A COMPARATIVE ANALYSIS OF THE EFFECTIVE ELASTIC CONSTANTS OF LAMINATED COMPOSITE PLATES OBTAINED BY VARIOUS CLOSED-FORM AND SERIES SOLUTIONS

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ABSTRACT:

Results of various finite element and closed form models developed in the attempt to evaluate and establish accurate values of the Young's modulus, E; the shear modulus G; and the Poisson's ratio,< for laminated composite plates having soft matrix and high fibre volume fraction are discussed in this paper. Their merits and limitations highlighted. The Finite Element Energy Method (FEEM) as a tool for the prediction of effective elastic constants for Flexible Matrix composites is proffered here as a model having no better alternative yet.

Keywords: Flexible-matrix-composites Effective-elastic-constants; Finite-element-energy-method

INTRODUCTION:

There are several closed form solutions that exist for the determination of effective elastic constants of a uniaxial composite layer. The works of Hashin, chamis and sendecky, [1,2] discuss several such solutions. However, it has been discovered that different methods give different results for some of the elastic constants. Apart from the two elastic constants, E_L and v_{TL} which seem to agree for all methods, experimental results grossly indicate that the rest of the elastic constants hardly ever agree as obtained from any one method with another, hence the need to develop a reliable closed form solution for this purpose [3].

Table 1: Effective elastic constants by different methods

E_f = 2.075 x 10^5 N/mm^2, E_m = 68.98N/mm^2, v_f = 0.3, v_m = 0.49444, V_f = 0.5

Method	E_L N/mm^2 (x10^4)	E_T N/mm^2	v_{TL}	V_{TT}	G_{LT} N/mm^2	G_{TT} N/mm^2
Rule of mixtures	10.35	138.0	0.39722	----	46.15	----
Halpin-Tsai	10.35	275.9	0.39722	----	69.20	----
Ekvall	10.35	195.2	0.39722	----	63.47	----
Greszezuk	10.35	293.2	0.39722	----	1759.0	----
Whitney and	10.35	1021.0	0.39668	0.87210*	69.20	----
Riley		63.7		0.92016*		
	10.35	1690.0	0.39722	0.63900	72.92	----
Foye	10.35	270.4	0.39722	----	69.20	68.70
Hahn	10.35	270.4	0.39668	0.87210	69.20	68.70
Hashin (LB)	10.37	1093.0	0.39668	0.99657	26560.0	275.3
Hashin (UB)						

Notes: {1}* Assumed V_{TT} For which E_T are given
{2} LB = Lower Bound, UB = Upper Bound

Symbols/Notations:

C = elastic stiffness of the composite
e = modified strains
E_f = Young's modulus of the fibre material
Em = Young's modulus of the matrix material
E_L and E_T = Longitudinal and transverse Young's modulus of the composite
G_{LT} and G_{TT} = Longitudinal and transverse shear moduli of the composite
U = Strain energy of the model
u = applied displacements
V_f = fibre volume fraction
V_{TL} and V_{TT} = Longitudinal and transverse Poisson's ratios
v_f = fibre Poisson's ratio
v_m = matrix Poisson"s ratio
, = strains
(= engineering shear strains.

The tendency for disparities to appear in the results of these methods become even more prominent in composites consisting of very stiff fibre with very soft matrix such as urethane. Here, the fibre volume fraction is usually very high. Such a composite material is usually employed in the manufacture of bearingless rotor systems and drive shafts [4,5]

Table 1 shows the results of elastic constants obtained from some existing closed form solutions working with flexible matrix composites.

Table 2: Effective elastic constants by different methods

E_f = 2.075×10^5 N/mm^2, E_m = 7.6N/mm^2, v_f = 0.3, v_m = 0.49908, V_f = 0.74

Method	E_L N/mm^2 (x10^4)	E_T N/mm^2	v_{TL}	V_{TT}	G_{LT} N/mm^2	G_{TT} N/mm^2
Rule of mixtures	15.31	29.18	0.3518	----	9.73	----
Halpin-Tsai	15.31	72.36	0.3518	----	16.97	----
Ekvall	15.31	72.78	0.3518	----	43.74	----
Greszezuk	15.31	227.40	0.3518	----	6623.0	----
Whitney and Riley	15.31	1518.0	0.3517	0.8580*	16.97	----
Foye	15.31	1725.0	0.3518	0.4811	38.98	----
Hahn	15.31	67.41	0.3518	----	16.94	16.90
Hashin (LB)	15.31	67.41	0.3516	0.8577	16.94	16.90
Hashin (UB)	15.31	1559.0	0.3517	0.9995	46700.0	391.50

Notes: {1}* Assumed V_{TT} For which E_T are given
{2} LB = Lower Bound, UB = Upper Bound

We note here, as we stated before, that all the methods give the same values for E_L and v_{TL}, indicating that even the Rule of Mixtures is quite appropriate for these properties. However, on the contrary, the shear moduli, G_{LT} and the transverse properties G_{TT} differ quite significantly.

For a composite made of <u>much softer matrix</u> and <u>much higher fibre volume fraction,</u> comparative results (see table 2 please) show that the differences in the values obtained by the different methods <u>are much higher than those given in table 1</u>. We also note the large difference between the Hashin lower and upper bounds of G_{LT}, G_{TT} and E_T.

In their various methods, Halpin Tsai [6,7] used some empirical factors for E_T and G_{LT} just as did Ekvall [8,9]. Greszczuk [10] modified the matrix moduli by assuming that the matrix is under plane stress in one of the transverse directions for its elastic modulus and that it is restrained in both the transverse directions for its shear modulus. Riley and Whitney [11] assumed values for V_{TT} in their work to obtain E_T whereas Hashin [12] developed equations for these constants by employing minimum potential energy, giving upper bounds and minimum complimentary energy giving low bounds on a model consisting of a cylindrical fibre enclosed by a cylindrical matrix {fig1}. He proposed that for stiff fibre the upper bounds of E_T and G_{TT} can be used but for fibre <u>much</u> stiffer than the matrix [13], the lower bounds should be used. In his investigations, Hahn [14] employed Hashins lower bounds to obtain his values whereas Foye [15] used, instead, a finite element method and obtained stresses for applied strains, which, in turn were used with the other orthotropic stress-strain relations to determine the elastic constants. Next, he compared his results with the existing solutions and selected Whitney and Riley solutions for E_T .The big limitation in his approach is that the stress distribution on a finite element model would be found *non-uniform* for composites made of very stiff fibres and very soft matrix if they possess very high fibre volume fraction.

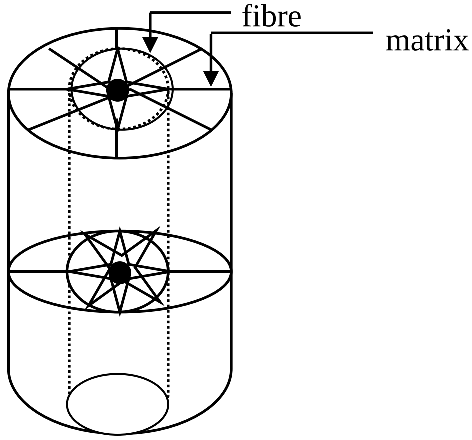

Fig 1: Minimum
Potential and minimum complimentary energy approach

So far, we see the great difficulty involved in trying to select one single method to predict the properties of soft matrix composites.

Purpose of the work:

The purpose of this work is to obtain by the interaction of the works of Hashin and Rosen; Iwona, Lee and Middya et al [16-20] and also those of Gupta [21], a single model that would effectively predict, <u>to a very reasonable extent</u>, all the elastic constants in a composite with very soft matrix. This model is found in the Finite Element Energy Method (FEEM).

THE FINITE ELEMENT ENERGY METHOD (FEEM)

Hashin and Rosen in the development of their equations employed a cylindrical fibre enclosed by a cylindrical matrix (see fig 1) and by applying the minimum potential and minimum complimentary energy approaches arrived at the values of the elastic constants found in tables 1 and 2, Gupta, improving on the works of Hashin and Rosen suggested the introduction of one Representative Volume Element (RVE) for this analysis. He applied to it boundary conditions subjected to controlled displacements (see fig 2).

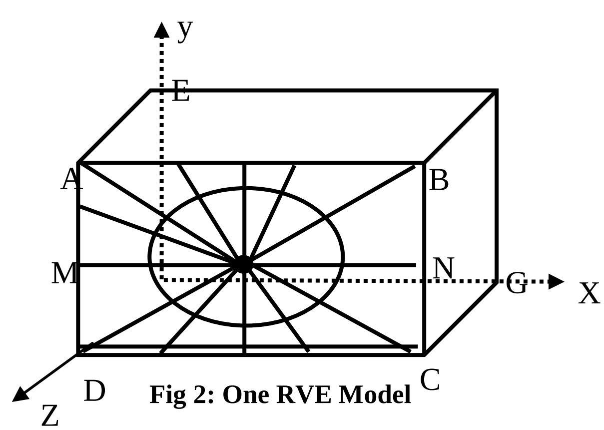

Fig 2: One RVE Model

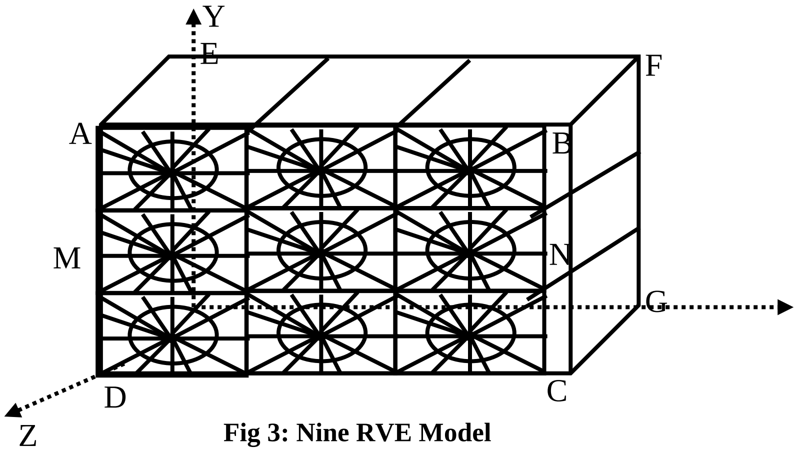

Fig 3: Nine RVE Model

By employing Nine Representative Volume Element, instead, (Fig 3) as a cross-check to the results of one Representative Volume Element given, the strain energy of the model is obtained as follows:

$$U = \frac{1}{2} C_{ijkl}\,\varepsilon_{ij}\,\varepsilon_{kl}, \qquad (1)$$

with boundary conditions:

$$U_i = \varepsilon_{ij}\; X_j \qquad (2)$$

Equation (2) produces the following strain relations:

$$\varepsilon_{11} = \breve{u}_I / X_I,\ \varepsilon_{22} = \breve{u}_2 / X_2,$$

and $\varepsilon_{33} = \breve{u}_3 / X_3$ (3)

when only normal strains are expected to exist and,

$$\Upsilon_{12} = \breve{u}_I / X_2,\ \breve{u}_2 = 0, \text{ etc} \qquad (4)$$

when only one shear strain is to exist.

STRAIN-STIFFNESS-ENERGY RELATIONS:

Equation (1) can be re-written in a more convenient form:

$$U = \frac{1}{2} C_{ij}\,\varepsilon_i\,\varepsilon_j,\ I, j = 1, 2, \ldots, 6 \qquad (5)$$

Here ε_1, ε_2, and ε_3 are normal strains and ε_4, ε_5 and ε_6 are shear strains.

Normal Stiffness: For all shear strains equal to zero, we get by expanding equation (5),

$$U = \frac{1}{2} C_{11}\varepsilon_1^2 + C_{22}\varepsilon_2^2 + C_{33}$$

$$\varepsilon_3^2 + 2C_{12}\varepsilon_1\varepsilon_2 + 2C_{13}\varepsilon_1\varepsilon_3 + 2C_{23}\varepsilon_2\varepsilon_3 \qquad (6)$$

Let us further simplify the notations by denoting,

$\frac{1}{2} C_{II} = C_I$, $\frac{1}{2} C_{22} = C_2$, $\frac{1}{2} C_{33} = C_{3,}$

$C_{12} = C_4$, $C_{13} = C_5$, $C_{23} = C_6$, and

$\varepsilon_1^2 = \mathbf{e}_1$, $\varepsilon_2^2 = \mathbf{e}_2$, $\varepsilon_3^2 = \mathbf{e}_3$, $\varepsilon_1\ \varepsilon_2 = \mathbf{e}_4$,

$\varepsilon_1\ \varepsilon_3 = \mathbf{e}_5$, $\varepsilon_2\ \varepsilon_3 = \mathbf{e}_6$ (7)

Equation (6) then becomes,

$$U = C_i\ \mathbf{e}_i,\ i = 1,2, \ldots, 6 \qquad (8)$$

Here $\mathbf{e}_i$ are applied modified strains, U is the resulting strain energy, and C_i are to be determined.

If we denote $\mathbf{e}_{pi}$ = Prescribed modified strains for the displacement boundary condition, p, where p = 1, 2, …, 6, and U_p= resulting strain energy due to $\mathbf{e}_{pi.,}$ equation (8) then becomes:

$$U_p = \mathbf{e}_{pi}\ \ C_i \qquad (9)$$

The following six sets of displacements boundary conditions to produce desired normal strains and energies are prescribed ($\varepsilon_1 = \varepsilon_{x,}$ $\varepsilon_2 = \varepsilon_y$ $\varepsilon_3 = \varepsilon_z$):

p = 1: $\epsilon_1 = \epsilon_{x,} \epsilon_2 = \epsilon_3 = 0$, giving $\mathbf{e}_{II} = \epsilon^2_{x,}$

and all other $\mathbf{e}_{Ii} = 0$,

p = 2: $\epsilon_2 = \epsilon_{y,} \epsilon_3 = \epsilon_1 = 0$, giving $\mathbf{e}_{22} = \epsilon^2_{y,}$

and all other e_{2i} =0,

p = 3: $\epsilon_3 = \epsilon_z, \epsilon_1 = \epsilon_2 = 0$, giving $e_{33} = \epsilon^2_z$,

and all other $e_{3i} = 0$,

p = 4: $\epsilon_1 = \epsilon_x, \epsilon_2 = \epsilon_y, \epsilon_3 = 0$, giving $e_{41} = \epsilon^2_x$, $e_{42} = \epsilon^2_y$, $e_{44} = \overline{\epsilon_x}\ \overline{\epsilon_y}$, and all other $e_{4i} = 0$,

p = 5: $\epsilon_1 = \epsilon_x, \epsilon_2 = 0, \epsilon_3 = \epsilon_z$, giving $e_{51} = \epsilon^2_x$, $e_{53} = \epsilon^2_z$, $e_{55} = \overline{\epsilon_x}\ \overline{\epsilon_z}$, and all other $e_{5i} = 0$, and

p = 6: $\epsilon_1 = 0, \epsilon_2 = \epsilon_y, \epsilon_3 = \epsilon_z$, giving $e_{62} = \epsilon^2_y$, $e_{63} = \epsilon^2_z$, $e_{66} = \overline{\epsilon_y}\ \overline{\epsilon_z}$, and all other $e_{6i} = 0$.

When these strains are substituted, equation (9) becomes.

$$\begin{Bmatrix} U_1 \\ U_2 \\ U_3 \\ U_4 \\ U_5 \\ U_6 \end{Bmatrix} = \begin{Bmatrix} e_{11} & 0 & 0 & 0 & 0 & 0 \\ 0 & e_{22} & 0 & 0 & 0 & 0 \\ 0 & 0 & e_{33} & 0 & 0 & 0 \\ e_{41} & e_{42} & 0 & e_{44} & 0 & 0 \\ e_{51} & 0 & e_{53} & 0 & e_{55} & 0 \\ 0 & e_{62} & e_{63} & 0 & 0 & e_{66} \end{Bmatrix} \begin{Bmatrix} C_1 \\ C_2 \\ C_3 \\ C_4 \\ C_5 \\ C_6 \end{Bmatrix} \quad (10)$$

The normal stiffness vector can then be determined by $\{C\} = [e]^{-1}\{U\}$ (11)

By using the relations giving by equation (7) the normal stiffness matrix can then be determined. Shear stiffness.

For $\epsilon_1 = \epsilon_2 = \epsilon_3 = 0$, equation (5)

$$U = (C_{44}\,\epsilon_{42} + C_{55}\,\epsilon_{52} + C_{66}\,\epsilon_6) \quad (12)$$

Where ϵ_4, ϵ_5, and ϵ_6 are engineering shear strains, γ_{12} , γ_{13} and γ_{23} (or γ_{xy}, γ_{xy}, γ_{yz}), respectively.

The following three sets of displacement boundary conditions are prescribed to produce these strains separately:

p=7: $\epsilon_4 = \gamma_{xy}$, $\epsilon_5 = \epsilon_6 = 0$, giving, $C_{44} = 2U_7/\gamma^2_{xy}$,

p = 8: $\epsilon_5 = \gamma_{xz}$, $\epsilon_4 = \epsilon_6 = 0$, giving, $C_{55} = 2U_8/\gamma_{xy}$ and

p = 9: $\epsilon_6 = \gamma_{yz}$, $\epsilon_4 = \epsilon_5 = 0$, giving,

$$C_{66} = 2U_9/\gamma^2_{yz}, \quad \{13\}$$

By using the matrix [C] thus determined can be used to compute the effective elastic constants of the composite.

Table 3: Applied displacements for different boundary conditions
(2D = plane strain two dimensional, 3D = three dimensional)

Load	Analysis	Applied displacements on sides (surfaces)					
Cond. (*P*)	type	AD(ADHE)	BC(BCGF)	AB(ABFE)	CD(CDHG)	(ABCD)	(EFGH)
1	2D	U_x=0	U_x=0.01	U_y=0	U_y=0	---	---
2	2D	U_x=0	U_x=0	U_y=0.01	U_y=0	---	---
3	3D	U_x=0	U_x=0	U_y=0	U_y=0	U_z=0.01	U_z=0
4	2D	U_x=0	U_x=0.01	U_y=0.01	U_y=0	---	---
5	3D	U_x=0	U_x=0.01	U_y=0	U_y=0	U_z=0.01	U_z=0
6	3D	U_x=0	U_x=0	U_y=0.01	U_y=0	U_z=0.01	U_z=0
7(A)	2D	U_x=0 U_x=Linear	U_y=0 U_x=linear	U_y=0 U_x=0.01	U_y=0 U_x=0.01	--- ---	--- ---
(B)	2D	U_x=0	U_y=0	U_y=0 U_x=0.01	U_y=0 U_x=0.01	--- ---	--- ---
(C)	2D	---	---	U_y=0 U_x=0.01	U_y=0 U_x=0.01	--- ---	--- ---
8(A)	3D	U_x=0 U_z=0.01	U_x=0 U_z=0.01	U_y=0	U_y=0	U_x=0	U_x=0
(B)	3D	U_x=0 U_z=0.01	U_x=0 U_z=0.01	U_y=0	U_y=0	---	---
9(A)	3D	U_x=0	U_x=0	U_z=0.01 U_y=0	U_z=0.01 U_y=0	U_y=0	U_y=0
(B)	3D	U_x=0	U_x=0	U_z=0.01 U_y=0	U_z=0.01 U_y=0	---	---

Table 4: Comparison of effective elastic constants by different methods

E_f = 2.075 x 10^5 N/mm^2, E_m = 68.98N/mm^2, v_f = 0.3, v_m = 0.49444, V_f = 0.5

Method	E_L N/mm^2 (x10^4)	E_T N/mm^2	v_{TL}	V_{TT}	G_{LT} N/mm^2	G_{TT} N/mm^2
Rule of mixtures	10.35	138.0	0.39722	----	46.15	----
Halpin-Tsai	10.35	275.9	0.39722	----	69.20	----
Ekvall	10.35	195.2	0.39722	----	63.47	----
Greszezuk	10.35	293.2	0.39722	----	1759.0	----
Whitney and Riley	10.35	1021.0 63.7	0.39668	0.87210* 0.92016*	69.20	----
Foye	10.35	1690.0	0.39722	0.63900	72.92	----
Hahn	10.35	270.4	0.39722	----	69.20	68.70
Hashin (LB)	10.35	270.4	0.39668	0.87210	69.20	68.70
Hashin (UB)	10.37	1093.0	0.39668	0.99657	26560.0	275.3
FEEM (1 RVE)	10.42	643.0	0.39615	0.92029	71.12	(A) 400.0, (B)77.8, (C)73.1
FEEM (9 RVE)	10.42	644.0	0.39620	0.92016	71.12	(A)82.4, (B)59.7, (C)47.8

Notes: (1) (A), (B), and (C) represent different boundary conditions described in Table 3

{2}* Assumed V_{TT} for which E_T are given

{3} LB = Lower Bound, UB = Upper Bound

Table 5: Comparison of effective elastic constants by different methods

E_f = 2.075 x 10^5 N/mm^2, E_m = 7.6N/mm^2, v_f = 0.3, v_m = 0.49908, V_f = 0.74

Method	E_L N/mm^2 (x10^4)	E_T N/mm^2	v_{TL}	V_{TT}	G_{LT} N/mm^2	G_{TT} N/mm^2
Rule of mixtures	15.31	29.18	0.3518	----	9.73	----
Halpin-Tsai	15.31	72.36	0.3518	----	16.97	----
Ekvall	15.31	72.78	0.3518	----	43.74	----
Greszezuk	15.31	227.40	0.3518	----	6623.0	----
Whitney and Riley	15.31	1518.0	0.3517	0.8580*	16.97	----
Foye	15.31	1725.0	0.3518	0.4811	38.98	----
Hahn	15.31	67.41	0.3518	----	16.94	16.90
Hashin (LB)	15.31	67.41	0.3516	0.8577	16.94	16.90
Hashin (UB)	15.31	1559.0	0.3517	0.9995	46700.0	391.50
FEEM (1 RVE)	14.90	1550.0	0.3512	0.8580	25.77	(A)1147.0, (B)41.0, (C)39.0
FEEM (9 RVE)	14.90	1550.0	0.3512	0.8580	25.77	(A)91.O, (B)22.3, (C)19.1

Notes: (1) (A), (B), and (C) represent different boundary conditions described in Table 3

{2}* Assumed V_{TT} For which E_T are given

{3} LB = Lower Bound, UB = Upper Bound

DISCUSSIONS/RESULTS

Table 3 gives the various applied displacements on the sides/surfaces of these models according to the strain-boundary conditions explained earlier. This set of displacements represents nine separate problems to be analyzed, yielding nine independent elastic constants. Where the transverse dimensions along X and Y axes of the model are equal, which could be the case for most general composites, the boundary condition (1) is the same as (2), (5) and (6), and (8) as (9), leaving only six independent boundary conditions. This reduced set of boundary conditions would give six independent elastic constants, E_L ,

E_T , $V_{TL,}$, V_{TT} , G_{LT} , and G_{TT} . Figures 2 and 3 show the two-dimensional finite element grids employed. The three-dimensional grids are exactly the same except that there are three elements in the Z-direction. Six separate finite element analyses were performed according to the boundary conditions given in table 3 which give six values of strain energy for the six sets of applied strains.

The six elastic constants were computed by using equations (11) and (13) and the other relations. These results are set into tables 4 and 5 to compare with results of the other methods as formerly obtained in tables 1 and 2 (see and compare for yourself). In tables 1 and 4 , a steel-urethane composite was employed, where Young's modulus for urethane is 68.98 N/mm^2 and fibre volume fractions is 50 percent and in tables 2 and 5, a steel-urethane composite having Young's modulus for urethane, 7.6 N/mm^2 and the fibre volume fraction , 74 percent.

CONCLUSIONS:

It is worthy of *note* that all elastic constants by the FEEM for one RVE and nine RVEs are found the same except for the values of G_{TT}.[see tables 4 and 5 to confirm]. This actually goes to prove the effectiveness of the work of Gupta. The little disparity in the values of G_{TT} can easily be understood if we recognize that more energy is required for the condition at (A), in the case of one RVE, than in nine RVE where the energy is reduced drastically because the comparatively rigid fibres are allowed to rotate freely under the application of the classical boundary conditions. Since the resulting values should not be less accurate, better results would be given by more RVE's in the model, with the boundary conditions at (A). In conclusion, therefore, the FEEM can be regarded as the ***all-in-one-tool*** so far available for the effective prediction of elastic constants in composites possessing *very stiff fibres* and *very soft matrix.*

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