FORMULA FOR FORCED VIBRATION ANALYSIS OF STRUCTURES USING STATIC FACTORED RESPONSE AS EQUIVALENT DYNAMIC RESPONSE

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ABSTRACT

This Paper proposed and examined a formula for forced vibration analysis of structures using static factored response as equivalent dynamic response. Some methods of dynamic analysis are based on using static factored response as equivalent dynamic response thereby excluding the formulation of the equations of motion for forced vibration. These methods obtain dynamic response by the magnification of static response using the dynamic magnification factor or a modified form of the dynamic magnification factor. Dynamic response obtained by such methods give stress values which differ greatly from the actual. The flaw of these methods consist in magnifying stress values with factors obtained from displacement consideration on a false assumption of direct linear variation in the stress-displacement relationship. Based on the flexible frame model and stiffness formulation a formula for forced vibration analysis of structures using static factored response as equivalent dynamic response was developed with forced vibration equations and appropriate stress-displacement relationship. Though the use of this formula excludes the formulation of the equation of motion for forced vibration, the results obtained by its application to an MDOF Frame agree with that of the exact method using the flexible frame model. The formula can be used in practice for forced vibration analysis of structures or serve as control for the exact methods.

1.0 INTRODUCTION

One of the exact methods of dynamic analysis of structures involves the formulation of a set of equations governing the motions of the structures using the flexible frame model with either the flexibility or stiffness formulation. The solution of the system gives the Natural Frequencies, Displacements, Bending Moments, Shear Force and Axial Force.

However there exit some methods in which forced vibration analysis is based on using static factored response as equivalent dynamic response thereby avoiding the formulation of equations of motion for forced vibration used in the exact method. In one of the methods dynamic response is obtained by multiplying the static equivalents, directly, by the dynamic magnification factor [1,8]. In another method the actual value of the dynamic magnification factor or dynamic magnifier is then used to multiply the pseudo-static response (i.e deflections and stress) to obtain their dynamic equivalents [2].

The dynamic magnification factor could be obtained as the ratio of the amplitude of dynamic deflection to maximum static deflection [4,7]. Alternatively this magnification factor

could also be obtained as a function of frequency ratios [3,6] in the case of SDOF (Single degree of Freedom) systems or as a function of the maximum frequency ratio[5] in the case of MDOF (Many Degrees of Freedom) systems.

The idea of dynamic magnification in structures, using the methods enumerated above, does not yield the desired results in dynamic response other than translation. Consequently Dynamic Bending moment, shear Force, Axial Force and joint rotation obtained by using such methods do not agree with the results of the exact methods. The flaw of these methods consist in using the governing relation-ship between the dynamic and static deflections for the dynamic and static stresses as well on a false assumption of direct linear variation in the stress-displacement relationship. The generalization of this biased relationship leads to inaccuracy of results.

This paper therefore proposed a rational formula for forced vibration analysis of structures using static factored response as equivalent dynamic response. Based on the flexible frame model and stiffness formulation the formula was developed with forced vibration equations and appropriate stress-displacement relationship using some acceptable assumptions. The use of this formula precludes the formulation of the equation of motion for forced vibration analysis . The accuracy of this formula shall be verified by its application to the forced vibration analysis of an MDOF frame and comparing the results so obtained to that of the exact method of forced vibration analysis of the same frame.

2.0. ASSUMPTIONS FOR FORMULA DERIVATION

The following assumptions are made for the formula derivation. They include,

(i) The ideal structures used for the derivation is a flexible frame.

(ii) The motion of the frame is considered to be simple harmonic and forcing function is of steady-state.

(ii) For steady-state response the dynamic magnification factor for each floor is practically the same.

(iv) Equation of motion for forced vibration analysis and its associated stress-displacement relations are considered appropriate for the formula derivation.

(v) Equation of motion for forced vibration analysis could be used for static analysis by setting the forcing frequency equal to zero.

(vi) The Dynamic Response R_a is a function of the following
(a) The dynamic magnification factor
(b) The static response R_s of the ideal frame
(c) The static response R_c of the conjugate frame

(vii) A conjugate frame is an indeterminate structure with imaginary horizontal translational restrictions.

3.0 DERIVATION OF THE FORMULA

Using the Lumped – Mass procedure and the flexible frame model with stiffness formulation, the equation of motion for forced undamped vibration of MDOF frames can be represented in a condensed matrix form as,

$$[K] \, [X] + [R_p] = 0 \qquad (1)$$

where,

K = Dynamic structure stiffness matrix

X = Displacement vector for forced vibration

R_P = Load vector obtained from the bending moment diagram due to the application of the external load to the conjugate frame.

The solution of equation (1) yields the values of the unknown displacements (translations) X at each floor level such that

X_i = Translation at the ith floor level due to forced vibration

for i = 1, 2, 3, …, n.

where, n = the dynamic degrees of freedom. The stress – displacement relationship is given by

$$R_a = \sum_{i=1}^{n} R_i X_i + R_c \qquad (2)$$

where, R_i = Response due to the application of unit translation at the ith floor level of the conjugate frame.

R_c = Response due to the application of the external load to the conjugate frame

R_a = The actual dynamic response.

X_i remain as previously defined.

When the forcing frequency θ is zero the solution of equation (3) becomes equal to the static equivalent. Thus

$$X_i = \Delta_i \text{ (for } \theta = 0) \qquad (3)$$

where, θ = the forcing frequency

Δ_i = static translation at the ith floor level

Substituting equation (3) into equation (2) gives the static response R_s of the ideal frame i.e

$$R_s = \sum_{i=1}^{n} R_i \Delta_i + R_c \qquad (4)$$

from which we obtain

$$R_s - R_c = \sum_{i=1}^{n} R_i \Delta_i \qquad (5)$$

Also, $\qquad R_i X_i = \rho_i R_i \Delta_i \qquad (6)$

where, $\rho_i = X_i / \Delta_i$ = The Dynamic Magnification factor for the ith floor level (7)

For steady – state response the dynamic magnification factors of an MDOF frame are practically the same at every floor level for a given forcing frequency and so a common value may be used for all floors without any appreciable error. The Common value adopted here for the dynamic magnification factor is ρ. Therefore,

$\rho = \rho_i = X_i/\Delta_i$ = The Dynamic Magnification Factor (8)

Alternatively, the Dynamic Magnification Factor of a frame could be obtained as a function of the maximum frequency ratio. i.e. $\qquad \rho = 1/(1 - \beta^2) \qquad (9)$

where β = Maximum Frequency Ratio

$\quad = \theta/\omega_1$

$\quad \theta$ = Forcing Frequency

$\quad \omega_1$ = Minimum Natural Frequency

Substituting equation (8) into (6) gives

$$R_i X_i = \rho R_i \Delta_i \qquad (10)$$

Substituting equation (10) into (2) gives

$$R_a = \rho \sum_{i=1}^{n} R_i \Delta_i + R_c \qquad (11)$$

Substituting equation (5) into (11) gives

$$R_a = \rho R_s + (1 - \rho) R_c \qquad (12)$$

Equation (12) is the desired formula for forced vibration analysis of structures using static factored response as equivalent dynamic response.

4.0   USE OF THE FORMULA

The formula could be used to determine dynamic stresses (i.e Bending moments, shear force and Axial force) and Displacements (i.e Rotations and

Translations). However in using the formula the static Response R_s of the ideal frame, the static response R_c of the conjugate frame   and the Dynamic Magnification factor ρ are obtained as follows:

4.1 Static Response R_s of the Ideal Frame

This could be obtained with any of the methods of analysis like the stiffness, flexibility, slope-deflection, method etc. it does not matter whichever method was used.

4.2 Static Response R_c of the Conjugate Frame

A conjugate frame is an intermediate structure with imaginary translational horizontal restrictions. The determination of the static response R_c of the conjugate frame is best suited for the displacement method.

4.3 Dynamic magnification Factor ρ

For MDOF Frames the dynamic magnification factor ρ could be obtained as a function of the maximum frequency ratio [5]. Thus,

$$\rho = 1 \, (1 - \beta^2) \tag{13}$$

Where β = Maximum frequency Ratio
$$= \theta/\omega_1$$
   θ = Forcing frequency
   ω_1 = Minimum Natural Frequency.

5.0 NUMERICAL EXAMPLE

An MDOF frame subjected to dynamic loads, P_1 and P_2, as shown in figure 1 is used for the numerical application. The given structure has two lumped masses, M_1 and, M_2 at the first and second floors

respectively and therefore has two dynamic degrees of freedom using the lumped mass procedure. Figure 2 shows structure displacement of the ideal frame while figure 3 shows the structure displacement of the conjugate frame. Horizontal translational restrictions are provided for the conjugate frame as shown in figure 3. Neglecting shear and Axial deformations, it is required to determine

i.   Static response R_s of the ideal frame
ii.   Static response R_c of the conjugate frame
iii. Dynamic magnification factor ρ
iv.   Dynamic responses using equation (12)
v.   Dynamic responses using direct forced vibration analysis.

Given that, Flexural rigidity,
$$EI = 3 \times 10^4 \text{ kNm}^2$$
Forcing Frequency,
$$\theta = 0.5248 \text{ rad/sec}$$
Acceleration due to gravity,
$$g = 9.81 \times 10^{-3} \text{ Kms}^{-2}$$
Responses to be considered are Bending Moment, Shear Force, Joint Rotation, and Floor Translation.
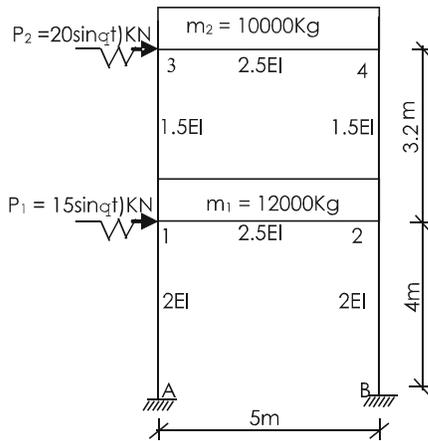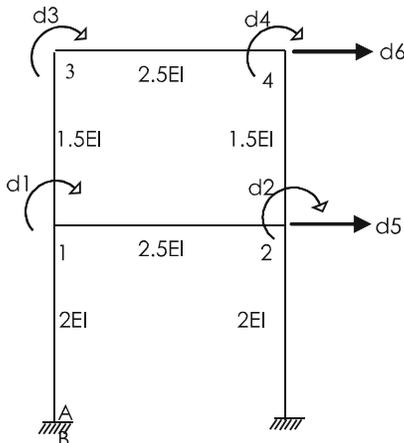
Fig.1 MDOF Frame
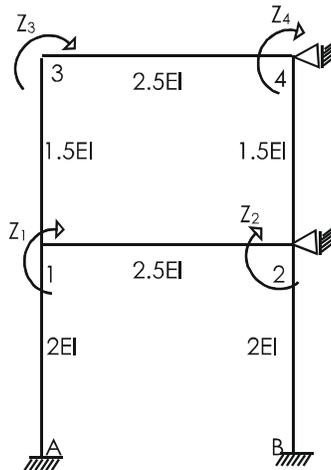

Fig.2 Structure displacement of the ideal frame

5.1

Fig.3: Structure displacement of the conjugate frame

onvention

Displacements and forces are taken positive in the direction indicated in figure 4 otherwise they are negative.
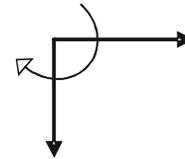

Fig. 4.0

5.2 Determination of the Dynamic Magnification Factor ρ

Using free vibration analysis the natural frequencies of the frame are

$\omega_1 = 0.7001$ rad/sec

$\omega_2 = 2.5056$ rad/sec

Given that the forcing frequency,

$\theta = 0.5248$ rad/sec

$\beta = \theta/\omega_1 = 0.5248/0.7001$

The Dynamic magnification Factor, ρ is thus given by

$\rho = 1/(1- \beta^2) = 2.28$

5.3 Determination of Static Response R_s of the Ideal frame and Static Response R_c of the conjugate frame

The static response R_s of the ideal frame and static response R_c of the conjugate frame are determined using stiffness method and the results are presented in tables 1-4.

5.4 Determination of Dynamic Response Using Equation (12)

Having obtained the dynamic magnification factor ρ, the static response R_s of the ideal frame and the static response R_c of the conjugate frame, equation (12) is now applied for the determination of dynamic response, the results of which are presented in tables 1-4

5.5 Determination of Dynamic Response Using Direct Forced Vibration Analysis
Using the lumped-mass procedure and stiffness method the direct forced vibration analysis is used to formulate the equations of motion for forced vibration whose solution yield the responses. The results are also presented in table 1-4.

6.0   RESULT OF ANALYSES

Table 1 Bending Moment (KNm)

Bending moment	Static analysis	Conjugate system	Forced vibration Analysis	
			formula Applied	Direct Analysis
M_{A1}	-33.99	7.82	-87.50	-87.35
M_{1A}	-12.56	15.64	-48.65	-48.45
M_{12}	-0.39	-41.23	51.88	52.02
M_{13}	12.95	25.60	-3.23	-3.61
M_{31}	9.85	29.21	-14.93	-15.28
M_{34}	-9.85	-29.21	14.93	15.28
M_{43}	48.56	29.21	73.34	73.70
M_{42}	-48.56	-29.21	-73.34	-73.70
M_{24}	-38.24	-25.60	-54.43	-54.80
M_{21}	82.07	41.23	134.34	134.48
M_{2B}	-43.83	-15.64	-79.92	-79.72
M_{B2}	-49.62	-7.82	-103.14	-102.98

Table 2 Joint Rotation (10^{-4} rad)

Joint	Static Analysis	Conjugate system	Forced vibration Analysis	
			Formula Application	Direct Analysis
Joint 1	7.14	2.16	12.95	12.97
Joint 2	1.93	-2.61	7.74	7.75
Joint 3	6.04	3.89	8.79	8.83
Joint 4	-1.74	-3.89	1.01	1.05

Table 3 Shear Forced (KN)

Shear Force	Static Analysis	Conjugate system	Forced vibration analysis	
			Formula Application	Direct Analysis
Q_{A1}	-11.64	5.86	-34.04	-33.95
Q_{1A}	11.64	-5.86	34.04	33.95
Q_{12}	42.52	58.86	21.62	21.56
Q_{13}	7.13	17.13	-5.67	-5.89
Q_{31}	-7.13	-17.13	5.67	5.89
Q_{34}	41.31	49.05	31.40	31.25
Q_{43}	56.79	49.05	66.70	66.85
Q_{42}	27.13	17.13	39.93	40.15
Q_{24}	-27.13	-17.13	-39.93	-40.15
Q_{21}	75.20	58.86	96.10	96.16
Q_{2B}	23.36	5.86	45.76	45.67
Q_{B2}	-23.36	-5.86	-45.76	-45.67

Table 4 Floor Translation (mm)

Floor Level	Static Analysis	Conjugate System	Forced vibration Analysis	
			Formula Application	Direct Analysis
1st Floor	2.46	0	5.61	5.61
2nd Floor	4.14	0	9.44	9.46

7.0   DISCUSSION OF RESULTS

Table 1, 2, 3, and 4 show the tabulated results of bending moment, joint rotation, shear force and floor translation respectively using the proposed formula and the direct forced vibration analysis. The proposed formula depends on the dynamic magnification factor and the results of both the static analysis of the ideal frame and that of the conjugate frame due to the application of the external load. The dynamic magnification factor was obtained as a function of the maximum frequency ratio. The direct analysis of forced vibration involves the formulation of the equation of motion for forced vibration whose solution yields the response.

The results obtained by the application of the proposed formula show reasonable agreement with that of the direct forced vibration analysis using the flexible frame model. This confirms the validity of the proposed formula and the assumptions made for its derivation. Thus the formula shows an exact relationship between the dynamic response and static response of frames and can therefore be used for the determination of both dynamic stresses and displacements.

8.0 CONCLUSION

The formula is recommended for force vibration analysis of structures in practice. Additionally it could be used as control for the exact methods. Except for the case of translation the formula, in fact, depicts a non-linear variation This will help to change the erroneous impression of a direct linear variation in the stress-displacement relationship, which leads to the wrong application of the dynamic magnification factor in the dynamic analysis of structures using static factored response as equivalent dynamic response. The proposed formula was applied to frames in this work but the principles involved could be extended to other structures. It should be noted that the application of this formula precludes the formulation of the equations of motion for forced vibration.

REFERENCES

1.   Biggs, J.M   Structure Dynamic, McGraw Hill, Inc; USA, 1964, p.38
2.   Bolton, A., Structural Dynamics in Practice, McGraw Hill, England, 1994, pp. 207-208
3.   Clough, R., and Penzien, J., Dynamic structures, McGraw-Hill, 1975, pp.53-54.
4.   Coates, R. C., Coutie, M.G., Kong F.K, Structural Analysis 2nd Ed; ELBS, 1980, p.343.
5.   Ezeokpube, G.C; Dynamic Response of Frames with Stiffened Joints, M. Eng (Struct. Engrg.) project, UNN, 2002, p.241.
6.   Gould, P.L., and Abu-sitta, S.H., Dynamic Response of Structures to Wind and Earthquake Loading, Pentech Press, London, 1980, pp. 6-11.
7.   Nelson, H.M., Bhatt, P., Marshall and Nelson's structures, Longman group Ltd., UK. 1990, p.460.
8.   Smith, J. W., Vibration of Structures, Chapman and Hall, USA, 1988, p. 264.