APPRAISAL AND RELIABILITY OF VARIABLE ENGAGEMENT MODEL PREDICTION FOR FIBRE REINFORCED CONCRETE

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ABSTRACT

The variable engagement model based on the stress - crack opening displacement relationship and, which describes the behaviour of randomly oriented steel fibres composite subjected to uniaxial tension has been evaluated so as to determine the safety indices associated when the fibres are subjected to pullout and with or without bending during fracture. Results indicate that the model as used is very safe for the prediction of fibre reinforced concrete and with associated small or negligible failure probabilities, since safety indices ranged between 25 and 160 for fibres fracture without bending, while it ranges between 2 and 4 for fibres fracture with pullout and bending. Also, the associated failure probabilities of fibres with bending are from 0.5×10^{-4} to 0.14×10^{-2}.

Keywords: Fibre reinforced concrete, model, reliability.

1.0 INTRODUCTION

Incorporation of steel or other fibres in concrete has been found to improve several of its properties, primarily cracking resistance, impact and wear resistance and ductility. For this reason fibre reinforced concrete (FRC) is currently being used in increasing amounts in structures such as airport and highway pavements, bridge decks, machine foundations and storage tanks. Early studies by Romualdi and Batson [1] indicated that the tensile strength of concrete can be improved by providing suitably arranged and closely spaced wire reinforcement.

The low tensile strength of concrete matrix is primarily due to the propagation of internal cracks and flaws. If these flaws can be locally restrained from extending into the adjacent matrix, the initiation of tension cracking can be retarded and a higher tensile strength of the material achieved, the inclusion of fibres may also enhance a number of other material properties such as fatigue resistance [2], energy absorption and toughness, ductility, durability and improve the service life of the material [3].

The addition of fibres to a concrete mix is objectively to bridge discrete cracks providing for some control to the fracture process and increases the fracture energy. The ideal of introducing or using discrete, ductile, fibre to reinforce brittle materials such as concrete is not new. Some of the major studies in the field include those of Gray [4, 5], Gopalaratnam and Shah [6], Mandel et al. [7], Naaman et al [8], Nanmur and Naaman [9], Wang [10], Wang et al [11, 12], Li [13] and many others [for example, 14, 15, 16], in the past.

Despite numerous publications on fibre concrete behaviour, limited research has been undertaken on developing general design models for fibre reinforced composites in tension. Visalvanich and Naaman [17] derived a semi-empirical model for the tension–softening curve in discontinuous randomly distributed steel fibre-reinforced mortar by assuming a purely frictional effect called the snubbing effect. Li [13] derived an analytical model named the fibres pullout model (FPM) that predicts the complete bridging stress-Crack Opening Displacement (COD) relationship for fibre reinforced brittle-matrix composite. One limitation of this model is that it does not account for the potential fracture effect of fibre in the composite [18].

A micromechanical model known as the fibres pullout and rupture model (FPRM) was developed by Maalej et al. [19]. In their model, the FPM model of Li [13] was extended to account for the possibility of fibre rupture in the composite. The model is able to predict the composite bridging stress-COD relationship, account for fibre pullout, fibre rupture and the local frictional effect or snubbing. A limitation of this model is that it does not account for interaction between neighboring fibres and the modification of the matrix spalling at the exit point of fibres inclined to the cracking plane.

Marti et al. [20] developed a simple parabolic model to describe the stress-COD relationship of randomly orientated fibre reinforced composites with the tension stress of the fibre composites. It was assumed in their model that after cracking of the matrix there is zero contribution of tensile stress stringent from the matrix and that the shear stress is constant along the shorter embedded length. An experimental investigation justifying the position of this model was carried out by Abejide [21].

Voo and Foster [22] developed a Variable Engagement Model (VEM). The VEM considers the slip between the fibres and the matrix that occurs before the full bond stress is developed and includes the condition where the fibres can fracture before being pulled out across a crack. The essence of the model is to provide a means that mathematically predicts the behaviour under load of fibre reinforced concrete in tension.

Many other models have been proposed such as those of Romauldi and Batson [1], Aveston and Kelly [23], Naaman et al. [24], Pakotiprapha et al. [25], Gray [4], Brandt [26], Lim et al. [27], Easley and Faber [28] and Gilles [29]; but these models are generally limited in their use as tools for structural designers due either to limitations of the models or their complexities. The VEM as suggested by Voo and Foster [22] thus seems flexible, can be easily applied while it predicts adequately the deterministic behaviour of fibre reinforced concrete in tension over a three-dimensional space. A good correlation in the results of finite element analysis for reactive powder concrete members with prestressed reactive powder beam failing in shear was obtained by Voo and Foster [30] experimentally and theoretically.

However, in engineering design there is always a failure surface and of course a safety region and with associated costs beyond the deterministic criteria. How reliable is the mathematical model given by Voo and Foster [22, 30] in a probabilistic setting. This is because the safety of an engineering design must suffice not only in a deterministic environment but also in a probabilistic setting. This paper reviews current studies of models for fibre reinforced concrete in tension controlled structures in order to substantiate and justify the choice of the VEM. Also, it evaluates the Variable

Engagement Model (VEM) with a probabilistic viewpoint. That is, it determines the reliability and confidence of the VEM developed by Voo and Foster [22] for engineering design purposes.

2.0 INFLUENCE FACTORS ON FIBRE-MATRIX BOND

Many experimental and analytical investigations on bond between fibres and a concrete matrix have been undertaken but the results are sometimes contradictory. Naaman and Shah [31] reported that the bond efficiency in a pullout test of steel fibres inclined with respect to the line of stress is at least as good as that of fibres parallel to the direction of stress. They also found that efficiency of the bond is inversely proportional to the number of fibres being pulled out across a plane. In tests on fibre-matrix specimens, Gray and Johnston [32] reported that the direction of casting has a substantial influence on the bond strength between the fibres and the mortar-matrix. They stated that vertically cast specimens have interfacial bond strengths higher than that of horizontally cast specimens. They also reported that an increase in the sand-cement ratio in the mortar matrix leads to a decrease in interfacial bond for the vertically cast specimens and an increase in bond for the horizontally cast specimens.

Maage (1978) found that the bond properties between steel fibres and cement-based matrixes are mechanical in nature and the anchorage of the fibres in the matrix is more important than the adhesion. He also noted that the number of fibres crossing the failure surface does not affect the mean per fibre pullout load. Maage [33] stated that Based on the weakest link theory, it should be reasonable that the pullout per fibre would decrease when the number of fibres across an area is increased. This is in contradiction with the results reported by Naaman and Shah [31] and an indication of the variability often encountered in fibre bond tests.

Pinchin and Tabor [34] carried out tests on wire fibres and showed that compaction of the concrete surrounding a fibre gives an increase in the mechanical bond and, thus, the pullout load. It was determined that the pullout load increases linearly with confinement and that it is proportional to the fibre-matrix misfit, which they defined as the difference between the radius of the wire and that of the hole in the matrix when subject to shrinkage.

Gopalaratnam and Abu-Mathkour [35] studied the effect of the fibre embedment length, fibre diameter and matrix quality on fibre pullout characteristics. From their experiments, Gopalaratnam and Abu-Mathkour [35] observed that the average bond strength is inversely related to the embedment length and that the average bond strength of the fibre-matrix interface increases with an increase in fibre diameter. Gopalaratnam and Abu-Mathkour [35] also reported that the strength of the concrete does not significantly influence the fibre pullout load. Their reasoning was that the frictional bond strength may be unrelated to the matrix compressive strength.

From the studies undertaken so far on fibre-matrix bond mechanics, it can be seen that a degree of contradiction exists in the literature. Further, the natural variability between different concrete mixes and the vast number of concrete-fibre combinations makes the formation of a model suitable for general design problematic. Nevertheless for design, engineers require a simple yet reliable approach that explains the mode of fracture and models with sufficient accuracy, the behaviour under load of fibre reinforced concrete in tension. A brief discussion of the variable engagement model is given for clarity in the next section since it is this approach that is probabilistically treated herein.

2.1 Development of the Variable Engagement Model

It is well established that for quasi-brittle materials, such as concrete, loaded in tension that localization dominates the behaviour beyond the peak load and that this behaviour can be described by the load versus crack opening displacement (w). For plain concrete the critical crack opening displacement, that is, the crack opening displacement (COD) for which the stress is zero, occurs at the point where the last of the mortar-aggregate matrix bridges the macro-crack. At this point, typical CODs are of the order of 0.4 - 0.5 mm as suggested by Peterson [36]. In the case of fibre reinforced concrete, assuming pullout of the fibre from the matrix and that the fibre pulls out from the side with the shortest embedment, the critical COD is half the length of the fibre. This is typically one to two orders of magnitude greater than that for plain concrete.

In addition, when a matrix crack is bridged by discontinuous, weakly bonded, fibres, further extension of the crack is inhibited as energy has to be supplied for fibre de-bonding, fibre pullout against interfacial traction and deformation of any fibres lying at oblique angles to the crack surface. In the development of the design model, the following assumptions are made: (i) behaviour of a fibre reinforced composite may be obtained by a summation of the individual component. That is, the effects of each individual fibre can be summed over the failure surface to yield the overall behaviour of the composite; (ii) the geometric centres of the fibres are uniformly distributed in space and all fibres have an equal probability of being oriented in any direction; (iii) all fibres pullout from the side of the crack with the shorter embedded length while the longer side of the fibre remains rigidly embedded in the matrix; (iv) displacements due to elastic strains in the fibres are small relative to displacements resulting from slip between the fibres and the matrix; and (v) the bending stiffness of a fibre is small and energy expended by bending of fibres may be neglected.

2.2.1 Determination of Fibres Engagement in Pullout

For mechanically anchored fibres, after the adhesion between the fibres and the matrix is broken some slip between the matrix and the fibres must occur before the anchorage is engaged. In addition, in the modified pulley theory (as noted by Voo and Foster [22] the COD is greater than the slip between the fibre and the matrix albeit the difference is small. The COD for which the fibre becomes effectively engaged in the tension carrying mechanism is termed the engagement length and denoted as w_e. Assuming the engagement-length versus fibre-slip relationship can be described using a continuous function then the boundary criteria dictates that for a fibre angle of $\theta = 0$, $w_e = 0$ and the function is to be asymptotic to $\theta = \pi/2$. One such a function is;

$$w_e = \alpha \tan \theta \qquad (1)$$

where w_e is the COD at the point of engagement of the fibre and, a, is a material parameter obtained from fibre pullout tests for varying θ. To avoid variations along the plateau of the load versus COD curves in the determination of w_e, the fibre is taken to be effectively engaged at the point corresponding to 50% of the peak load. The model resulting from the engagement equation (equation 1) is called the Variable Engagement Model (VEM). For the VEM, the force in a single fibre is

$$w \prec w_e \text{ and } w \geq l_a : \quad P_f = 0 \qquad (2a)$$

$$w_e \leq w \prec l_a : \quad P_f = \pi\, d_f\, \tau_b (l_a - w) \qquad (2b)$$

where d_f is the diameter of the fibre, l_a is the initial length of embedment of the fibre and τ_b is the mean shear stress between the fibre and the matrix measured along the remaining portion of embedded fibre (l_a -w). In the analyses that follow τ_b is taken as constant for a given fibre-matrix structure. However, while some adhesion exists for low CODs, significant slip is needed before mechanical locking occurs.

2.2.2 Fibres Engagement Angle

Using the concept of a fibre engagement length, discussed above, we can infer that for a randomly orientated fibre composite material, cracked in tension, at any point in the load-COD path there can be defined a critical angle for which fibres are becoming active. We term θ_{crit} as the point where fibres orientated at $\theta < \theta_{crit}$ carry load while fibres at $\theta \geq \theta_{crit}$ are yet to be engaged. Then from equation (1) we do obtain,

$$\theta_{crit} = \tan^{-1}\left(\frac{w}{\alpha}\right)$$

The model represented by equations (1) and (3) shows that, for a given COD, as α increases θ_{crit} decreases and, hence, α is a material measure of the resistance to slip between the fibre and the matrix. In equation (3) it is seen that θ_{crit} is a function of the current COD. Substituting the maximum possible fibre slip before engagement, w = l_f /2, into equation (3) gives the limiting angle;

$$\theta_{lim} = \tan^{-1}\left(\frac{l_f}{2\alpha}\right)$$

It is noted that not all fibres at $\theta < \theta_{lim}$ may be engaged (this depends on the initial embedded length, l_a), however, no fibres at $\theta \geq \theta_{lim}$ can ever be engaged.

2.2.3 Stress-COD Model Excluding Fibre Fracture

In fibres that are randomly orientated in three dimensions, Aveston and Kelly [23] have shown that the number of fibres crossing a plane of unit area is p_f /2 where ρ_f is the volumetric ratio of fibres. For fibres of length, l_f and diameter, d_f passing through a cracking plane with the fibre pulling out from the side with the smaller embedded length, Marti, et al. [20] noted that when w = 0 the average length of embedment is l_f / 4; and also, that the number of bonded fibres decreases linearly with increasing COD. Hence rewriting equation (2) in the form

$$P_f = k\,\pi\, d_f \tau_b \left(\frac{l_f}{2}\right)$$

gives

$$k = 0: \qquad \text{when } w \prec w_e \text{ and } w \geq l_a \quad (6a)$$

$$k = \left(\frac{2(l_a - w)}{l_f}\right) \qquad \text{when } w_e \leq w \prec l_a \quad ($$

(3)

where k is denoted as the local orientation factor.

Now, integrating equation (5) over a plane of unit area, we will obtain the tension stress in the fibres as

$$\sigma = k_f \, k_d \, \alpha_f \, \rho_f \, \tau_b \qquad (7a)$$

where $\alpha_f = l_f / d_f$ is the aspect ratio of the fibre, k_f is the global orientation factor and k_d is a damage factor or fibre efficiency factor and ρ_f is the volumetric fraction of fibres. The damage factor (k_d) in equation (7a) accounts for a loss of efficiency in bond of the fibres when the region around an individual fibre is affected by the pullout of adjacent fibres. As the relative volume of fibres increases the local damage in the region bounded by l_f /2 from the crack increases as the crack opens. Thus, as the volume of fibres increases the damage factor decreases. Also, for the case where fibre balling occurs the efficiency of the fibre is reduced. It can be reasonably inferred that the damage factor is a function of the quantity of fibres, fibre type, strength of the adjacent concrete-matrix and the COD. However, in conventional volumes of fibres as used in practice and where no fibre balling occurs, k_d may be taken as unity. Thus,

$$\sigma = K_f \, \alpha_f \, \rho_f \, \tau_b \qquad (7b)$$

Note that $K_f = k_f \, k_d$ and is also called the global orientation factor for the fibres since k_d is unity.

Various formulae and corresponding values for the determination of k_F have been proposed in literature. The orientation factor can be determined by probability and is affected by the shape of the domain over which the orientation is considered. Romualdi and Mandrel [37] gave its value as 0.405, Parimi, et al [38] as 0.637, Aveston and Kelly [23] as 0.50, Pakotiprapha [39] as 0.20 and Foster [40] as 0.375. Taking all fibres as effectively engaged upon cracking of the matrix, Marti et al.[20] showed that, in general $k_F = 0.5\,(1 - 2\,w/l_a)$ while at the point of initial cracking $k_F = 3/8$. In the model adopted by Foster [40] where only fibres at pullout were considered effective at the point of matrix cracking, using the fibre engagement model described by equations (1) and (2) it can be deduced that

$$K_f = \frac{1}{N}\sum_{i=1}^{N} k_i = N \xrightarrow{\lim} \infty \; \frac{1}{N}\left\{\sum_{0}^{\theta_{crit}} k(w)\right\}$$

where N is the number of fibres crossing a plane of unit area and is the local orientation factor for the i[th] fibre. If we consider a random distribution of fibres with equal probability that any given fibre crossing a crack has a shorter embedded length of between zero and l, the average value of the local orientation factor for all engaged fibres will be given as;

$$k_{ave} = \left(\frac{1}{2} - \frac{w}{l_f}\right)$$

Also, if all fibre orientations have equal probability and noting that the proportion of bonded fibres decreases linearly with increasing w, then from equation (8) we obtain,

$$K_f = \frac{2\theta_{crit} k_{ave}}{\pi}\left(1 - \frac{2w}{l_f}\right)$$

The term in parenthesis in equation (10) is the proportion of fibres that have not pulled out from the matrix for a given COD. Substituting equation (3) and (9) into (10) we obtain,

$$K_f = \frac{\tan^{-1}(w/\alpha)}{\pi}\left(1 - \frac{2w}{l_f}\right)^2$$

By equation (11) the orientation factor K_f is variable and can be seen to be $0 \leq k_f \leq 0.50$; all fibres are pulled out from the matrix and there is no fibre fracture. Thus equation (12) applies;

$$l_f \prec l_c = \left(\frac{d_f}{2}\right)\left(\frac{\sigma_{fu}}{\tau_b}\right) \qquad (12)$$

where l_c is the critical fibre length and σ_{fu} is the ultimate tensile strength of the fibre. If the inequality of equation (12) is violated then a portion of the fibres will fracture and equation (11) does not apply.

2.2.4 Stress-COD Model Including Fibre Fracture (excluding bending)

Let us assume a constant bond shear stress along the fibre length then, by force equilibrium, any arbitrarily orientated fibre will fracture if

$$l_a \geq \left(\frac{d_f}{4}\right)\left(\frac{\sigma_{fu}}{\tau_b}\right) + w_e \qquad (13)$$

Hence for a given COD the global orientation factor may be expressed as;

$$K_f = \left[\left(\frac{2}{\pi}\right)\frac{1}{\left(\left(\frac{l_f}{2}\right) - w\right)}\int_0^{\theta_{crit}}\int_w^{l_{a,crit}} k(l_a, \theta)\, dl_a\, d\theta\right] \cdot \left[1 - \frac{2w}{l_f}\right] \qquad (14)$$

where $l_{a,\mathrm{crit}}$ is the critical fibre embedment length for fracture and is given by

$$l_{a,\,\mathrm{crit}} = \min\left(\frac{l_c}{2} + w_e,\ \frac{l_f}{2}\right) \qquad (15)$$

Therefore, substituting equation (6) into equation (14) gives

$$K_f = \left(\frac{4}{\pi l_f^2}\right)\int_0^{\theta_{\mathrm{crit}}}\left[\max\left(l_{a,\mathrm{crit}} - w,\ 0\right)\right]^2 d\theta \qquad (16)$$

where θ_{crit} is given by equation (3). Equation (16) may be solved by numerical integration. For the case of $l_c < l_f$ no fibres fracture and equation (16) reduces to equation (11).

In equations (12) and (13) the ultimate tensile strength of the fibres σ_{fu} is evaluated excluding the effect of bending stresses on the performance of the fibres. Whilst the effect of bending is commonly ignored in the calculation of the critical length (equation 12), fibre fracture strains induced by bending of the fibres can reduce the axial capacity of the fibre, particularly for fibres having limited ductility such as glass and carbon fibres. The opinion followed in this study is that fibre fracture can occur by only pullout or in conjunction with bending of fibres. Therefore equation (11) is readily applied for only pullout, while equations (17) to (24) will apply for those in association with bending.

The fibre engagement parameter, α, was safely taken as in equation (17) after calibration and verification against a wide range of test data and attainment of theoretical experimental correlation. Thus,

$$\alpha = \frac{d_f}{3.5} \qquad (17)$$

2.2.5 Fibre Fracture due to Bending and Pullout of Fibres

In the formulation of FRC in tension, developed above, where fibre fracture is included it was assumed that bending of the fibres has only a small effect on the overall behaviour when using ductile fibres. However, where brittle fibres are used or where a large portion of fibres that cross the cracking plane fracture, the stresses induced through fibre bending can not be ignored. The data of Banthia and Trottier [41, 42, 43] for their twin cone fibres compared the average axial stress in the fibres, normalized for the fracture stress for fibres at θ = 0 degrees, versus fibre angle. In these tests the strength was limited by fibre fracture. It was obtained that as the fibre angle increases the average stress in the fibres, at fracture, decreases. Also Kanda and Li [44] obtained the same results for their PVA fibre tests where the fibres fractured. In this section a general model is developed that includes fibre bending effects on the influence of fibre fracture for the determination of the stress - COD curve for FRC. Two cases can be considered in the derivation of the limiting axial fracture strength ($\sigma_{\alpha u}$) for fibre fracture with bending. The first model is based on linear elastic-brittle behaviour and the second case being of material behaviour for a rigid-plastic material. Taking plane sections to remain plane, the axial strain on the fibre and the bending strains can be superimposed to give the total strain. A fibre will fracture when the extreme tensile fibre strain reaches its limiting fracture strain, ε_{fu}. The limiting axial strain of the fibre at any angle of inclination, ε_{au}, can therefore be written as

$$\varepsilon_{au} = \varepsilon_{fu} - \varepsilon_{bu} \tag{18}$$

where ε_{bu} is the bending strain at the point of fibre fracture for a fibre orientated at an angle θ to the crack lane. For a perfectly elastic-brittle fibre the limiting average axial fracture strength (σ_{au}) is

$$\sigma_{au} = \sigma_{fu} - \varepsilon_{bu} E_f \quad \text{for} \quad 0 \le \varepsilon_{bu} \prec \varepsilon_{fu} \tag{19a}$$

$$\sigma_{au} = 0 \quad \text{for} \quad \varepsilon_{bu} \ge \varepsilon_{fu} \tag{19b}$$

where σ_{fu} is the fracture stress of the fibre and E_f, the elastic modulus of the fibre. In the case of a rigid-plastic material, the limiting average axial fracture strength ($\sigma_{\alpha u}$) of a fibre of circular cross-section subject to bending as derived by Voo and Foster (2003) is given by

$$\sigma_{au} = \sigma_{fu} \quad \text{for} \quad 0 \le \varepsilon_{bu} \prec \varepsilon_{fu}/2 \tag{20a}$$

$$\sigma_{au} = \left(\frac{\sigma_{fu}}{\pi}\right)\left[2\bar{d}\sqrt{1-(\bar{d})^2} + 2\sin^{-1}(\bar{d})\right]$$

$$\text{for} \quad \varepsilon_{fu}/2 \le \varepsilon \prec \varepsilon_{fu} \tag{20b}$$

where $\bar{d} = \dfrac{2d_0}{d_f}$ and d_o is the distance from the plastic centroid of the section to the neutral axis and is given by

$$d_0 = \frac{d_f}{2}\left(\frac{\varepsilon_{fu}}{\varepsilon_{bu}} - 1\right)$$

Assuming the part of the fibre for which bending occurs has constant moment over an arc of constant radius of curvature (r), the relationship between the bending strain and the diameter of the fibre d_f can be written as

$$y'' = \kappa = \frac{2\varepsilon_{bu}}{d_f} \tag{22}$$

where k is the curvature ($\kappa = 1/r$).

The boundary conditions dictate that any function describing the radius of the curvature (r) must be asymptotic to the θ = 0 axis for $\sigma_{au} = \sigma_{fu}$. One function meeting this criteria is, r cot(θ). For the analyses that follow, let us adopt the following function.

$$r = \frac{\beta d_f \cot\theta}{2} \qquad \kappa = \frac{2\tan\theta}{\beta d_f} \quad \text{or}$$

where β is an empirical constant. Therefore, with equations (22) and (23) we obtain the following:

$$\varepsilon_{bu} = \frac{1}{\beta}\tan\theta \tag{24}$$

The strength interaction prediction as in equations (19) to (24) for elastic ductile fibres in bending and pullout has been verified by Kanda and Li [44] theoretically; and experimentally by Voo and Foster [22] when β was taken as 200. The experimental results correlate very well with the theoretical prediction of these fibres. Also, Li, et al [45] has proved that PVA fibre materials can be successfully used for practical construction situations and with excellent durability criteria.

3.0 RELIABILITY ANALYSIS

Reliability based design is founded on the concept that one can estimate the probability of an undesirable event such as a fracture, occurring over the lifetime of a structure, despite the uncertainties involved. It is a design method that provides an assumed level of safety by reducing the probability of such an occurrence below the target value. The quality of design is judged by comparing the failure probability with the target failure probability, and this use of the failure probability makes it possible to evaluate the safety of a structure quantitatively. Above all, the design method enables one to gain a firm grasp of the safety level desired and to design a structure so as to attain the prescribed reliability independently of other design requirements.

The First Order Reliability Method (FORM) is a convenient tool to assess the reliability of structural elements. It also provides a means for calculating the partial safety factors. FORM uses a combination of analytical and approximation methods and comprises three stages [46]. Firstly, independent of whether each parameter has been defined as a Normal or Weibull distribution all variables are first transformed into equivalent space with zero mean and unit variance [47]. The original limit state surface is then mapped unto the new limit state surface. Secondly the shortest distance between the original and the limit state surface, termed the reliability index, β, is evaluated. This is termed the design point, or point of maximum likelihood, and gives the most likely combination of basic variables to cause failure [48]. Finally, the failure probability associated with this point is then calculated. FORM can be easily extended to non-linear limit states and has reasonable balance between ease of use and accuracy [49].

First Order Reliability Method (FORM) and Monte-Carlo Simulation (MCS) are the most common basic techniques and are applicable to all probabilistic problems [50]. Of these FORM is usually a preferred method, as it does not depend on the number of simulations carried out [51]. For complex limit states, FORM may not converge and an answer may not be obtained. In this case MCS is used but a large number of simulations must be made when failure probability is low, thus requiring extended computing time [51].

4.0 RESULTS AND DISCUSSIONS

The stochastic models generated by the equations of fibres fracture with and without bending and their associated parameters in equations (1) to (24) are analyzed using the First Order Reliability Method (FORM) as suggested by Gollwitzer, et al [46], to give values of safety index (β) and probability of failure (P_f). The Crack Opening Displacement (COD) and the mean bond shear stress were varied for each of the engagement constants. The engagement constants used were 0.5, 1.0, 1.5, 2.0 and 2.5 convergence was achieved when β values ranging from 25 to 165 were obtained. It is noted from results (see figures 1 to 3) that the mean bond shear stress has no effect on the β value. (i.e., the β value remained the same when the values of the mean bond shear stress was varied; but while the COD was varied the β value changes.

Figures 1 and 2 give the results of the reliability analysis for the fibre fracture without bending. From the results it is observed that the mean bond shear stress, τ_b, when varied does not have effect on the safety index (β) value which are given from the theoretical analysis. It is also observed that the crack opening displacement (COD) increases with increase in the safety index (β). Figure 3 gives the result of the reliability analysis for the fibre fracture with bending. It was observed that the ultimate tensile strength of fibre of 5,000, 6,000, 8,000, 9,000 and 10,000(N/mm) gave positive values of safety indices (β), while the ultimate tensile strength of fibre at 7,000 N/mm had a probability of failure of 1.0, thereby giving negative values of safety index. It was also observed that the diameter of fibres when varied from 0.1 to 1.0 at an increment of 0.1 had no effect on the value of the safety index.

The safety indices obtained from the probabilistic analysis as shown in figure 3 show that they range between 2.5 to 4.2 with a corresponding probability of failure, P_f of 0.000014 to 0.005, which is in agreement with Ellingwood [48] suggestions.

5.0. CONCLUSION

Design engineers require a simple yet reliable approach that explains the mode of fracture and models with sufficient accuracy, and which predicts the behaviour under load of fibre reinforced concrete in tension. This presentation evaluates a model for fibre reinforced concrete in tension, which was developed by Voo and Foster [22]. A variable engagement model (VEM) based on the stress – crack opening displacement relationship and which describes the behaviour of randomly oriented steel fibre reinforced composite subject to uniaxial tension was develop by integrating the behaviour of single, randomly oriented, fibres over three dimensional space. The model is capable of describing the peak and post-peak response of fibre-cement-based composite in tension. The VEM was analyzed to check its reliability in a probabilistic setting because in engineering design there is always a failure surface and safety region.

The value of safety indices obtained from the probabilistic analysis for fibre fracture without bending as shown in figures 1 and 2 shows that it ranges between 25 and 165 with a corresponding probability of failure P_f of 0.000 to 0.351 - 240. These values are quite in excess of Ellingwood [48] suggestions. Ellingwood [48] had suggested that safety index,β, values for structural members can safely be between 2 and 4 inclusively. It is thus clear from the results obtained that the following conclusion can be safely made.

(1) The VEM model as proposed is suitable for predicting tensile strength of steel fibre in fibre reinforced concrete with ductile fibres.

(2) The model can be applied to assess the fatigue performance of composite material structural elements, such as fibre reinforced concrete beams and slabs in tension controlled structures.

(3) The fibres were effective in retarding the propagation of cracks in concrete thereby providing a better control energy absorption than ordinary concrete.

(4) Another potential application of the variable engagement model is to serve in the design and optimization of fibre

reinforced concrete materials with superior fatigue properties.

(5) The values of safety indices of between 2 and 4 as obtained for fibres when bending is considered shows a sharp contrast when only pullout of fibres is considered. It is the consideration of bending that should govern the safety of the VEM as sometimes bending may occur in the fibre reinforced concrete section. Thus, although the fibre reinforced concrete section is safe, it is definitely not economical. In practical cases the worst exposure to load of a structure governs its design. Thus it can be safely concluded that fibre reinforced concrete structures are safer when no bending is associated with the ductile fibres.

REFERENCES

1. Romualdi, J.P., and Batson, G.B. (1963). *Behaviour of Reinforced Concrete Beams with Closely Spaced Reinforcement*. Journal of the American Concrete Institute, Proceedings Vol. 60, No. 6, June, pp: 775-789.
2. Romualdi, J.P., Ramey, M and Sanday, S. C. (1968). *Prevention and Control of Cracking by Use of Cracking in Concrete*. Detroit, ACI, Publication SP-20. pp: 179 – 203.
3. Shah, S. P and Rangan, B. V. (1971). Fibre Reinforced Concrete Properties. Journal of the American Concrete Institute, Proceedings Vol. 68, No. 2, February. pp: 126 - 135.
4. Gray, R. J. (1984a). "Analysis of Effect of Embedded Fibre Length on Fibre Debonding and Pullout from an Elastic Matrix; Part 1, Review of Theories". Journal of Materials Science, Vol. 19, No. 3. pp: 861-870.
5. Gray, R. J. (1984b). *Analysis of Effect of Embedded Fibre Length on Fibre Debonding and Pullout from an Elastic Matrix; Part 2, Application to Steel Fibre - Cementatious Matrix Composite System*. Journal of Materials Science, Vol. 19, No. 5. pp: 1680-1691.
6. Gopalaratnam, V.S., and Shah, S.P. (1987) *Tensile Failure of Steel Fiber-Reinforced Mortar*. Journal of Engineering Mech., ASCE, Vol. 113, No. 5, pp: 635-652.
7. Mandel, J. A., Wei, S and Said, S. (1987). *Studies of the Properties of the Fibre Matrix Interface in Steel Fibre Reinforced Mortar*. ACI Material Journal, March – April. pp: 101-109.
8. Naaman, A. E., Namur, G., Najm, H. S and Alwan, J. (1989). *Bond Mechanisms in Fibre Reinforced Cement-Based Composites*. Report No. UMCE-89-9, University of Michigan. August. pp: 253.
9. Namur, G. G and Naaman, A. E. (1989). *A Bond Stress Model for Fibre Reinforced Concrete Based on Bond Stress Slip Relationship*. ACI Material Journal, Vol. 86, No. 1. pp: 45-57.
10. Wang, Y. (1989). *Mechanics of Fibre Reinforced Cementitious Composites*. PhD Thesis, Department of Mechanical Engineering, Massachusetts Institute of Technology, Cambridge USA. pp: 306.
11. Wang, Y., Li, C. V and Backer, S. (1990a). *Experimental Determination of Tensile Behaviour of Fibre Reinforced Concrete*. ACI Materials Journals, Vol. 87, No. 5. pp. 461-468.
12. Wang, Y., Li, C. V and Backer, S. (1990b). *Tensile Properties of Synthetic Fibre Reinforced Concrete Mortar*. Cement and Concrete Composites, Vol. 12. pp. 29-40.
13. Li, C. V. (1992). *Post-Cracking Scaling Relations for Fibre Reinforced*

Cementitious Composites. Journals of Materials in Civil Engineering, ASCE, Vol. 4, No. 1. pp. 41-57.

14. Hughes, B.P., and Fattuhi, N.I. (1975) Fibre Bond Strength in Cement and Concrete. Magazine of Concrete Research, Vol. 27, No. 92, pp: 161-166.
15. Burakiewicz, A.. (1978) *Testing of Fibre Bond Strength in Cement Matrix. Testing and Test Methods of Fibre Cement Composites*, RILEM Symposium. The Construction Press, Lancaster, pp: 355-365.
16. Christenson R. N. and Waals, F. M. (1972) *Effective Stiffness of Randomly Oriented Fibres Composite*. Journal of Concrete Institute. October 1979. pp 72-89.
17. Visalvanich, K and Naaman, A. E. (1983). *Fracture Model for Fibre Reinforced Concrete*. ACI Journal, Vol. 80, No. 2. pp. 128-138.
18. Li, Z., Li, F., Chang, T. Y. P and Mai, Y. W (1998) *Uniaxial Tensile Behaviour of Concrete Reinforced with Randomly Distributed Short Fibres*. ACI Materials Journals, Vol. 15, No. 5, Sept – Oct. pp. 564-574.
19. Maalej, M., Li., C.V., and Hashida, T. (1995) Effect of Fibre Rupture on Tensile Properties of Short Fibre Composites. Journal of Engineering Mechanics, Vol. 121, No. 8, August, pp: 903-913.
20. Marti, P., Pfyl, T. S., Viktor, and Ulaga, T. (1999) *Harmonized Test Procedure for Steel Fibre-Reinforced Concrete*. ACI Materials Journal, Vol. 96, No. 6, Nov – Dec. pp: 676-685.
21. Abejide O. S. (2000) Application of fibre concrete to tension controlled structures. Spectrum Journal, Vol. 7, No. 2. pp 66 – 76.
22. Voo, J. Y. L. and Foster, S. J. (2003). *Variable Engagement Model for Fibre Reinforced Concrete in Tension*. UNICV Report R-420. School of Civil and Environmental Engineering, University of New South Wales England.
23. Aveston , J and Kelly, A. (1973) *Theory of Multiple Fracture of Fibrous Composites*. Journals of Materials Science, Vol. 8. pp. 352 - 362.
24. Naaman, A. E., Argon, A. S and Moavenzadeh, F. (1973) *Fracture Model for Fibre – Reinforced Concrete*. Journal of Structural Division, Proceedings of ASCE, Vol.102, No. ST8, August. pp. 1537 – 1548.
25. Pakotiprapha, B., Pama, R.P., and Lee, S.L.(1974) Mechanical Properties of Cement Mortar with Randomly Oriented Short Steel Wires**.** Magazine of Concrete Research, Vol. 26, No. 86, March, pp: 3-15.
26. Brandt, A.M. (1985) *On the Optimal Direction of Short Metal Fibres in Brittle Matrix Composites***.** Journal of Materials Science, Vol. 20, pp: 3831-3841.
27. Lim, T.Y., Paramasivam, P., and Lee, S. L. (1987). *Analytical Model for Tensile Behavior of Steel-Fibre Concrete*. ACI Materials Journals, July-Aug, pp: 286-298.
28. Easley, T.C., and Faber, K.T. (1999) *Use of Crack-Bridging Single-Fiber Pullout Test to Study Steel Fibre-Cementitious Matrix Composite***.** Journal of American Ceramic Society, Vol. 82, No. 12, pp: 3513-3520.
29. Gilles, C. (1999) *Modeling the Pullout of Wire-Drawn Steel Fibres*. Cement and Concrete Research, Vol. 29, pp: 1027-1037.
30. Voo, J. Y. L. and Foster, S. J. (2005). *Variable Engagement Model for the Design of Fibre Reinforced Concrete Structures*. Proceedings, 2003 ECI Conference on Advanced Materials for

Construction of Bridges, Buildings and Other Structures III, Davos, Switzerland.

31. Naaman, A.E., and Shah, S.P. (1976) *Pull-Out Mechanism in Steel Fibre-Reinforced Concrete*. Journal of the Structural Division, Proceedings of ASCE, Vol. 102, No. ST8, August, pp: 1537-1548.

32. Gray, R.J., and Johnston, C.D. (1984) *Effect of Matrix Composition of Fibre/Matrix Interfacial Bond Shear Strength in Fiber Reinforced Mortar***.** Cement and Concrete Research, Vol. 14, pp: 285-296.

33. Maage, M. (1978). *Fibre Bond Friction in Cement and Concrete, Testing and Test Methods of Fibre Cement Composites*. RILEM Symposium, Construction Press, Lanchester. pp: 329-336.

34. Pinchin, D.J., and Tabor, D. (1978). *Interfacial Contact Pressure and Frictional Stress Transfer in Steel Fibre*. Proceeding, RILEM Symposium on Testing and Test Methods of Fibre Cement Composites, R.N. Swamy, ed., The Construction Press, pp: 337-334.

35. Gopalaratnam, V.S., and Abu-Mathkour, H.J. (1987) *Investigation of the Pull-Out Characteristics of Steel Fibres from Mortar Matrices***.** Proceedings, International Symposium on Fiber Reinforced Concrete, Madras, India, pp: 201-211.

36. Petersson, P.E. (1981). *Crack Growth and Development of Fracture Zones in Plain Concrete Transfer in Steel Fibre*. Proceedings from Advances in Cement Matrix Composites Mat. Res. Soc., Annual Meeting, Boston, pp: 95-106.

37. Romualdi, J. P and Mandrel, J. A. (1964) T*ensile Strength of Concrete Affected by Uniformly Distributed and Closely Spaced Short Length Wire Reinforcement*. ACI Journal, Vol. 61, No. 6, June. pp. 657 – 671.

38. Parimi, S. R., Sridha, K and Rao, J. K. (1973) *Fracture Toughness of Fibre Concrete*. International Symposium of Fibre Reinforced Concrete, ACI, Publication No. SP-44, Oct. pp. 80 – 92.

39. Pakotiprapha, B. (1976) *A study of Bamboo Pulp and Fibre Cement Paste Composite*. D. Engineering Dissertation, Asian Institute of Technology, Bangkok.

40. Foster, S.J. (2001) *On the Behaviour of HSC Columns: Cover Spalling, Steel Fibres and Ductility***.** ACI Structural Journal, July-August, pp: 583 - 589.

41. Banthia, N., and Trottier, J.F. (1991) *Deformed Steel Fibre - Cementitious Matrix Bond Under Impact*. Cement and Concrete Research, Vol. 21, pp: 158 - 161.

42. Banthia, N., and Trottier, J.F. (1992) *Micro-mechanics of Steel Fibre Pullout, Rate Sensitivity at Very Low Temperatures*. Cement and Concrete Composites, Vol. 14, pp: 119 - 130.

43. Banthia, N., and Trottier, J.F. (1994) *Concrete Reinforced with Deformed Steel Fibres, Part 1: Bond - Slip Mechanisms*. ACI Materials Journal, Sept – Oct. pp: 435 - 446.

44. Kanda, T and Li, V. C. (1999) *A New Micromechanics Design Theory for Pseudo Strain Hardening Cementitious Composite*. Journal of Engineering Mechanics, ASCE, Vol. 125, No. 4. pp. 373 – 381.

45. Li, V. C; Horikoshi, T; Ogawa, A; Torigoe, S and Saiti, T (2004) *Micromechanics – Based Durability Study of Poly Vinyl Alcohol – Engineered Cementitious Composites*. ACI Materials Journal, Vol. 101, No. 3, May / June. pp. 242 – 248.

46. Gollwitzer, S; Abdo, T and Rackwitz,R

(1988) First *Order Reliability Method (FORM) Manual*. RCP, Munich Germany.

47. Fiorato, A. C. (1973) *Geometric Imperfections in Concrete Structures*. National Building Research Report, D5, Stockholm, Sweden.
48. Ellingwood, B. (1978) *Reliability Bases of Load and Resistance Factor for Reinforced Concrete Design*. National Bureau of Standards. Building Science Series 110, Washington, D.C.
49. Augusti, G; Baratta, A and Casciati, F. (1984) *Probabilistic Methods in Structural Engineering*. Chapman and Hall, New York, USA.
50. Melchers, R. E. (1987) *Structural Reliability: Analysis and Prediction*. Ellis Horwood Series in Civil Engineering, John Wiley and Sons, New York, USA.
51. Webster, and Bannister, A,C (2001) *Method And Application of Software for Reliability Assessment*. Corns UK Limited.

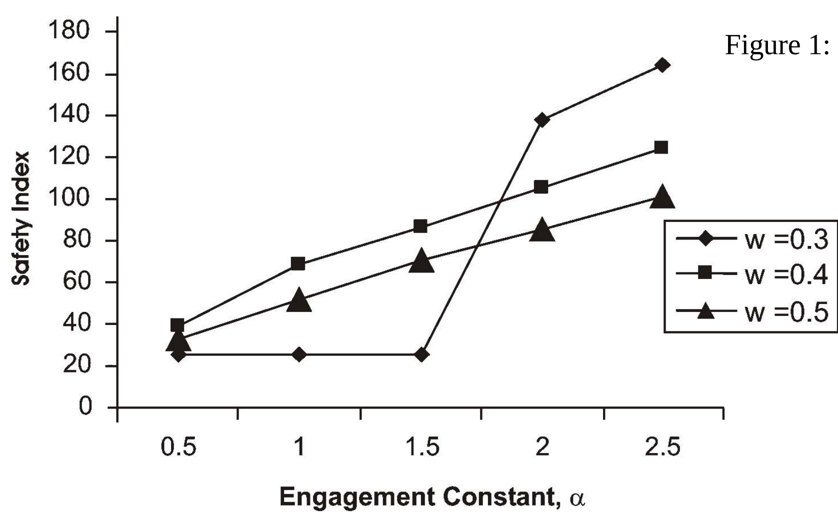

Figure 1: Reliability of engagement constant for fibres fracture without bending (pullout only).

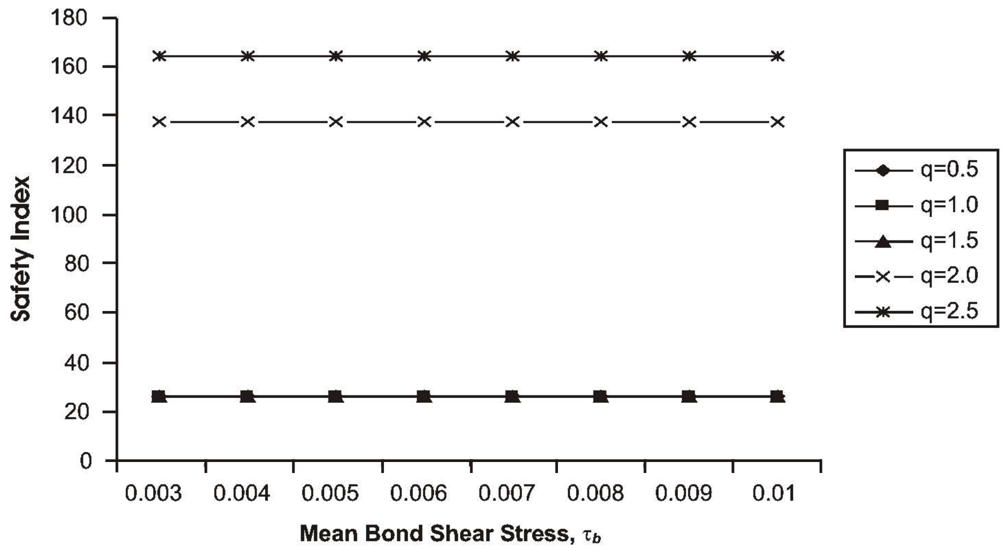

Figure 2: Reliability of mean bond shear stress for fibres fracture without bending (pullout only).

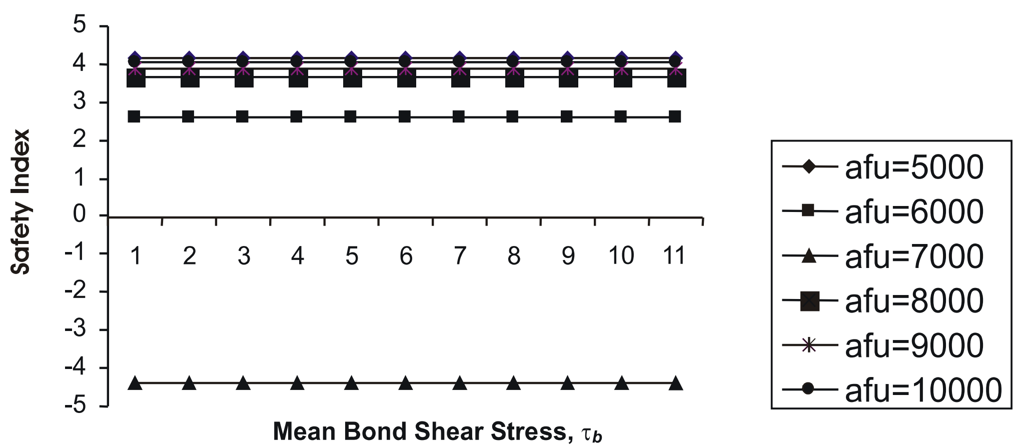

Figure 3: Reliability of fibres diameter in bond for fibres fracture with bending.