plate of Varying Conditions

	Fixed supported all round ω, (rad/sec)			Simply supported ω, (rad/sec)			Mixed support condition ω, (rad/sec)		
b/a	[7]	Present study	% Difference	[7]	Present study	% Difference	[7]	Present study	% Difference
0.5	5.311507	5.324414	+ 1.2907	10.63443	10.67997	+4.5540	7.756025	7.740264	-1.5761
0.6	3.882301	3.896874	+ 1.4573	8.034906	8.080188	+4.5282	5.139736	5.122782	-1.6954
0.7	3.043266	3.059546	+ 1.6280	6.467473	6.512604	+4.5131	3.550316	3.532231	-1.8085
0.8	2.518011	2.535903	+ 1.7892	5.450148	5.495181	+4.5033	2.503932	2.484141	-1.9791
0.9	2.173582	2.192906	+ 1.9324	4.752673	4.797639	+4.4966	1.767031	1.744719	-2.2312
1.0	1.939488	1.960027	+2.0539	4.253774	4.298692	+4.4918	1.211627	1.185061	-2.6566
1.1	1.775622	1.797157	+2.1535	3.884645	3.929528	+4.4883	0.7512761	0.7153127	-3.59634
1.2	1.657954	1.680286	+2.2332	3.603892	3.648748	+4.4856	0.2441158	0.1232622	-12.0853
R	-0.92069	-0.92039		-0.93308	-0.93308		-0.94658	-0.94921	

DISCUSSION OF RESULTS

From the tables, it can be seen that there is an increase in the fundamental frequencies obtained when the flexural rigidity of the reinforcements is taking into account in all the cases. This is substantiated by higher correlation coefficients, r obtained by Product Moment Method between the span ratio, *b/a* and w computed when the steel effects are considered. Hence, indicates that the reinforcing bars in the reinforced concrete slabs have effect on the fundamental natural frequency of reinforced concrete slabs.

Interestingly, in Table 1, the simply supported RC slab gave highest value of ω of about 9.0 rad/s. The value decreases rapidly as the span ratio, b/a increases down the table. The effect of the steel on co obtained is also high in the simply supported case, ranging from 0.0241 % - 0.0713%. Slabs of mixed support condition gave ω of 6.53073 rad/s, but decreases in value spontaneously as b/a ratio increases down the table. The effect of the steel is about 0.00164 % - 0.0520 %. Fixed supported slab gave the lowest value of ω , 4.4724 rad/s, but as *b/a* increases down the table, ω decreases gradually. This trend reveals the usual suitability and stability of fixed supported RC slabs in large slab spans. The effect of steel is between the range of 0.0111 - 0.0355. Here, the slab is heavily reinforced, thereby yielding to higher value in the fundamental natural frequencies, w obtained when the effect of the steels t flexural rigidity are taking into account. The difference is very appreciable in the simply supported slab to about 4.5 %. In fixed slab, the difference increases to about 2.2332 % down the table, as the *b/a* ratio increase. While in the mixed support condition, the difference increases geometrically to about -12.1 %. The negative sign is due to the free support end unsupporting the steels. This at the same time reveals the gross danger the member would have been subjected to under dynamic regime, when its fundamental natural frequency is estimated neglecting the flexural rigidity of the reinforcements.

CONCLUSIONS

The following main conclusions are drawn from the study:

1. The flexural rigidity of the bars has significant effect on the fundamental natural frequency of heavily reinforced concrete sections.
2. The negligence of flexural rigidity of the reinforcing bars in the estimation of the fundamental natural frequency of slabs gives results that are inaccurate; and in some cases unsafe leading to unstable structure.
3. Inculcating the flexural rigidity of the reinforcing steels in estimation of the fundamental natural frequency of slabs will give the actual fundamental natural frequency of the slab in its self-excited dynamic regime, without approximation.
4. The assumption of uniform flexural rigidity of reinforced concrete slabs would appear adequate only in singly reinforced sections, for calculating the natural fundamental frequency of slabs.
5. In the case of heavily reinforced slabs, the present formulations would appear adequate, safe and economic for predicting the dynamic regime of reinforced concrete plates.

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