AN IMPROVED STRUCTURAL MODEL FOR SEISMIC ANALYSIS OF TALL FRAMES

N. N. OSADEBE[1] and M. E. ONYIA[2]
Department of Civil Engineering
University of Nigeria, Nsukka. Nigeria

ABSTRACT

This paper proposed and examined an improved structural model that overcomes the deficiencies of the shear frame model by considering the effects of flexible horizontal members and column axial loads in seismic analysis of multi-storey frames. Matrix displacement method of analysis is used on the basis of the stiffness coefficients of axially-loaded beam element. These stiffness coefficients are obtained as a product of the conventional stiffness coefficients and relevant stability functions which serve as modification factors. The results show that the improved model is more realistic and gives results that are significantly lower than those obtained using the shear frame model.

NOTATION

MDOF Multi-degree of freedom
[M] Mass matrix
$\{\ddot{U}\}$ Acceleration vector
[K] Stiffness matrix
{Ű} Displacement vector
[C] Damping matrix
{U} Velocity vector
[ϕ] Mode shape matrix
M_n $\{\phi_n^T\}[M]\{\phi_n\}$ = Generalized mass
Ln $\{\phi_n^T\}[M]\{\phi_n\}$ = Earthquake Participation factor
$U_g(t)$ Earthquake ground motion

INTRODUCTION

The type of model used in the dynamic analysis of rigid frames greatly influences the results obtained as the stiffness distribution of the frame is dependent on the model used, [1]. Earlier works on seismic analysis of ideal frames employed mainly the shear frame (vertical pole) model, [2].

A major shortcoming of the shear frame model, however, is that it disregards the effect of joint rotations. It also disregards the effect of the column axial loads on the bending stiffnesses of the columns, [3,4], The latter simplification is acceptable as long as the axial loads remain small in comparison with the critical loads of the columns. When the ratio of the axial loads to the critical loads becomes sizeable, as may be case in the lower columns of tall rigid buildings, the bending stiffnesses are markedly reduced by the presence of axial compression and it may no longer be reasonable to neglect the axial loads in determining the bending stiffnesses of the columns, [5].

Verbanov and Capitanov [6] and Osadebe [7, 8] improved on the vertical pole model by taking into consideration the axial load deformation in addition to the lumped masses. The presence of the axial loads necessitated the use of the stability functions or so-called modified stiffness coefficients which incorporated the effects of the axial loads. But the model still shared the disadvantage that the rigidities of the horizontal members were not considered in the stiffness analysis. It merely summed up the rigidities of the vertical members at each storey level to give the rigidity of the vertical pole at that storey level.

This paper therefore aims at the development and application of an improved structural model for seismic analysis that includes the effects of axial loads and flexibility of joints. The results are compared with those obtained using the shear frame model.

STIFFNESS COEFFICIENTS

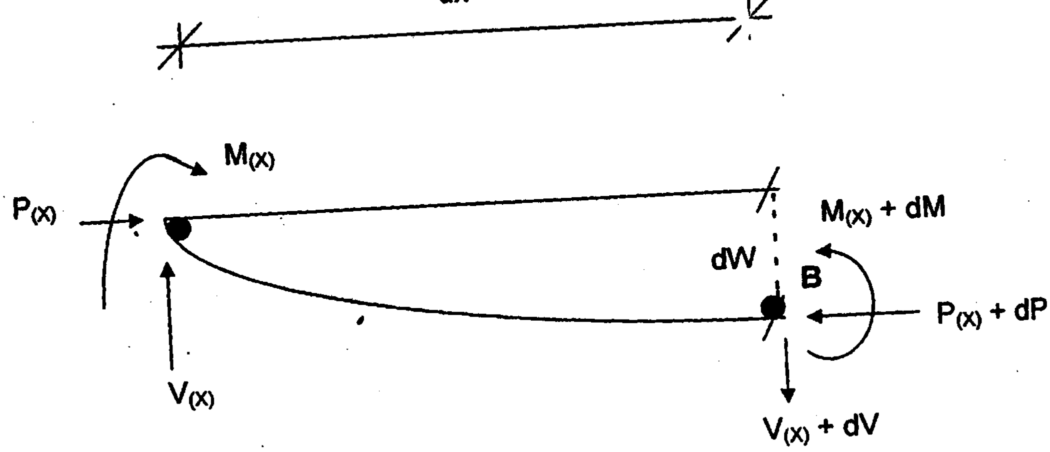


Fig. 1: A section of a Beam-Colum

By considering the equilibrium of the beam-colum section shown in Fig. 1 above, the deflection curve is shown to be: [9]

$$W_{(\times)} = W_0 + \frac{\alpha_0}{K}\sin K\times + \frac{M_0}{EIK^2}(\cos K\times - 1) + \frac{V_0}{EIK^3}(\sin KX - KX) \qquad (1)$$

where $K^2 = P/(EI)$, and W_0, α_0, M_0 and V_0 are the inital values.

By applying relevant boundary conditions, the rotational stiffnesses for a fixed-ended beam-column of length L are shown to be:

$$V_o = \frac{6EI}{L^2} \times F_2(A) = V_L \qquad (2)$$

$$where\ F_2(A) = \frac{A^2}{6}\ \frac{(-\cos A)}{(2 - 2\cos A - A\sin A)}\ and\ A^2 = PL^2/(EI)$$

$$M_o = \frac{4EI}{L} \times F_1(A) \qquad (3)$$

$$where\ F_1(A) = \frac{A}{4}\ \frac{(\sin A - A\cos A)}{(2 - 2\cos A - A\sin A)}$$

$$M_L = \frac{2EI}{L} \times F_3(A) \qquad (4)$$

$$where\ F_3(A) = \frac{A}{2}\ \frac{(A - sin\ A)}{(2 - 2\cos A - A\sin A)}$$

For the same fixed-ended beam column, the translational stiffnesses are shown to be:

$$V_o = -\frac{12EI}{L^3} \times F_5(A) = V_L \qquad (5)$$

$$\text{where } F_5(A) = \frac{A^3 \sin A}{12(2 - 2\cos A - A\sin A)}$$

$$M_o = \frac{6EI}{L^2} \times F_4(A) \qquad (6)$$

$$where\ F_4\,(A) = \frac{A^2(1-\cos A)}{6(2-2\cos A - A\,sin\,A)}$$

$$M_{LO} = -\frac{6EI}{L^2} \times F_4(A) \qquad (7)$$

It is noted that all the stiffness coefficients derived above assume their conventional expressions if the stability functions F_j (A) =1. The actual beam and column stiffnesses are used in computing the stiffness matrix elements

L^2

SEISMIC RESPONSE

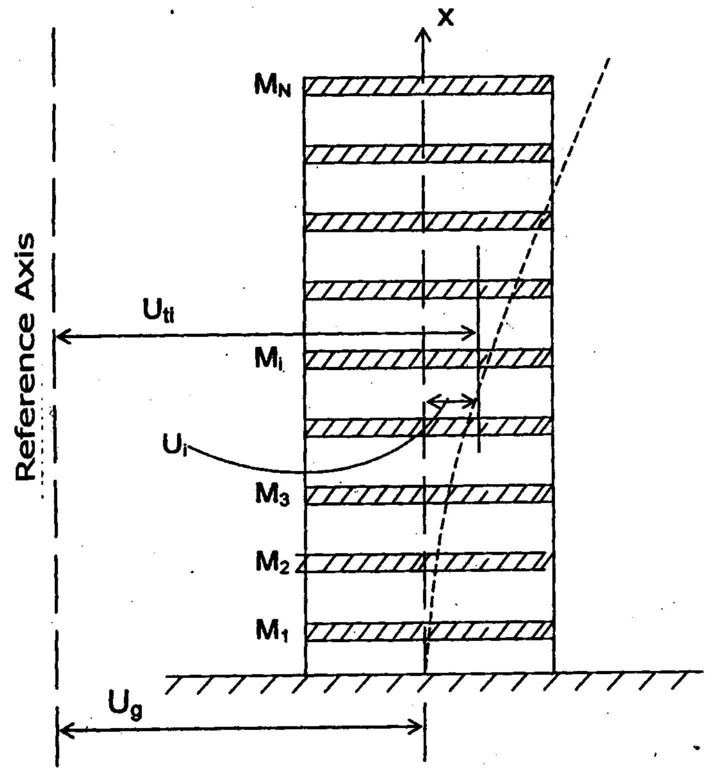


Fig. 1: **Lumped MDOF System with Rigid Base Translation**

The equation of motion of multI-storey building shown in fig. 2 can be written as [10]

$$[M]\{\ddot{U}_t\} + [C]\,\{\dot{U}\} + [K]\,\{U\} = \{0\} \ ...... \qquad (8)$$

The effective earthquake force can be derived by expressing the total displacements as the sum of the relative motions and the displacements resulting directly from the support motions.

$$\text{i.e.}\quad \{U_t\} = \{U\} + \{I\}\,U_g \qquad (9)$$

in which {I}represents a column of ones. Substituting equation (9) into (8) leads to the relative response equations of motion:

$$[M]\{\ddot{U}\} + [C]\,\{U\} + [K]\,\{U\} = \{F_{eff}\,(t) \qquad (10)$$

$$\text{in which}\ \{F_{eff}(t)\} = -[M]\,\{I\}\ddot{U}_g\,(t)$$

The transformation to normal coordinates leads to a set of N uncoupled modal equations of the form:

$$\ddot{Y}_n + 2\mathcal{E}_n\omega_n Y_n + \omega_n^2 Y_n = \frac{\{\phi_n^T\}\{F_{eff}(t)\}}{\{\phi_n^T\}[M]\{\phi_n\}} \qquad (11)$$

Y_n is the amplitude of this modal response. The generalized force resulting from the earthquake excitation (neglecting the negative sign) is given by:

$$F_n(t) = \{\phi_n^T\}\{F_{eff}(t)\} = L_n \ddot{U}_g(t) \quad (12)$$

where the earthquake participation factor is

$$L_n = \{\phi_n^T\}[M]\{I\}\text{..................} \quad (13)$$

in which {I} is a unit column vector of dimension n. The earthquake participation factor is different for each mode because it contains the mode shape ϕ_n.

Thus, in terms of earth ground motion $\ddot{U}_g^{(t)}$,

$$\ddot{Y}_n + 2\mathcal{E}_n\omega_n Y_n + \omega_n^2 Y_n = Ln \frac{\ddot{U}_g(t)}{\{\phi_n^T\}[M]\{\phi_n\}} \quad (14)$$

The response of the nth mode at any time t of the MDOF system demands the solution of this equation for Y_n. This may be done by evaluating the Duhamel integral:

$$\ddot{Y}_n(t) = L_n.V_n(t)M_n\omega_n \quad (15)$$

where M_n is the generalized mass associated with mode n given by:

$$M_n = \{\phi_n^T\}[M]\{\phi_n\}$$

V_n (t) is the modal earthquake- response integral given by:

$$V_n(t) = \int \ddot{U}_g(\tau)\, e^{-\varepsilon_n\omega_{n(t-\tau)}} \mathrm{Sin}\omega_n(t-\tau)d\tau \quad (16)$$

The relative displacement vector due to all modal responses is obtained by superposition, that is,

$$U(t) = [\phi]\{Y(t)\}\left\{\frac{L_n}{M_n\omega_n} V_n(t)\right\} \quad (17)$$

in which [ϕ] is made up of all mode shapes for which the modal response is excited significantly by the earthquake, and the term in braces represents a vector of such terms defined for each mode considered in the analysis.

The elastic forces associated with the relative displacements can be obtained directly by premutiplying by the stiffness matrix.

$$\{F_s(t)\} = [K]\{U(t)\} = [K][\phi]\{Y(t)\}$$

$$= [M][\phi]\left\{\frac{L_n\omega_n}{M_n} V_n(t)\right\} \quad (18)$$

Equation (18) is a completely general expression for the elastic forces developed in a damped structure subjected to arbitrarily varying ground motions.

NUMERICAL EXAMPLE

A computer program 'QUAKE' developed for seismic analysis of multi-storey frames possessing axially-loaded vertical and flexible horizontal members [9], is used to perform a seismic analysis of frames of 6,10, 15 and 16 storey respectively. The North-South component of the EI Centro (California) earthquake time-history is used to simulate the input ground motion, [11]

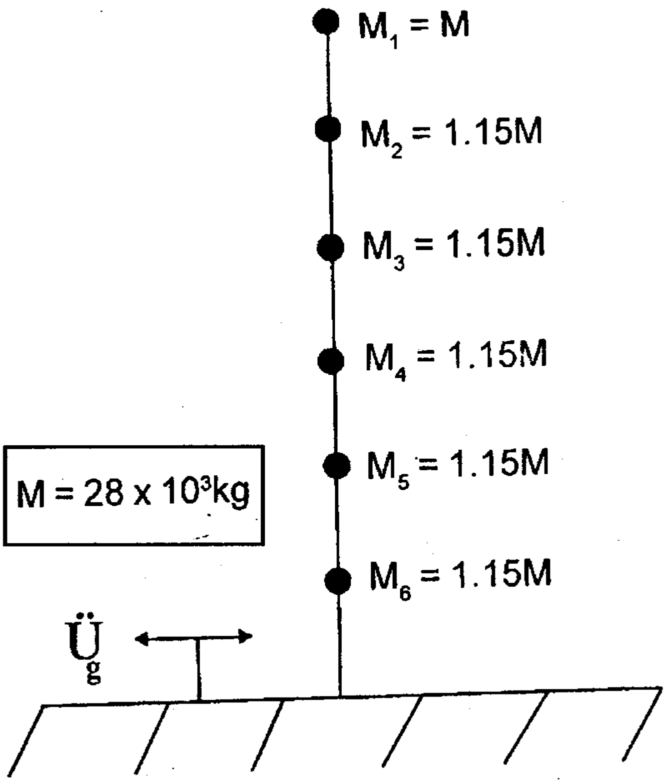


Fig. 3 A 6-DOF System subject to ground motion.

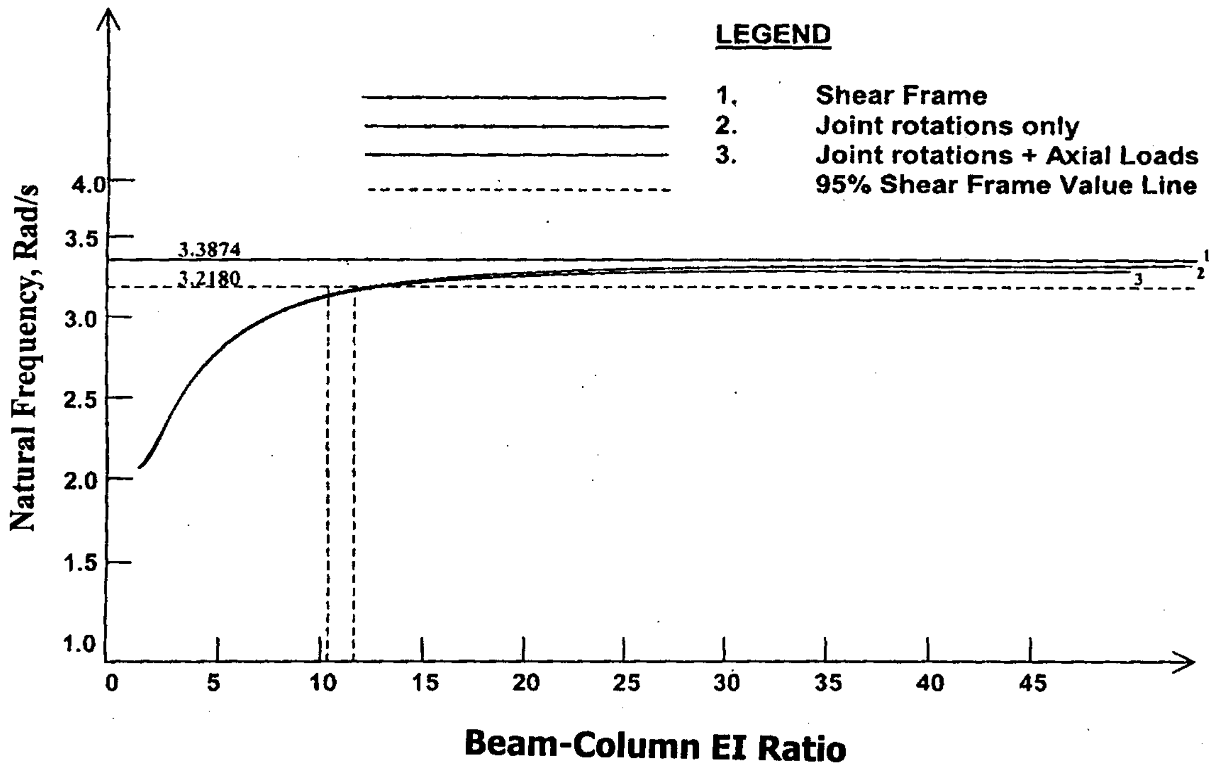
The results are show in Tables 1 and 2

Table 1 Variation of Fundamental Frequency with Number of Storeys

NO. OF STOREYS	FUNDAMENTAL FREQUENCY, RAD /S		
	SHEAR FRAME MODEL	IMPROVED MODEL (FLEXIBLE JOINTS +AXIAL LOADS)	% DIFFERENCE
3	15.3048	10.2816	32.8
6	8.1556	5.0163	37.9
10	5.0181	3.0163	39.9
12	4.2080	2.5091	40.4
15	3.3874	2.0037	40.8
16	3.1807	1.8776	41.0

Table 2 Variation of First-Storey Force with Number of Storeys

NO.OF STOREYS	FIRST-STOREY FORCE, KN		
	SHEAR FRAME MODEl	IMPROVED MODEL (Flexible Joints + Axial Loads)	% DIFFERENCE
6	104.1912	68.1142	34.6
10	60.2023	38.5401	36.0
12	53.6290	34.1698	36.3
15	43.4705	26.1383	36.9
16	36.6918	21.0243	42.7



ig4: Plot of Natural Frequency Against Beam-Column EI Ratio for the 15-Storey Frame.

DISCUSSION OF RESULT AND CONCLUSION

A 3-storey frame was first of all solved manually, [4, 11, 12] using the improved model, i.e. with joint rotations allowed and column axial loads included. The results compared accurately with those obtained using the developed earthquake computer program 'QUAKE'. This serves to confirm the validity of 'QUAKE'

From table 1, it is observed that the modal vibration frequencies of the shear frame decrease when joint rotations are permitted and axial loads included (37.9% drop in the fundamental mode for a 6- storey frame). From the plot of Fundamental Frequency against Beam- Column EI-Ratio in Fig. 4, it is observed that when the beam-column EI ratio is less than say 12, the shear frame gives results that differ by more than 5% from those obtained using the improved model, i.e. when the horizontal members are assumed to be flexible and column axial loads are included.

It can be concluded that the

improved model is more realistic and should therefore be used for seismic analysis of tall frames. As observed, the use of this model will ensure that the vibrating structure is taken further off the resonance range, thereby reducing vulnerability in the event of an earthquake. Accordingly Seismic codes should reflect the fact that the flexibility of horizontal members of tall frames and the presence of column axial loads can have significant effect on their seismic behavior.

REFERENCES

[1] CHOPRA, A.K. *Dynamics* of *structures: theory and Applications to Earthquake Engineering,* 2nd Edition, (2000) Prentice- Hall N.J.

[2] POLYAKOV, SV *Design* of *Earthquake Resistant structures,* (1985): Mir publishers, Moscow.

[3] PAZ, M. *Structural Dynamics: Theory and Computation,* 2nd ed (1985): van

[4] CLOUGH, R.W and PENZIEN,J: *Dynamics* of *Structure,* (1982) McGraw- Hill Book Company, New York.

[5] COATES, R.C COUTIE, MG. and Kong, F.K. *Structural Analysis* English Language Book Society. (1987)

[6] VERBANOV, C.P and KAPITANOV, N *Effect* of *Axial Forces on the Dynamics Characteristics* of *Multi- Storey Buildings,* Annuire De L' institute D' Architure et DE Genie civil, Sofia, vol. XXVIII, NO.3, (1980) pp. 7-14.

[7] OSADEDE, N.N.); *An Improved Performance* of *Multi-storey Frames with Flexible Horizontal Members,* ***Technical Transactions of Nigerian Society of Engineers*** Vol 33, (1990), pp.30-4.

[8] OSADEDE, N.N *Dynamics Analysis of Mass Loaded Axially Compressed Clamped Timoshenko Beam,* ***Proceedings, Second International Conference on Structural Engineering Analysis and Modeling (SEAN 2) KUMAS****,* VOL 1, (1998) pp 237-247.

[9] ONYIA, M.E. (2005) *Seismic Analysis* of Multi-*Storey Frames Possessing Axially-loaded Vertical and Flexible Horizontal Members,* Unpunished Ph. D Thesis, University of Nigeria, Nsukka, Nigeria.

[10] VERBANOV, C. P *Stability and Dynamism* of *Elastic Systems,* (1989) Technika Press, Sofia 3rd ed.

[11] HART, G.C and WONG, K. *Structural Dynamics for Structural Engineers, (2000)* John Wiley & Sons, New York.

[12] KRISHNA,J. CHANDRASEKARAN A.R and CHANDRA, B. *Elements* of *Earthquake Engineering* 2nd (1994): ed. South Asian Publishers PVT. LTD New Delhi