AN OPTIMIZATION MODEL DEVELOPMENT FOR LATERIZED-CONCRETE MIX PROPORTIONING IN BUILDING CONSTRUCTIONS

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ABSTRACT

Lateritic soil is abundant in most construction sites in Nigeria. The potentials of this indigenous material are still being sourced. The effective utilization of this material as a component in concrete depends on the mix proportioning of the various components of laterized concrete. In this study, a mathematical model was developed and was used to optimize the mix proportion that will produce the maximum strength of laterized concrete using Scheffe's simplex lattice approach.

The model formulated compares favourably with the experimental data. It also satisfies the Student's t and χ^2-tests. The optimum value of strength predicted by this model is 27.151 N/mm^2 corresponding to a mix ratio of 1:1:2 of cement, laterized oil and gravel respectively at a water-cement ratio of 0.650.

1. INTRODUCTION

In many parts of the country, Nigeria sand which is the second-largest of the concrete volume is a limited resource, and in some areas, it is completely unavailable. Efforts are therefore being directed towards sourcing out material that can partly or completely replace sand in concrete production. According to Osunde [1], sand is the second most costly item per unit volume of concrete produced. Thus, construction cost can be reduced by replacing sand with more readily available and cheap material, such as lateritic soils. Lateritic soils are abundant in most construction sites, particularly areas where there is no sand deposit in Nigeria.

Studies have been made by some researches [2], [3], [4]; to determine the usefulness of lateritic soils in the building and allied industries. In the recent years, lateritic soils have been introduced as a component in concrete, replacing sand as the fine aggregate [5]. It is however, well understood that the strength characteristics of concrete is a function of the proportion of the various constituent materials. One of the major problems that still remain is to optimize the mix ratio that will produce maximum strength of laterized concrete. This will enable the correct mix to be adopted for jobs that require high strength laterized concrete or as case may be.

The objective of this work is specifically to optimize the mix ratio of the constituent materials that will result in maximum strength by developing a mathematical model which relates the strength of laterized concrete and its component ratios.

2.0 MODEL DEVELOPMENT

Simplex lattice design proposed by Scheffe (1958) was used to formulate a mathematical model which relates compressive strength of laterized concrete and its component ratios of cement, laterite, gravel and water cement ratio.

2.1 SIMPLEX LATTICE DESIGN

In mixture experiment involving the study of properties of a q- component mixture which are dependent on the component ratio only, the factor space is a regular, (q – 1)-simplex. The relationship that holds for the component of the mixture is given as:

$$\sum_{i=1}^{q} X_i = 1 \text{...}(1)$$

where:

$X_i \geq 0$ is the component Concentration

q is the number of components

Therefore, for a 4-comonent mixture the sum of all the proportions of the components must be unity. That means that:

$$X_1 + X_2 + X_3 + X_4 = 1 \text{……..}(2)$$

where in this case:

X_1 is proportion of cement

X_2 is proportion of sand

X_3 proportion of gravel

X_4 is proportion of water-cement ratio

For quaternary system, q = 4, the regular simplex is a tetrahedron.

where each vertex represents a straight component, an edge represents a binary system, and a face a ternary one. Points inside the tetrahedron correspond to quaternary systems (see Fig. 1). Each point in the tetrahedron therefore represents a certain composition of the quaternary system.

The component X_1 is therefore, absent in the face X_2, X_3, X_4, but as tetrahedron sections parallel to the face approach vertex X_1, component X_1 in them grows in concentration.

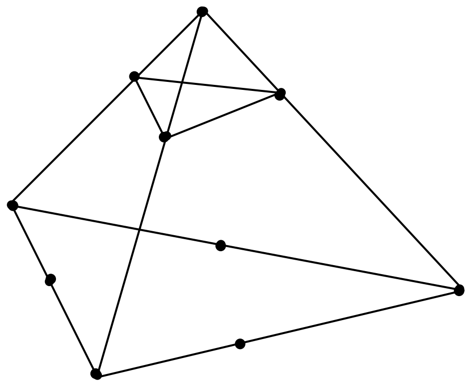

Fig. 1: Tetrahedron and representative Points

Scheffe [7] showed that the response function (property) in multi-component system can be approximated by a polynomial. To describe such function adequately, high degree polynomials are required and hence a great many experimental trials. According to Scheffe [7], a polynomial of degree n in q variable has C^n_{q+n} coefficients and is in the form:

$$\hat{y} = b_0 + \sum_{1\leq i\leq q} b_i X_i + \sum_{1\leq i\leq j\leq q} b_{ij} X_i X_j + \sum_{1\leq i\leq j\leq k\leq q} b_{ijk} X_i X_j X_k + \ldots + \sum b i_1 i_2 \ldots, i_n x i_1 x i_2 x i_n \quad \text{….}(3)$$

The relationship given in equation (1) enables the equation component to be eliminated and the number of coefficients reduced to C^n_{q+n-1}. But it is required that all the q components be introduced into the model.

Scheffe [7] suggested to describe mixture properties by reduced polynomials from equation (3) subject to the normalization condition of equation (1) for a sum of independent variables. The reduced second-degree polynomial for quaternary system is derived as follows:

$$\hat{Y} = b_0 + b_1X_1 + b_2X_2 + b_3X_3 + b_4X_4 + b_{12}X_1X_2 + b_{13}X_1X_3 + b_{14}X_1X_4 + b_{23}X_2X_4 + b_{24}X_2X_4 + b_{34}X_3X_4 + b_{11}X_1^2 + b_{22}X_2^2 + b_{33}X_3^2 + b_{44}X_4^2 \quad \ldots\ldots\ldots\ldots(4)$$

Since $X_1 + X_2 + X_3 + X_4 = 1$ ……….(5)

Then

$$b_0X_1 + b_0X_2 + b_0X_3 + b_0X_4 = b_0 \quad \ldots\ldots (6)$$

Multiplying Equation (4) by X_1, X_2, X_3, and X_4 in succession gives:

$$\left.\begin{aligned} X_1^2 &= X_1 - X_1X_2 - X_1X_3 - X_1X_4 \\ X_2^2 &= X_2 - X_1X_2 - X_2X_3 - X_2X_4 \\ X_3^2 &= X_3 - X_1X_3 - X_2X_3 - X_3X_4 \\ X_4^2 &= X_4 - X_1X_4 - X_2X_4 - X_3X_4 \end{aligned}\right\} \quad ..\ (7)$$

Substituting equation (6) and equation (7) into equation (4) and transforming it gives:

$$\hat{Y} = (b_0 + b_1 + b_{11})X_1 + (b_0 + b_2 + b_{22})X_2 + (b_0 + b_3 + b_{33})X_3 + (b_0 + b_4 + b_{44})X_4 + (b_{12} - b_{11} - b_{22})X_1X_2 + (b_{13} - b_{11} - b_{33})X_1X_3 + (b_{14} - b_{11} - b_{44})X_1X_4 + (b_{23} - b_{22} - b_{33})X_2X_3 + (b_{24} - b_{22} - b_{44})X_2X_4 + (b_{34} - b_{33} - b_{44})X_3X_4 \quad \ldots\ldots\ldots(8)$$

Denoting:

$$\beta_i = b_0 + b_i + b_{ii}\ ;\ \beta_{ij} = b_{ij} - b_{ii} - b_{jj} \quad \ldots..(9)$$

The reduced second-degree polynomial in four variables is then arrived at as:

$$\hat{Y} = \beta_1X_1 + \beta_2X_2 + \beta_3X_3 + \beta_4X_4 + \beta_{12}X_1X_2 + \beta_{13}X_1X_3 + \beta_{14}X_1X_4 + \beta_{23}X_2X_3 + \beta_{24}X_2X_4 + \beta_{34}X_3X_4 \quad \ldots\ldots\ldots\ldots\ldots (10)$$

The solution of equation (9) as given by Scheffe [7] for the coefficients of the polynomial is:

$$\beta_i = Y_i \text{ and } \beta_{ij} = 4Y_{ij} - 2Y_i - 2Y_j \quad \ldots\ldots(11)$$

where;

$\beta_i = \beta_1, \beta_2, \beta_3, \ldots, \beta_4$

$\beta_{ij} = \beta_{12}, \beta_{13}, \beta_{14}, \ldots, \beta_{34}$

Y_i and Y_{ij} = response (property)

Equation (10) is the governing equation.

Scheffe's simplex lattice designs provide a uniform scatter of points over the a (q -1)-simplex. The points form a (q -1)-lattice on the simplex where q is the number of mixture components, 'n' is the degree of polynomial. Scheffe [7] showed that for each component, there exist (n + 1) similar levels, $X_i = 0, \frac{1}{n}, \frac{2}{n}, \ldots, 1$ and all possible mixtures are derived with such values of component concentration. So for (4, 2)-lattice the proportion of every component that must be used are 0, ½ and 1. He also showed that the number of points in (q, n) lattice is given as :

$$\frac{q(q+1)\ldots(q+n-1)}{n!} \quad \ldots\ldots\ldots\ldots(12)$$

where n is a digit number. This implies that for a (4, 2) lattice, the number of points (coefficients)

$$\frac{4(4+1)}{2\times1} = 10$$

The (4, 2) lattice is shown in Fig. 2

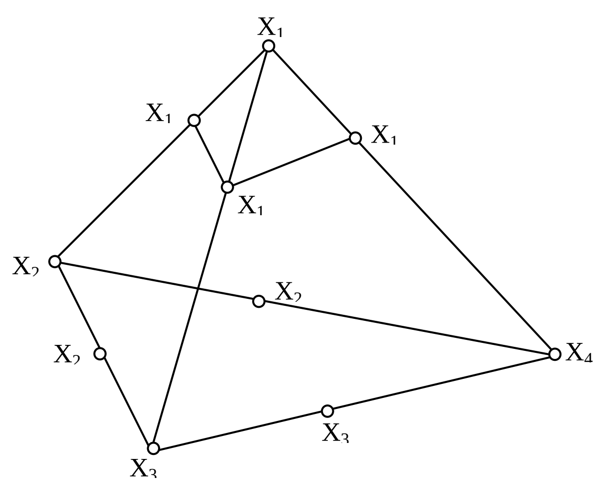

Fig. 2: The (4, 2) - Lattice

2.2 MIX DESIGN

In this design, the relationship that holds for the components of the mixture as given in equation (1) above was transformed to establish the actual component concentration. The transformed proportions X_i(i = 1 – 4) for each experimental points are called "pseudo components". For actual component Z_i the pseudo components X is given by X = BZ

where B is the inverse of Z matrix. Similarly, the inverse transformation from pseudo components to Z_i (actual components) is expressed as

Z = AX

where A is the inverse transformation matrix.

The actual components for the first four points are chosen arbitrarily for the tetrahedron vertices (see Fig. 3).

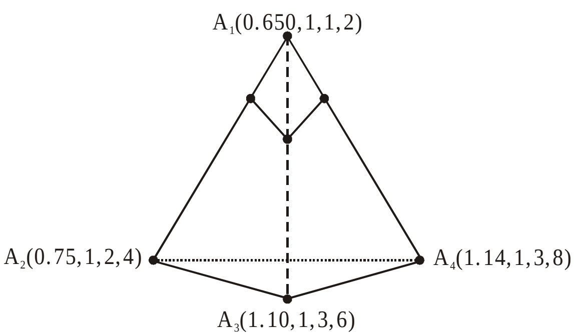

Fig. 3: Tetrahedron vertices for (4, 2) lattice

The inverse transformation matrix 'A' is obtained since the Z_i (actual components) values and X_i (pseudo component) values are known. Thus for any pseudo component, the actual component is given by:

$$\begin{bmatrix} Z_1 \\ Z_2 \\ Z_3 \\ Z_4 \end{bmatrix} = \begin{bmatrix} 0.650 & 0.750 & 1.10 & 1.14 \\ 1 & 1 & 1 & 1 \\ 1 & 2 & 3 & 3 \\ 2 & 4 & 6 & 8 \end{bmatrix} = \begin{bmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \end{bmatrix} \quad (13)$$

This is employed to determine the actual components for all the experimental and control points. The ten control points were chosen such that they could be incorporated in the new design e.g. The (4, 3) lattice should the (4, 2) lattice not fit adequately, thus the model can be refined.

The pseudo components (X_i) and the actual co,ponents (Z_i) for the ten experimental and ten control points are shown in Tables 1 and 2.

3. MATERIALS AND METHOD

The lateritic soil for this investigation was collected from a borrow-pit at Nsukka in Enugu State, Nigeria. The coarse aggregate used was crushed granite of igneous origin with size range of 9 – 14mm. Ordinary Portland cement the properties which conformed to the British Standard BS [8] part 2 of 1970 was used in this study while water was drawn from the nearest clean water source.

3.1 Sample Preparation

The lateritic soil was sieved so as to exclude the clay content as well as the coarse aggregate contents of lateritic soils. The size range of the sample needed was obtained by passing the sample through the upper sieve with size opening of 4.75mm and retained above the lower sieve of size opening, 0.3 mm. The crushed granite with almost uniform size of 12 mm was prepared to BS [9].

3.2 Batching and Mixing of Specimens

Batching was by weight using Avery weighing scale. Different mixtures of cement, lateritic soil and gravel, were prepared and 'worked' manually using shovel to stir. The working process involved the gradual addition of predetermined quantity of water to the mixtures already made and the continuous stirring with a shovel until a workable mix was obtained.

Table 1: Actual (Z_i) and Pseudo (X_i) Components for the ten Experimental Points of (4, 2) lattice

N	X_1	X_2	X_3	X4	Response	Z_1	Z_2	Z_3	Z_4
1	1	0	0	0	Y_1	0.65	1	1	2
2	0	1	0	0	Y_2	0.75	1	2	4
3	0	0	1	0	Y_3	1.10	1	3	6
4	0	0	0	1	Y_4	1.14	1	3	8
5	0.5	0.5	0	0	Y_{12}	0.70	1	1.5	3
6	0.5	0	0.5	0	Y_{13}	0.875	1	2	4
7	0.5	0	0	0.5	Y_{14}	0.895	1	2	5
8	0	0.5	0.5	0	Y_{23}	0.925	1	2.5	5
9	0	0.5	0	0.5	Y_{24}	0.945	1	2.5	6
10	0	0	0.5	0.5	Y_{34}	1.12	1	3	7

Table 2: Actual (Z_i) and Pseudo (X_i) Components for the ten Control Points of (4, 2) lattice

N	X_1	X_2	X_3	X_4	C_{exp}	Z_1	Z_2	Z_3	Z_4
1	0.75	0.25	0	0	C_1	0.675	1	1.25	2.5
2	0	0.75	0	0.25	C_2	0.848	1	2.25	5
3	0.25	0	0.75	0	C_3	0.988	1	2.5	5
4	0	0.25	0.75	0	C_4	1.013	1	2.75	5.5
5	0.5	0.25	0.25	0	C_5	0.788	1	1.75	3.5
6	0.5	0	0.25	0.25	C_6	0.885	1	2	4.5
7	0.25	0	0.5	0.25	C_7	0.998	1	2.5	5.5
8	0	0.25	0.5	0.25	C_8	1.023	1	2.75	6
9	0.25	0	0.25	0.5	C_9	0.998	1	2.5	6
10	0.25	0.25	0.25	0.25	C_{10}	0.910	1	2.25	5

3.3 Compressive Strength Test

Cube specimens of size 150 mm x 150 mm were made and tested for compressive strength. The preparation of the test cubes was done in accordance with BS [10]. Each specimen was made by filling each mould in three layers and compacting manually with 25 mm diameter rod. On each layer, 35 strokes were delivered. Demoulding was performed in accordance with BS [10]. The specimens after being demoulded were submerged in a water or curing tank. The curing was in accordance with BS [10]. The cubes specimens were cured for 28 days. The cubes were weighed and then subjected to crushing using compression testing machine. The testing was in accordance with the specification of BS [10]. The load was applied without shock at a loading rate of 15 N/mm^2.min until no greater load could be sustained. The maximum load applied at crushing was recorded. Two replicates of each of the mixture composition were made. Therefore, for the ten experimental points and ten control points, a total of 40 cubes were tested. The compressive strength (response) of laterized concrete was estimated from the formular given as:

$$\frac{\text{Maximum Load}}{\text{Cross Sectional Area}} \; N/mm^2 \qquad \text{…..(14)}$$

4.0 RESULTS AND DISCUSSION

Table 3 shows the results of two parallel observations each, of the 10 design points and the 10 test points of the (4, 2) lattice. The comprehensive strength (response, Y) of each cube was obtained from equation 14 above.

4.1 The Regression Equation

From equation (11) and Table 3 the coefficients of the second degree polynomial equation are determined as follows:

$\beta_1 = 27$, $\beta_2 = 18.4$, $\beta_3 = 16.6$, $\beta_4 = 7.3$,

$\beta_{12} = 11.2$, $\beta_{13} = -16$, $\beta_{14} = 1.4$, $\beta_{23} = -2$,

$\beta_{24} = 16.2$, $\beta_{34} = -10.6$

Thus, from equation (10)

$$\hat{Y} = 27X_1 + 18.4X_2 + 16.6X_3 + 7.3X_4 + 11.2X_1X_2 - 16X_1X_3 + 1.4X_1X_4 - 2X_2X_3 + 16.2X_2X_4 - 10.6X_3X_4 \qquad (15)$$

Equation (15) is the regression equation for the compressive strength of laterized concrete, as obtained in this study.

Table 3: Compressive Strength Test Results of the (4, 2) Simplex Lattice

Expt. No (N)	Repli-Cation	Response (Y_1) N/mm^2	Response symbol	$\sum_{i=1}^{n} Y_i$	$\bar{Y} = \frac{(\sum_{i=1}^{n} Y_i)}{n}$
1	1 2	26.1 28.0	Y_1	54.1	27
2	1 2	18.6 18.2	Y_2	36.8	18.4
3	1 2	16.4 16.8	Y_3	33.2	16.6
4	1 2	6.6 8.0	Y_4	14.6	7.3
5	1 2	25.7 25.2	Y_{12}	50.9	25.5
6	1 2	18.0 17.6	Y_{13}	35.6	17.8
7	1 2	17.5 17.5	Y_{14}	35	17.5
8	1 2	16.9 17.2	Y_{23}	34.1	17
9	1 2	16.8 17.0	Y_{24}	33.8	16.9
10	1 2	8.8 9.7	Y_{34}	18.5	9.3
11	1 2	25.7 27.4	Yc_1	53.1	26.6
12	1 2	17.4 17.2	Yc_2	34.6	17.3
13	1 2	16.8 17.2	Yc_3	34	17
14	1 2	16.5 16.8	Yc_4	33.3	16.7
15	1 2	23.0 23.5	Yc_5	46.5	23.3
16	1 2	18.1 17.1	Yc_6	35.2	17.6
17	1 2	15.0 15.7	Yc_7	30.7	15.4
18	1 2	17.0 15.0	Yc_8	32.0	16.0
19	1 2	14.6 12.8	Yc_9	27.4	13.7
20	1 2	15.7 18.4	Yc_{10}	34.1	17.1

4.2 Tests of Adequacy of the Regression Model

The model was statistically analysed using student- t and χ^2- test. The adequacy of the models was tested against the experimental results of the control points. The t-table (= 3.15) is far greater than t-calculated in all the ten test points (see Table 4). Chi-square table (= 6.0) was also greater than chi-square calculated (0.896). The model is found adequate in both student-t and χ^2- tests.

Table 4: t-Statistics for the Test Points

N	Control points	$Y_{observed}$	$Y_{expt.}$	ΔY	t
1	C1	26.6	26.95	0.35	0.41
2	C2	17.3	18.7	1.4	1.7
3	C3	17	16.2	0.8	1.03
4	C4	16.7	16.6	0.1	0.13
5	C5	23.3	21.5	1.8	2.4
6	C6	17.6	17.0	0.6	0.81
7	C7	15.4	13.6	1.8	2.4
8	C8	16.0	14.2	1.8	2.4
9	C9	13.7	12.4	1.3	1.7
10	C10	17.1	17.3	0.2	0.28

4.3 Program Testing and Test Results

The optimization was achieved by a computer code written in Q-Basic. Fig. 4 shows a flow chart that was developed for the computation of laterized concrete mix proportions corresponding to a desired strength. The optimization of strength using this model, gives a model predicted optimum value of strength of 27.151 N/mm^2. This corresponds to the optimum mix proportions of 1:1:2 by weight of cement, laterite and gravel respectively, at a water-cement ratio of 0.650. Relevant data that are synthesized from the raw printed matching combinations for each desired strength are shown in Table 5.

5.0 CONCLUSION

The experimental data are very well fitted to the regression model, which satisfies the student-t and Chi-square tests. The model parameters estimated are therefore acceptable. The implication of this study is that there is distinction in the compressive strength property of laterized concrete resulting from the variations in the mix proportions of the constituent materials. Thus, for a job where a high strength concrete is specified, particularly in the construction of heavy loaded storage structures such as water reservoirs, silos etc, the data provided for the optimum mix proportion in this study can therefore be adopted in the analysis and design of such structures.

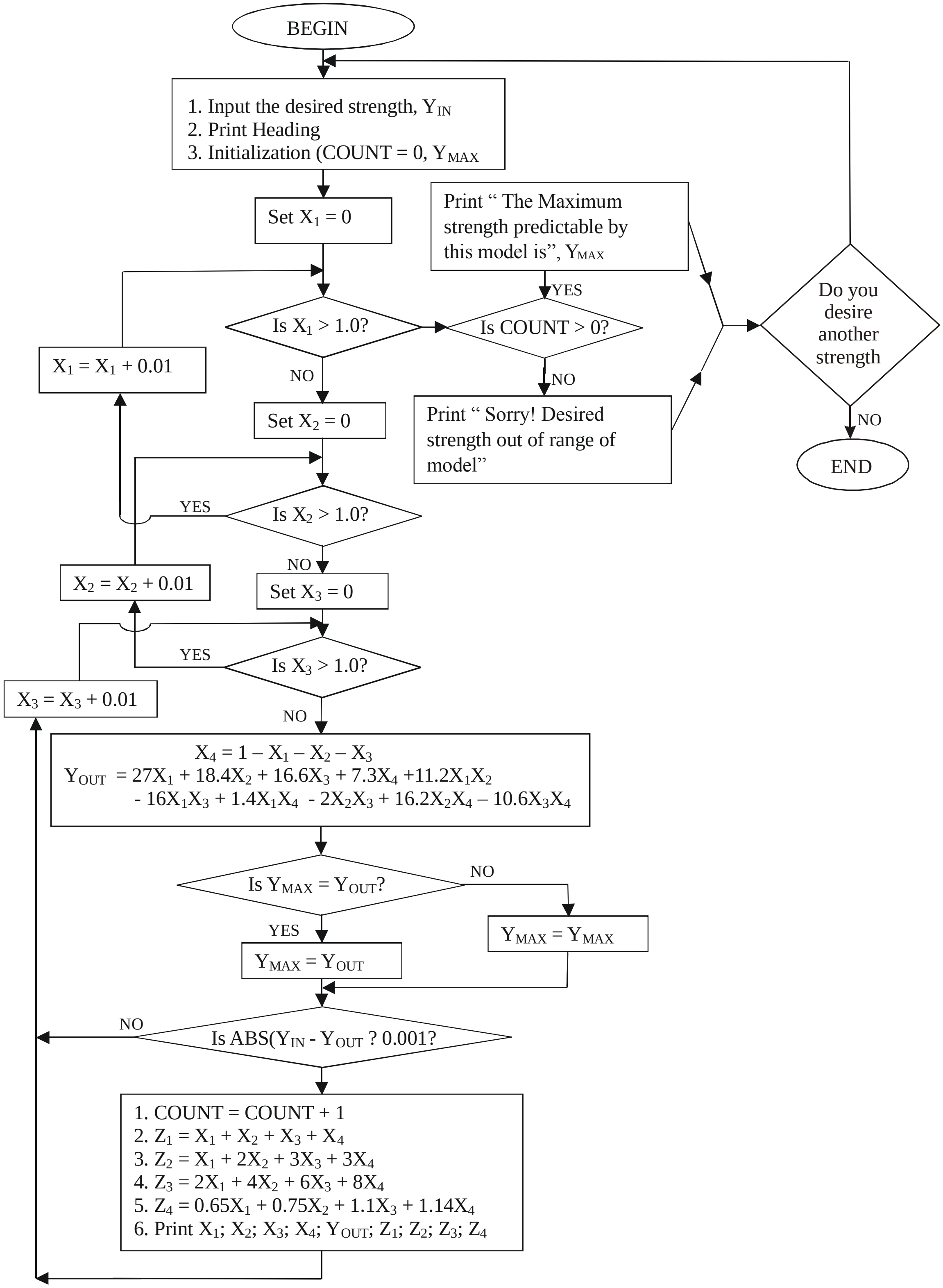

Fig. 4: Flow Chart for Computation of Laterized Concrete Mix Proportion Corresponding to a Desired Strength

Table 5: Program Test Results

Desired Strength N/mm^2	Classification	Mix Proportions(kg)			
		Cement	laterite	gravel	water-cement ratio
13	Low strength	1	2.860	6.280	1.062
14	,,	1	2.820	5.920	1.065
15	,,	1	2.910	5.980	1.082
16	,,	1	2.710	5.540	1.008
17	Moderate strength	1	2.440	4.880	0.956
18	,,	1	2.320	5.600	0.921
19	,,	1	2.180	5.200	0.892
20	Very moderate strength	1	2.030	4.640	0.855
21	,,	1	1.940	4.480	0.836
22	,,	1	1.850	4.140	0.806
23	,,	1	1.720	3.780	0.779
24	,,	1	1.640	3.500	0.751
25	High strength concrete	1	1.400	2.980	0.721
26	,,	1	1.360	2.760	0.697
27	,,	1	1.000	2.000	0.650

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