# APPLICATION OF THE REISSNERS PLATE THEORY IN THE DELAMINATION ANALYSIS OF A THREE-DIMENSIONAL, TIME-DEPENDENT, NON-LINEAR, UNI-DIRECTIONAL FIBRE REINFORCED COMPOSITE STRUCTURE.

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**ABSTRACT**

*The prediction of delamination using Damage Mechanics and Interface Modeling approaches were reviewed and their limitations noted. Target was set to obtain a model free from the limitations of the Damage Mechanics approach, which involved a non- linear, three-dimensional evolution problem and the Interface Modeling method, which involved results not tallying with experimental data. This target was achieved by employing the Reissne's Plate Theory Equations, which gave results that show extraordinarily good agreement with experimental data.*

## 1. INTRODUCTION

Majority of sensitive failures recorded in components made of fibre Reinforced Plastics- particularly, carbon-epoxy laminates are due to de-lamination [1]. Delamination however, can be defined as failure of laminates along the interface of two plies bonded together by a resin, which failure is usually a kind of slip movement about the plane of contact. What then causes delamination and how does it occur? This phenomenon, which consists of de-bonding of layers, is due to the occurrence of three-dimensional stress states in these regions, which may lead to high tensile or shear stresses on the interface of two adjacent layers [2, 3].

Different scholars have employed various approaches in the past in an attempt to model the de-lamination phenomenon. One of such approaches is the Damage mechanics approach [4- 6]. Here, the modeling involved the analysis of gradual degradation, which also involved simulation of micro-cracks and micro-voids in the vicinity of the laminate interface. The model included coupled anisotropic, unilateral damage and elastic-plastic behaviour. The interface presented a two-dimensional entity, which ensured displacements and traction transfer from one layer to another [4, 5]. However, this model had a difficulty attached to it because the computation of delamination initiation growth led to a non- linear, three-dimensional evolution problem.

Another very exciting model is the Interface modeling approach (see fig 1)

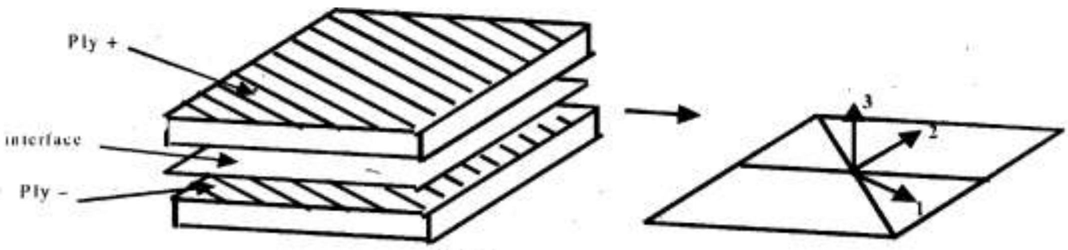


*Fig. 1*

**Symbols/Notations:**

*E* = *Young's modulus of elasticity*
*ν* = *Poisson's ratio*
*U* = *strain energy*
*u,v,w* = *displacement fields*
*δ* = *fibre normal stress*
G = *Young's modulus of rigidity*
*Ω* = *domain of interest*
*Ko* = *conditioner operator*

This model believes that the interface is a zero thickness medium which ensures stress and displacement transfers from one place to another and that elementary constituents is necessary in the vicinity of the edges [6]

Where (N1, N2) axes are the bisectors of the fibre directions (U representing the strain energy) and the displacement discontinuities are denoted by:

$$[\mathrm{U}] = \mathrm{U}^{+} - \mathrm{U}^{-} = [\mathrm{U}_1]\mathrm{N}_1 + [\mathrm{U}_2]\mathrm{N}_2 + [\mathrm{U}_3]\mathrm{N}_3 \qquad (1)$$

And the energy permit area is given by:

$$\mathrm{E_D} = \frac{1}{2}\frac{(-\delta_{33})_+^2}{\mathrm{K^o}} + \frac{1}{2}\frac{(-\delta_{33})_+^2}{(1-\mathrm{d})\mathrm{K^o}} + \frac{1}{2}\frac{{\delta_{32}}^2}{(1-\mathrm{Y}_2\mathrm{d})\mathrm{K}_2^{\mathrm{o}}} + \frac{1}{2}\frac{{\delta_{13}}^2}{(1-\gamma_1\mathrm{d})\mathrm{K}_1^{\mathrm{o}}} \qquad (2)$$

Where

$\mathrm{Y}_1, \mathrm{Y}_2$ = constants

$\mathrm{K^o}, \mathrm{K}_1^{\mathrm{o}}, \mathrm{K}_2^{\mathrm{o}}$ = elastic parameters of the interface

$\mathrm{K^o}, \mathrm{K}_1^{\mathrm{o}}, \mathrm{K}_2^{\mathrm{o}} \rightarrow > +\omega$ for a perfect to be obtained

The variable associated with damage evolution, d, is $\mathrm{Y_d}$ give as

$$Y_d = \frac{\partial E_D}{\partial d/_\delta} = ½\frac{(\delta_{33})_+^2}{(1-d)^2K^O} + Y_2\frac{\delta_{32}^2}{(1-d)^2K_2^O} + Y_1\frac{\delta_{13+}^2}{(1-d)^2K_1^O} \qquad (3)$$

Here, the fist term is associated with the first opening mode; the two other terms with the second and third modes such that damage evolution can be described as:

$$\mathrm{d} = \sup|\mathrm{t}' \leq \frac{\mathrm{Yd(t');}}{\mathrm{Y_O}}, if\ d < 1, otherwise\ d = 1 \qquad (4)$$

Where $\mathrm{Y_c}$ = critical energy, $\mathrm{Y_c} = \mathrm{Y_d}$ for d = 1.

These results were compared with the experimental parameters of Damage mechanics: $\mathrm{G_L}$: $\mathrm{G_{IL}}$; $\mathrm{G_{ILL}}$

However, this model presented some difficulties

$$\mathrm{Y} = (\upsilon\ (\mathrm{x}) - \mathrm{zv}\ (\mathrm{x})\ \mathrm{N}_1 + \mathrm{w(x)N} \qquad (5)$$

since comparisons between computations and experimental results did not tally [7].

**1.1 Purpose of the Work:**

The purpose of this work is to seek solution to a three-dimensional, time-dependent, non-linear, damage mechanics problem in a unidirectional Fibre Reinforced composite structure such that the model employed yields results comparable with the experimental figures as obtainable in Damage Mechanics.

## 2. METHODOLOGY:

### 2.1 Application of the Reissner's Plate Theory Equations

Consider the separation of two laminate plies, fig 2 If Ѓ represents the laminate interface; N, $\mathrm{N}_1$ $\mathrm{N}_2$, the various planes, and the displacement fields occur in the Ω domain for a given length L and the applied forces PN under the assumption of plane strain state in the ($\mathrm{N}_1$, N) plane, we may have for the displacement domain Ω = Ω+μΩ·strain, energy in the form:

Where the admissible displacement fields are restricted to

υs = {υ/(υ,v,w)} regular in Ω (6)

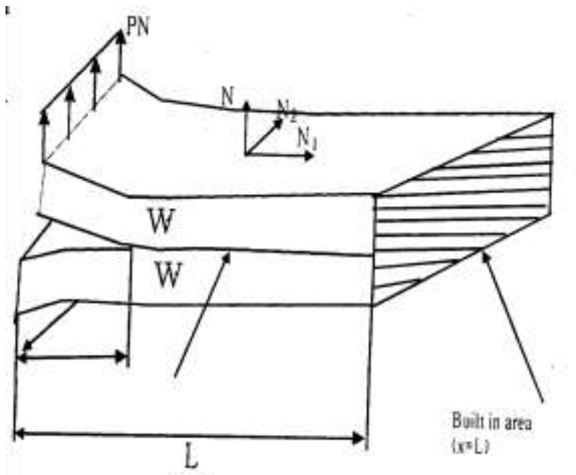


*Fig. 2*

The Reissners plate theory requires that we find the stress. δ and strain, e satisfying

υϵ υs

Appling the equilibrium equation:

$$\forall\upsilon^* \int_\Omega T_r(\delta\varepsilon(\upsilon^*)d\Omega + \int_{\mathrm{r}} \delta_{33}[\upsilon *]\mathrm{d}Ѓ = 2\mathrm{PW}^*(0) \qquad (7)$$

and the constitutive relations:

$$\delta = \mathrm{K_c}\varepsilon \text{ in } \Omega \text{ and } \delta_{33} = \mathrm{K_o}(1-\mathrm{d})[\mathrm{W}]\text{in}Ѓ \qquad (8)$$

The condition for delamination propagation is obtained from an instability situation reached, when d = 1. The load then is assumed to increase monotonously and the $G_1$ is calculated from the energy release rate G.

### 2.2. Problem formulation

Consider a situation where the analysis is restricted to the vicinity of the edges in the domain Ω, where the three-dimensional effects are significant. A link with the solution obtained by shell computation is made on the $\partial_1\Omega$ area through given displacement $U_d$. On the complimentary part of the boundary $\partial_2\Omega$ either displacement or forces may be imposed.

The problem is to find (δ,ε) values satisfying

$$-\forall \upsilon^{*} \epsilon U = \{U/U \text{ regular }\} \text{and } U/\partial_1\Omega = U_d(t)$$

$$\forall t\epsilon[0,T] \int_{\Omega} T_r(\delta\varepsilon(U^{*}) d\Omega + \sum_{i}^{n-1} \int_{r} \delta N[U^{*} *] d\ \acute{\Gamma} = \int \partial_{2\Omega} F_d(t) U^{*} d_s \quad (9)$$

and the constitutive relations in Ω and in

$$\acute{\Gamma} \forall t \in [0, T] \quad (10)$$

### 2.3 Computation Method:

To solve, with a reasonable cost, this non-linear, three-dimensional time-dependent problem (i) the large time increment method and (ii) a semi- analytical method, which requires the solution of two-dimensional problems, only are used. The time increment method developed by Ladeveze et al [8] breaks with the step by step scheme of all previous computation techniques (for example Newton Raphson). It proceeds by a single global iterative procedure on the whole loading history [O,T] between global linear steps (satisfying equation) and local linear steps (satisfying item (10), which reduces considerably the number of global solutions. The global 'step is then a linear but three-dimensional, time-dependent problem. This three-dimensional problem is reduced to two-dimensional problem in the strip with use of (i) conjugate gradient method and (ii) the fast Fourier transform. Here, an axially symmetric conditioned operator is used to solve non-axially symmetric problem [9]

### 2.4 Global Step Solving Method;

In a finite element scheme, the problem to be solved is state as:

$$K(t)X = F(t) \quad (11)$$

Where K stands for the search operator between local and global steps. Applying the conjugate gradient with conditioner iterative method (in time and space), equation (11) can be replaced by a series of intermediate problems such as:

$$K_O X(t) = F^{(i)}(t) \quad (12)$$

Where K is a conditioner and a constant. The conditioner, $K_o$ is a choice and the axially symmetric edge area allows us to develop the solution of equation (12) in the form of Fourier series in the base of cylindrical coordinates. The chosen transverse isotropic conditioner $K_o$ is:

$$K_O = \frac{1}{2\Pi} \int_{0}^{2\pi} K_e d\theta \quad (13)$$

(13)

Where $K_e$ is the elastic operator.

This choice allows one to separate the different modes of Fourier. By means of the fast Fourier transform, this displacement is developed in Fourier series and each of its components is computed as the solution of a two-dimensional elastic problem associated with K and is solved in a finite strip [10].

## 3. RESULTS:

### 3.1 Numerically Worked Example;

Consider a structure with the following configuration: (0, 45, -45, 90)s which is loaded by a radial pressure on the hole. Using the above semi-analytical method, this computation was made in 64 strips, which represented about 100,000 degrees of freedom for the whole problem, employing only 25 iterations. Figure 3 shows the displacements in strip 8 at 0°. Figure 4 shows the peeling stress in the strip at (90 degree, 90 degree) interface.

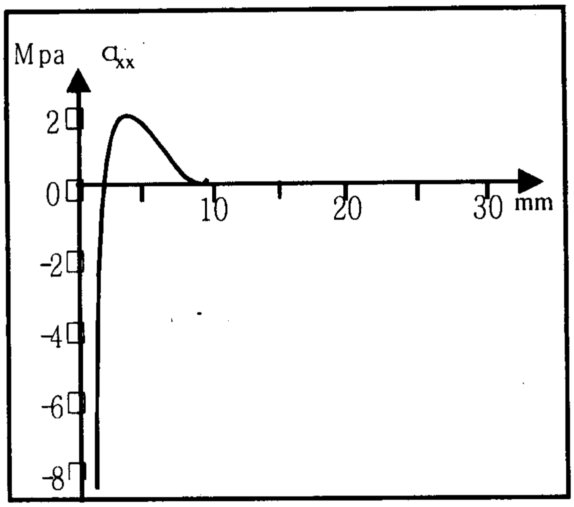


*Fig. 3*

A first non-linear example computation of a laminate (0,90) loaded in pure mode I is given. A normal displacement is prescribed on the edge of the hole (radius, $r_0$) under the following form $U(r_0, t) = \lambda(t)\,Uo$, where $\lambda(t) = t/T$. Fig 5 shows the

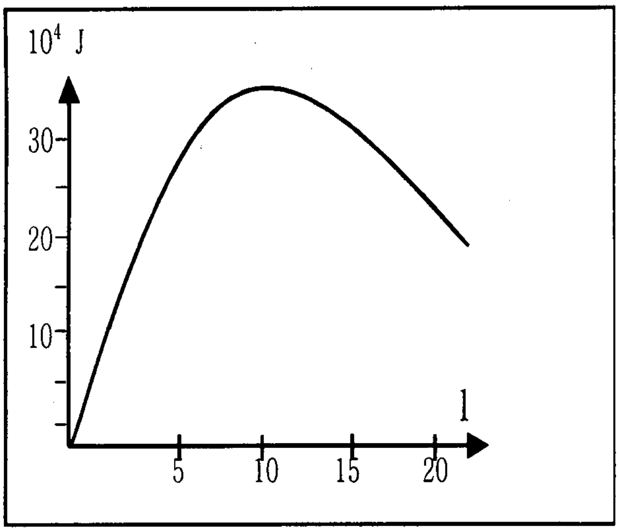


*Fig. 5*

Finally, the energy release rate G is calculated by employing the numerical method first introduced by Parks [11]. This method calculates the derivative of the stiffness matrix with respect to virtual extension, ∂a.

$$\text{i.e}\quad G = \frac{\partial Ep}{\partial a} \qquad \text{.....................(14)}$$

where Ep is the potential energy

In a finite element scheme we know that:

$$Ep(X) = \tfrac{1}{2} X^{r} KX - F^{r}X \qquad \text{...............(15)}$$

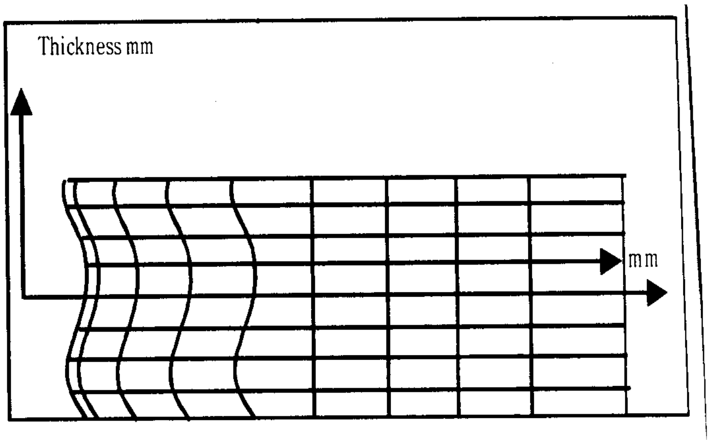


*Fig. 4*

work of outside forces divided by X, with respect to λ. Through a global instability condition, this curve allows one to predict the delamination initiation and the Gauss point at which it happens. Fig 6 shows the evolution of the Peeling stress at this Gauss point

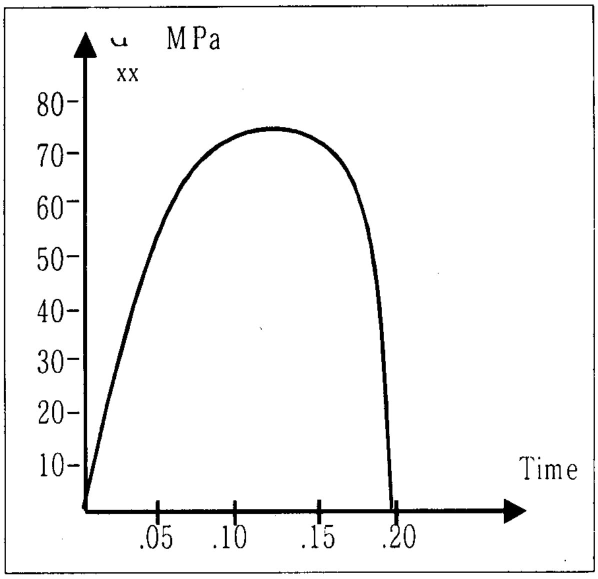


*Fig. 6*

Assuming the forces being constant and taking the equilibrium equation: KX = F into account, the energy release rate equation employed becomes

$$G = \tfrac{1}{2} X^{r} \frac{\partial K}{\partial a} X \qquad \text{......................(16)}$$

## 4. DISCUSSION:

Results obtained using the above relations (equations 14 - 16) have shown extra-ordinarily good agreement with the energy release rate figures obtained from experiments in Damage Mechanics. (please see table 1)

Table 1: For a given Damage Evolution variable, Yd corresponding values of Energy Release Rate, G using various approaches show that:

| Damage Evolution Variable, Yd | Interface Modeling approach | Damage Mechanics theoretical approach | Application of Reissner's Plate Theory approach (Present 'Work) | Existing experimental figures in Damage Mechanics |
|---|---|---|---|---|
| 0.1 | 2.01 | 2.05 | 2.24 | 2.35 |
| 0.2 | 3.07 | 3.09 | 3.32 | 3.47 |
| 0.3 | 4.20 | 4.43 | 4.97 | 5.02 |
| 0.4 | 6.08 | 6.34 | 6.72 | 6.89 |
| 0.5 | 8.10 | 8.33 | 8.65 | 8.79 |
| 0.6 | 9.09 | 9.21 | 9.45 | 0.66 |

## 5. CONCLUSION:

The application of the Reissners Plate Theory in the prediction of delamination initiation and propagation has provided a better alternative to the use of the Damage Mechanics prediction approach which led to a non-linear, three- dimensional evolution problem and also the Interface Modeling method whose computed results do not tally with experimental figures

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