DYNAMIC MODELING OF NATURAL CONVECTION SOLAR ENERGY FLAT PLATE COLLECTOR FOR AGRICULTURAL DRIERS II - VALIDATION OF SOLUTION EQUATIONS

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ABSTRACT

The analytical solutions to the dynamic model of an air-heating flat plate solar energy thermal collector were validated by direct measurement from a physical model constructed for that purpose, of the temperatures of the cover and absorber plates, the inlet and outlet fluids, and the ambient air from morning to evening for four different days at 1800s intervals.

A plot of the measured plates and fluid outlet temperatures showed the values to be very close to those of the analytical dynamic model, the small differences being attributable to the attenuation produced by cloud cover, mist, fog, and rain for the real collector and clear sky conditions for the model. The developed output expressions (in closed form) for the dynamic model of flat plate solar energy air heating collectors can easily be used for optimization studies and design of better air heating solar energy collectors. (SSSCA means "Single Glazing, Single Pass, and Single Flow Air Heating Collector with Flow between the cover and Absorber Plates".)

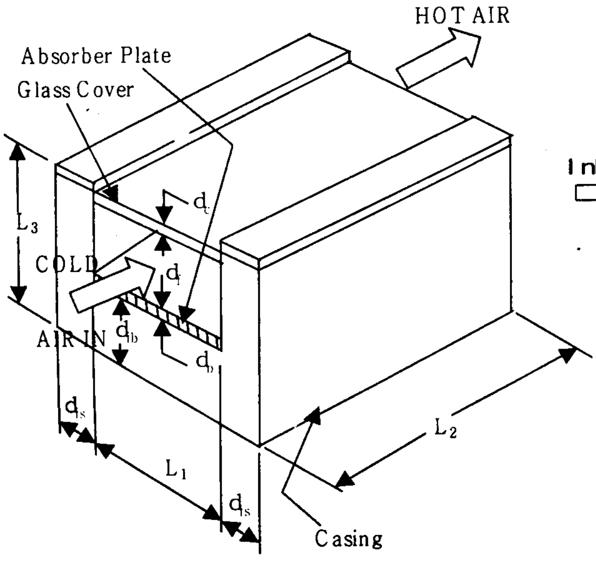

Fig. 1 Principal Dimensions of SSSCA Flat Plate Solar Energy Thermal Collector

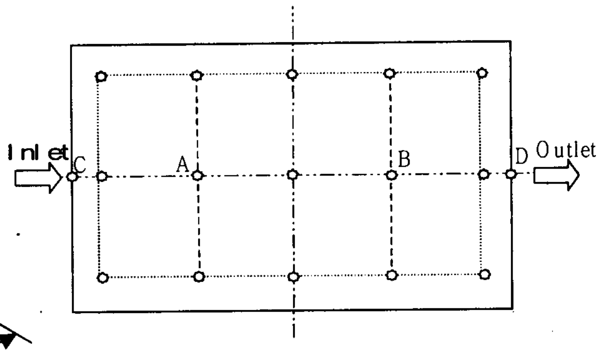

Fig. 2 Plan View of the Absorber Plate showing the Thermocouple Connections shown as ○ . Points A and B correspond to the Locations of the two Thermocouples on the Glass Cover. Points C and D correspond to the Thermocouple Locations in the Fluid Flow Channel.

1. INTRODUCTION

The analytical consideration of the dynamic model of the energy equations of a single glazing, single pass and single flow solar energy thermal collector (Onuoha, 2004) enabled expressions for the mean glazing, absorber and outlet fluid temperatures to be obtained. Expressions were also obtained for the collector instantaneous energy delivery rate, efficiency, heat removal factor, and combined plates coefficient of performance of the cover and absorber plates (formally called plate's efficiency (i.e. of the absorber plate only)). The energy equations are as follows for the:

COVER: $(\rho c_p\delta)_c\partial T_c/\partial t = I_T\eta_{oc} - u_c(T_c - T_a) - h_{rcp}(T_c - T_p) - h_{fc}(T_c - T_f)$ .... (1)

FLUID: $(\rho c_p\delta)_f\partial T_f/\partial t + (\dot{m}_f c_{pf}/L_1)\partial T_f/\partial x = -h_{fc}(T_f - T_c) - h_{fp}(T_f - T_p)$ .... (2)

ABSORBER: $(\rho c_p\delta)_p\partial T_p/\partial t = I_T\eta_{op} - u_p(T_p - T_a) - h_{rcp}(T_p - T_C) - h_{fp}(T_p - T_f)$ ....(3)

subject to the ***Initial Conditions:***

At $t = 0$, $\partial T_c/\partial t = \partial T_p/\partial t = \partial T_f/\partial t = 0$ .... (4)

$T_c(0) = T_{c0}$, $T_p(0) = T_{p0}$, $T_{fi}(0) = T_{fi0}$, and $T_{fo}(0) = T_{fo0}$ ....(5)

Also for operation from cold at t = 0s,

$$T_{c0} = T_{p0} = T_{fi0} = T_{fo0} = T_{a0} \quad (6)$$

where T_{a0} is the ambient air temperature at time, t = 0s.

The ***Boundary Conditions*** are:

At $x = 0$, $T_f = T_{fi} = T_a$, $t \geq 0$ .... (7)

At $x = L_2$, $T_f = T_{fo}$ $t \geq 0$ .... (8)

The analytical solutions to the energy equations (eqns. 1 – 3) are

$T_c = C_{AC}\cos 2\omega t + C_{AS}\sin 2\omega t + C_{BC}\cos\omega t + C_{BS}\sin\omega t + \lambda_c/e_m$ ....(9)

for the cover temperature,

$T_p = P_{AC}\cos 2\omega t + P_{AS}\sin 2\omega t + P_{BC}\cos\omega t + P_{BS}\sin\omega t + \lambda_p/e_m$ .... (10)

for the absorber temperature, and

$$T_{fo} = T_{fi} + [(1-\psi)/U_T]\{[f(t)/F_p\eta_{op} + C]\eta_{op}\} \quad (11)$$

for the outlet fluid temperature, where

$$\psi = \exp[-(h_{fc} + h_{fp})A_c/ \ c_{pf}] \quad (12)$$

$$\lambda_c = C(u_{pp}\eta_{oc} + h_r\eta_{op}) + (u_{pp}u_c + h_r u_p)T_a + \psi((u_{pp}h_{fc} + h_r h_{fp})T_{fi} \quad (13)$$

$$\lambda_p = C(u_{cp}\eta_{op} + h_r\eta_{oc}) + (u_{cp}u_p + h_r u_c)T_a + \psi((u_{cp}h_{fp} + h_r h_{fc})T_{fi} \quad (14)$$

C is the constant term in the expression for

$$I_T = A\cos 2\omega t + B\sin\omega t + C \quad (15)$$

$$F_p = h_f[u_{cp} + h_r + (u_{pp} + h_r)\eta_{oc}/\eta_{op}]/e_m \quad (16)$$

is the combined plates' coefficient of performance,

$$e_m = e_c e_p m_1 m_2 = e_c e_p u_2 = u_{cp}u_{pp} - h_r^2 \quad (17)$$

and $U_T = 2h_f/F_p$ .... (18)

is the collector overall heat transfer coefficient. The collector overall heat loss coefficient is given by

$$U_L = h_f[u_{pp}u_c + h_r u_p + u_{cp}u_p + h_r u_c]/F_p e_m \quad (19)$$

and the energy delivery rate is given by

$$Q_{uD} = \dot{m}_f c_{pf}(T_{fo} - T_{fi}) = F_R A_c[f(t)/F_p\eta_{op} + C]\eta_{op} \quad (20)$$

where

$$F_R = (\dot{m}_f c_{pf}/U_T A_c)\{1 - \exp[-F_p U_T A_c/\dot{m}_f c_{pf}]\} \quad (21)$$

is the collector heat removal factor and the temperature time factor

$$f(t) = h_f\{T_c + T_p - (\lambda_c + \lambda_p)/e_m\} \quad (22)$$

The collector instantaneous efficiency is given by

$$\eta_{ci} = Q_{uD}/I_T A_c = (F_R/I_T)[f(t)/F_p\eta_{op} + C]\eta_{op} \quad (23)$$

The optical efficiencies of the cover and Absorber plates are given by Onuoha (b, in press) respectively as

$$\eta_{oc} = 1 - \rho - \tau + \tau(1 - \alpha_{rp})(1 - \rho_d - \tau_d)/\{1 - \rho_d(1 - \alpha_{rp})\} \quad (24)$$

and $\eta_{op} = \tau\alpha_{rp}/\{1 - \rho_d((1 - \alpha_{rp})\}$ .... (25)

C_{AC}, C_{AS}, C_{BC}, C_{BS}, P_{AC}, P_{AS}, P_{BC}, P_{BS} are functions of the heat transfer coefficients of the materials of the collector, the optical efficiencies of the cover and absorber plates and the collector time constants m_1 and m_2, the global radiation frequency ù, and the constants A and B in eqn. (15) and are defined by eqns. (35)-(50) in Onuoha, 2004.

The above analytical expressions however, need validation by an experiment to find out their suitability for solar collector modeling and hence acceptability by future investigators and designers of air-heating, natural convection solar energy systems. The experimental set-up for measuring the temperatures and the results obtained are the subjects of this paper.

2. THE EXPERIMENTAL SET-UP

The expressions for the analytical solutions to the dynamic model equations of the flat plate collector considered were validated by temperature measurements on a physical model, which was constructed for that purpose. The absorber plate was a black galvanized iron, *0.0035m* thick, 1.225m long, and 0.95m wide with 0.94m of the width exposed to solar radiation, and was not given further black painting. The glazing material was clear window glass, 0.004m thick, 1.225m long, 0.95m wide, with 0.94m of the width exposed to solar radiation. The insulation material was sawdust obtained from a sawmill at Nsukka, Nigeria. The dimensions of the collector and thermo-physical properties of the material of construction are as given in (Onuoha, 2004, Section 4.2). The average temperature of the absorber plate was obtained by the average of the readings of fifteen thermocouples soldered at different points on its upper surface as shown in fig. 2. For the glazing material, the average of the readings of two thermocouples cellotaped on its upper surface at points A and B as indicated in fig. 2, gave its average temperature. Two thermocouples were also located at positions C and D in the fluid channel (0.02m deep, 0.94m wide, and 1.225m long) to measure the fluid inlet and outlet temperatures respectively. A liquid-in-glass mercury thermometer was also located at position D in the fluid channel to confirm the thermocouple readings. The thermocouples were initially calibrated using the liquid-in-glass mercury thermometer (Michalski et al ,1991; Benedict, 1984; Kent, 1993; Sato, 1971; White, 1959; Wilson, 1964) and a water bath, the voltage readings being taken at equal intervals during the heating and cooling of the water, which was continuously well stirred. The calibration equation of the thermocouple is

$$T = -1.458E+10e^4 + 1.022E+08e^3 - 7.085E+05e^2 + 2.4553383E+04e + 273.967192 \text{ K} \ldots\ldots\ldots\ldots(26)$$

where the emf e, is in volts, and was measured using ALDA precision digital multimeter model *DT*- 830B. This multi meter was also used to measure the emf output of the Eppley precision radiometer model PSP/ 17361 F3 used to measure the global radiation. The output of the radiometer is related to the global radiation by the equation:

$$I \text{ (or } I_T) = e/(9.6E-06) \text{ Wm}^2 \ldots\ldots\ldots\ldots(27)$$

for e in volts. A liquid-in-glass mercury thermometer, which was shielded from direct Sun's rays was used to measure the ambient air temperature, T_a which was found to be a function of time of day and its functional form from a least- square analysis for each model day is given by

$$T_a = T_{ao} + T_{a1}t + T_{a2}t^2 + T_{a3}t^3 + T_{a4}t^4, \text{ for } 0 \le t \le 43200\text{s} \ldots (28)$$

where T_{a0} T_{al}, T_{a2}, T_{a3} and T_{a4} are constants. The Global Radiatior s, H and H_T were obtained as summation or integration of a least-square approximation of the half-hourly values of **I,** (on a horizontal surface) and I_T (on the plane of the collector) respectively, obtained from Eppley PSP precision pyranometer (radiometer). Where a pyranometer is not available, it is expected that Insolation (global radiation) Graphs like that of Ezekwe and Ezeilo (1981) or Climatic Radiation Graphs like that of Ezekwe (1988) will be used in estimating H from $\widehat{H}$ and hence H_T The collector itself was mounted on a table at a tilt angle of β = 0.174533 rad., facing South. The latitude of Nsukka, Nigeria is L = 0.0119555 rad. All readings were taken at 1800s intervals as the model output and are displayed in figs. (3) (4), (5), and (6) for 14-03-02, 16-03-02, 21-03-02, and 23-03-02 respectively.

3.0 OBSERVATIONS

But for the attenuation produced by cloud-cover and to a lesser extent by fog and drizzle (rain), the outputs are similar to those of the model displayed in figs. (2a),(3a),(4a) and (Sa) respectively of Onuoha , 2004 and repeated here as figs *3b, 4b, 5b,* and 6b compared with the measured output, respectively and with the ordinate of the measured output, displaced to the right by 1800s. The measured maximum average temperatures as displayed in Table 1 are also very close to the model temperatures, their points of occurrence depending on the actual maximum global radiation and cloud cover duration. The closeness of the outlet fluid temperature to the cover temperature on 21-03-02 and to the absorber temperature on 14-03-02 is attributed to the closeness of the thermocouple (and thermometer) to the cover plate in the first case and to the absorber plate in the second. From the above, it appears that but for the attenuating factors, the analytical solutions accurately represent the operation of the real physical single cover, single pass, single flow air heating flat plate solar energy collector with flow between the cover and the absorber plates under clear sky conditions.

4.0 CONCLUSION AND RECOMMENDATIONS.

4.1. Conclusion

Equations have been developed that can be used to accurately predict the output parameters of a natural convection, single glazing, single pass, single flow, air heating flat plate solar energy collector with flow between the cover and the absorber plates. The small difference noticed between the analytical model solutions and the actual collector is attributable to the attenuation produced by cloud cover, mist, fog, and rain for the real collector and clear sky conditions for the model.

4.2. Recommendations.

It is recommended as follows:-

(i) The developed equations and given above by eqns. (9) - (25) and (28) should be used in determining the collector output if the configuration is of the form as the one analyzed.

(ii) Since the input to the model is the daily global radiation on a horizontal surface, H, or its monthly average ($\widehat{H}$), published graphs of $\widehat{H}$ (e.g. Ezekwe and Ezeilo, 1981) or Solar Radiation Climatic Maps (e.g. Ezekwe, 1988) of the location under consideration should be used to obtain H if a pyranometer is not available.

(iii) Since for a particular collector con figuration. $T_a = T_a(t)$ is used in conjunction with H as the only input to the model, ordinary mercury-in-glass thermometer should be used to determine the functional relationship. It is further suggested that further research be conducted to obtain the coefficients: T_{a0}, T_{al}, T_{a2}, T_{a3} and T_{a4} in eqn. (28) which future investigators can use without recourse to direct measurements like has been done for \$, and hence for $\widehat{H}_d, \hat{I}_o, \hat{I}_d, and\ \hat{I}_T$.

TABLE 1. MEASURED AND MODEL COLLECTOR MAXIMUM OUTPUT TEMPERATURES AND GLOBAL RADIATION ON ITS PLANE FOR FOUR DAYS.

DATE AND DAY NUMBER		14-03-02 73	16-03-02 75	21-03-02 80	23-03-02 82
GLOBAL RADIATION, H_T, MJm^{-2}		16.78125	15.825	14.85	12.30
MAX.INST.GLOBAL RAD., $I_{T,max}$, Wm^{-2} At time ts from 6.00 a.m.	Measured	833.332	916.67	656.25	656.25
		7000	23400	21600	25200
	Model	673.71	648.32	589.69	493.90
		21600	21600	21600	21600
COVER MAX. TEMP., $T_{c,max}$, K At time t s from 6.00 a.m.	Measured	-	329.74*	333.04	327.15
		-	30600	25200	23400
	Model	335.24	333.24	332.67	327.15
		25200	25200	25200	25200
ABSORBER MAX. TEMP, $T_{p,max}$, K At time t s from 6.00 a.m.	Measured	370.00	369.03	367.31	358.03
		30600	25200	23400**	25200
	Model	371.17	368.23	365.54	356.40
		23400	23400	23400	2340 0
MAX. FLUID OUTLET TEMP., $T_{fo,max}$, K,at time ts from 6.00a.m.	Measured	363.60	347.08	336.32	335.95
		28800	25200	21600***	23400
	Model	0.00684	0.00680	0.00632	0.00581
		21600	21600	21600	23400

NOTES: ** Also at t = 32400s; ** Also at t = 27000s; *** Also at t = 25200s.* $H_T = \int (I_T)dt \approx \sum(I_T)\Delta t \approx \Delta t \sum(I_T) = 0.0018\sum(I_T)\ MJ\ m^{-2}$

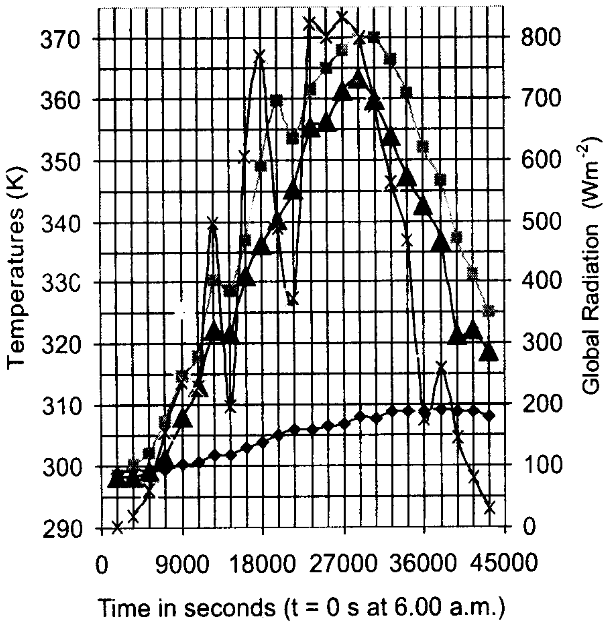

Fig. 3a Measured Input and Output Parameters of SSSCA Flat Plate Solar Collector on 14-03-02

- Ambient Temperature (K)
- Absorber Temperature (K)
- Fluid Outlet Temperature (K)
- Global Radiation on Collector Plane (W/sq.m)

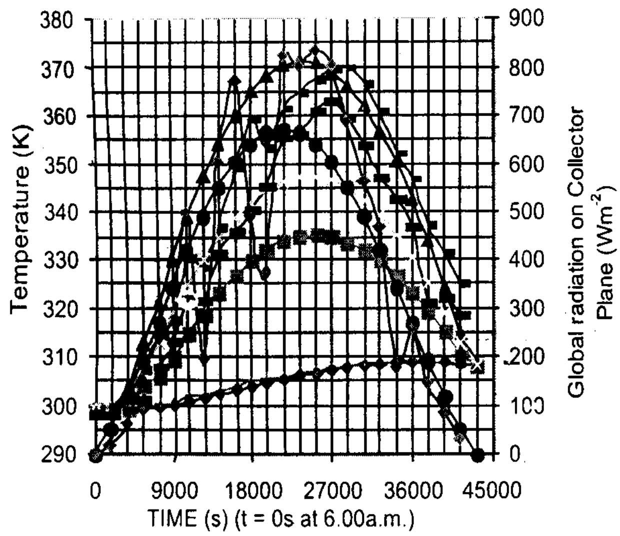

Fig. 3b Model and Measured Input and Output Parameters of SSSCA Flat Plate Collector on 14-03-02

$T_a = T_{fi} = 299.09883 - 1.323E{-}04t + 3.596E{-}08t^2 - 8.711E{-}13t^3 + 5.1751E{-}18t^4$ K, H = 17.0 MJm^{-2}

- Ta, K (model)
- Tc, K (model)
- Tp, K (model)
- Tfo, K (model)
- Ta, K (measured)
- Tp, K (measured)
- Tfo, K (measured)
- IT, Wm-2 (model)
- IT, Wm-2 (measured)

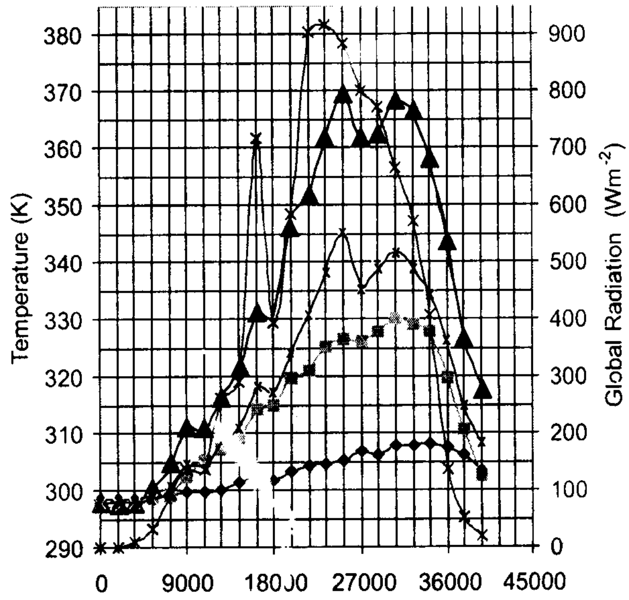

Fig. 4a Measured Input Output Parameters of SSSCA Flat Plate Collector on 16-03-02

- Ambient Temperature (K)
- Cover Temperature (K)
- Absorber Temperature (K)
- Fluid1 Outlet Temperature (K)
- Global Radiation (W/sq.m)

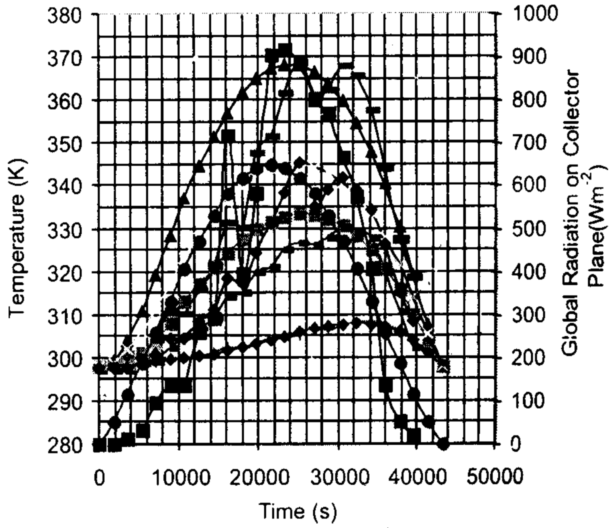

Fig. 4b. Model and Measured Input and Output Parameters of SSSCA Flat Plate Collector on 16 -03-02

$T_a = T_{fi} = 298.052408 + 2.262E{-}04t + 2.187E{-}08t^2 + 1.8432E{-}12t^3 - 3.385E{-}17t^4$ K, H = 16.4 MJm^{-2}

- Ta = Tfi, K (model)
- Tc, K (model)
- Tp, K (model)
- Tfo, K (model)
- Ta = Tfi, K (measured)
- Tc, K (measured)
- Tp, K (measured)
- Tfo, K (measured)
- IT, Wm^{-2} (model)
- IT, Wm^{-2} (measured)

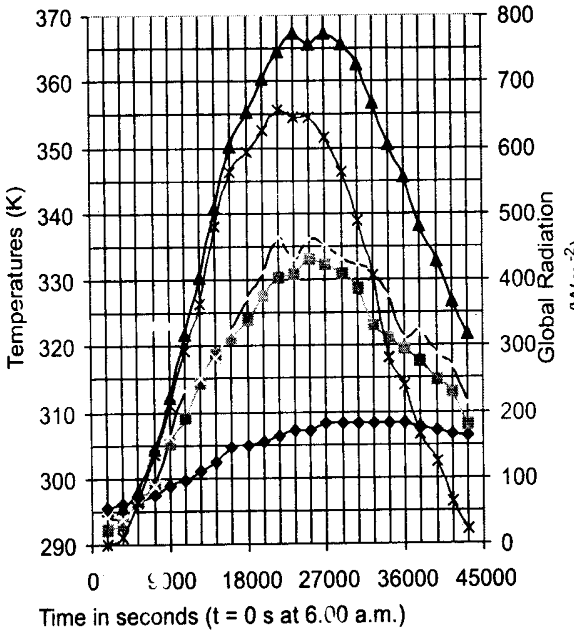

Fig. 5a Measured Input and Output Parameters of SSSCA Flat Plate Collector on 21-03-02

- Ambient Temperature (K)
- Cover Temperature (K)
- Absorber Temperature (K)
- Fluid1 Outlet Temperature (K)
- Global Radiation on Collector Plane (W/sq.m)

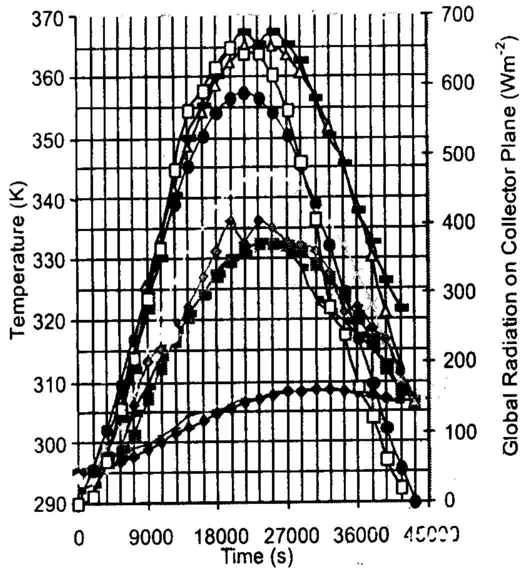

Fig. 5b. Model and Measured Input and Output Parameters of SSSCA Flat Plate Collector on 21-03-02

$T_a = T_{fi} = 295.247134 + 2.0434E\text{-}05t + 5.9981E\text{-}08t^2 - 2.163E\text{-}12t^3 + 2.0873E\text{-}17t^4$ K, H = 15.0 MJm^{-2}

Ta = Tfi, K (model)	Tc, K (model)
Tp, K (model)	Tfo, K (model)
Ta = Tfi, K (measured)	Tc, K (measured)
Tp, K (measured)	Tfo, K (measured)
I_T, Wm^{-2} (model)	I_T, Wm^{-2} (measured)

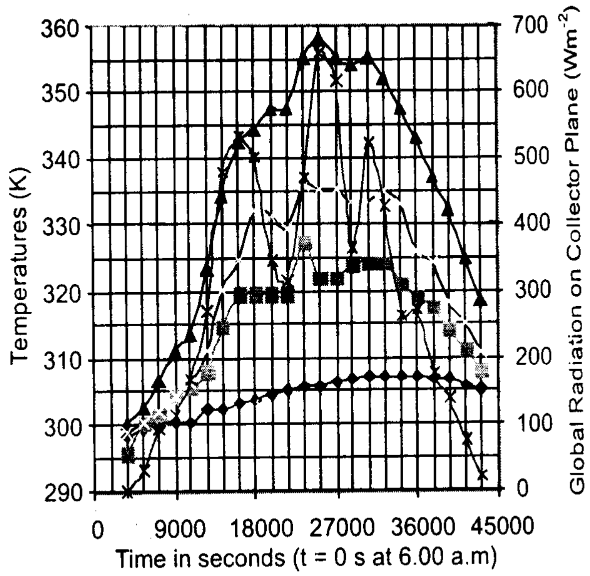

Fig. 6a Measured Input and Output Parameters of SSSCA Flat Plate Collector on 23-03-02

- Ambient Temperature (K)
- Cover Temperature (K)
- Absorber Temperature (K)
- Fluid Outlet Temperature (K)
- Global Radiation on Collector Plane (Wm^{-2})

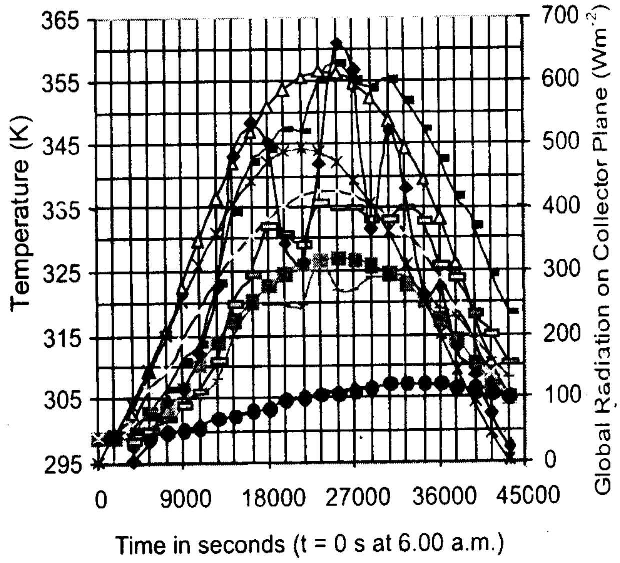

Fig. 6b Model and Measured Input and Output Parameters of SSSCA Flat Plate Collector on 23-03-02

Ta = Tfi= $298.83499 - 5.367E\text{-}05t + 3.1721E\text{-}08t^2 - 8.571E\text{-}13t^3 + 5.3463E\text{-}18t^4$ K, H = 12.6 MJm^{-2}

- Model Ambient Temperature (K)
- Model Cover Temperature (K)
- Model Absorber Temperature (K)
- Model Fluid Outlet Temperature (K)
- Measured Ambient Temperature (K)
- Measured Cover Temperature (K)
- Measured Absorber Temperature (K)
- Measured Fluid Outlet Temperature (K)
- Model Global Radiation on Collector Plane (Wm^{-2})
- Measured Global Radiation on Collector Plane (Wm^{-2})

NOMENCLATURE

A_c	collector area exposed to solar radiation, m^2
A_f	cross-section of fluid flow channel, m^2
D	hydraulic diameter, m
e	heat capacity per unit area, $JK^{-1}m^{-2}$,; emf, V.
Fp	collector combined plates' coefficient of performance
F_R	collector heat removal factor
g	acceleration due to gravity, ms^{-2}
G_r	Grashof Number
G_z	Graetz Number
h	Heat transfer coefficient, Wm^2K^{-1}
H	daily global radiation on a horizontal surface, MJm^2
H_d	daily diffuse ration on horizontal surface, MJm^2
I_T	instantaneous global radiation on a titled surface Wm^2
K	thermal conductivity, $Wm^{-1}K^{-1}$
L	latitude, rad; length, m
Nu	Nusselt number
P	pressure, Nm^{-2}
Pr	Prandtl Number
Q_{uD}	collector energy delivery rate, W.
$Q_{uD,T}$	collector total (daily) energy delivery, MJ
Re	Reynolds Nuber
t	times, s
T	Temperature, K
u	heat transfer/loss coefficient, $Wm^{-2}K^{-1}$
U_L	overall heat transfer coefficient, $Wm^{-2}K^{-1}$
U_T	overall heat transfer coefficient, $Wm^{-2}\,K^{-1}$
V_f	fluid velocity, ms^{-1}
α_{rp}	obsorber plate abosrptivity
β	collector tilt, rad
ε	emissivity
η	efficiency
μ	dynamic viscosity, Nsm^{-2}
π	3.141592654
ρ	density, kgm^{-3} reflectivity, ground albedo
σ	Stefan-Boltzmann constant, $Wm^{-2}K^4$
τ	solar transmitivity
ω	solar angular frequency, $rad.s^{-1}$, $= \pi/43200.0$

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