PREDICTION OF STRESS CONCENTRATION FACTORS IN UNLAPPED SQUARE HOLLOW *"K"* JOINTS BY THE FINITE ELEMENT METHOD

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ABSTRACT

The paper sets out to *discover numerically the effect of brace spacing on stress concentrations in welded square hollow section 'K' joints. Thin shell theory and isoparametric formulation are employed* to *obtain equilibrium equations, which were solved by the frontal solution code developed for this purpose. Thereafter, the effect of brace spacing was investigated by varying the spacing between the two braces and making computer runs. The results obtained for the present study show a gradual deterioration in strength as the spacing between the two braces was increased. However, due to geometrical differences between the square hollow section and the tubular 'K' joints, it was considered not necessary to compare results of present work directly with published ones on tubular 'K' joints in the literature. It was, however, observed that the trend in behaviour between the tubular and the square section 'K' joints is similar. Thus, the present investigation and others in the literature show that lapped tubular and hollow section 'K' joints are stronger than unlapped joints for the same cross-sections.*

NOTATIONS

U^e	element strain energy
W^e	external work done by the element during deformation
u	displacement
v	displacement
x	global coordinate axis in u direction
y	global coordinate axis in v direction
x	local (element) coordinate axis in u direction
y	local (element) coordinate axis in v direction
N_i	element shape functions
[B]	element strain matrix
[D]	element material matrix
[J]	Jacobian matrix
q	distributed pressure over element
$[Q^e]$	element load vector -
$[K^e]$	assembled system stiffness matrix
σ_f	stress from finite element analysis
σ_n	calculated stress from formula
Q	assembled system load vector
$[\delta]$	element displacement vector
$\{\Delta\}$	system displacement vector.
ζ	non-dimensional coordinate
β	non-dimensional coordinate
Π^s	system total potential energy functional
σ	Stress
ε	Strain

1.0 INTRODUCTION

Lapped joints generally, and lapped tubular joints in particular are known to be stronger than un-lapped joints following a heuristic reasoning. However, for the purpose of bracing, it is not always feasible or even possible to brace a structural system althrough with lapped joints. Hence, a strong case arises for using un-lapped tubular or square hollow *'K'* joints which are in many cases welded on site due to ease of construction and transportation. As usual, one of the problems with welded steel construction is the development of stress concentrations or "hot spot" stresses. The "hotspots" affect the strength (fatigue) of both lapped and un-Iapped joints in offshore structures and steel bridges [5, 9-12].

Kuang et. al [1] investigated the distribution of stress concentration factors in tubular joints using the finite element method but did not consider the effect of brace spacing on square hollow joints. Potuin et. al [2] carried out a similar study but they too did not consider the effect of brace spacing on the distribution of stress concentrations. In 1973, Gibstein [3] carried out a parametric study of T tubular joint, which precludes the study of brace spacing, and no square hollow section joints were used in' the study. Wordsworth and Smedley [4] also investigated the effect of stress concentrations in unstiffened tubular joints but did not investigate the effect of brace spacing on stress distribution in the tubular K joint. Jiki [8] also investigated the effect of brace spacing on stress concentrations in tubular K joints, but did not however, examine the same effect on square hollow K joints.

Some of the studies reviewed above consider the effect of brace spacing on strengths of tubular K joints without consideration of same effect on square section K Joints. This lack of studies design strengths of gapped square section K joints makes availability of information on stress concentration factors in same joints scarce. However, information on 'Hot Spot' stresses is useful in the fatigue design of such joints.

Thus, the purpose of the present study is to investigate the effect of brace spacing on the distribution of stresses' unlapped welded square hollow ‘K’ joints using the Finite Element Method. The Finite Element discretization of the proposed square *'K'* joints is shown in fig 1. The sketches for tubular section join are shown in references [2,5,8].

2.0 EQUILIBRIUM EQUATIONS

The usual curved thin shell iso-parametric element was used for the present investigation [5-7]. The shape the element is shown in Fig. 1. Using potential energy formulation and the Rayleigh-Ritz process, the equilibrium equations were derived as follows:

The element total potential energy functional is given as:

$$\pi^e = U^e - W^e \tag{1}$$

Where U^e is the strain energy of the element. W^e is the external work don by the element during deformation. The strain energy of the 8-node element is given as:

$$U^e = \frac{1}{2}\iiint_v \sigma_{ij} E_{ij} dvol \tag{2}$$

Where σ_{ij} is direct stress

ε_{ij} is direct strain.

Using iso-parametric formulation, the displacement are interpolated as

$$u = \sum_{i=1}^{8} N_i(\zeta,\beta)u_i \tag{3}$$

$$v = \sum_{i=1}^{8} N_i(\zeta,\beta)v_i \tag{4}$$

The 8 node (curved) isoparametric element is mapped into the normalized square space through the following transformations

$$X = \sum_{i=1}^{8} N_i(\zeta,\beta)x_i \tag{5}$$

$$y = \sum_{i=1}^{8} N_i(\zeta,\beta)y_i \tag{6}$$

Where N_i are the shape functions corresponding to node I with Cartesian coordinates (x_i, y_i) in X, y system of axis and non-dimensional coordinates ζ_1,β_i coordinate system where $\zeta_1,\beta_I = \pm 1$. For corner nodes and zero for mid nodes. Then the shape functions are given as:

$$N_i = [(1+\zeta\zeta_i)(1+\beta\beta_1)-(1-\zeta^2)-(1+\beta^2)(1+\zeta\zeta_1)\frac{\zeta_1^2\beta_1^2}{4}+(1-\zeta^2)(1+\beta\beta_1)$$
$$(1-\zeta_1^2)\frac{\beta_1^2}{2}+(1-\beta^2)(1+\zeta\zeta_1)(1-\beta_1^2)\frac{\zeta_1^2}{2}] \quad (7)$$

In terms of the shape functions Ni the strain matrix [B] is given as:

$$[B] = \begin{bmatrix} \frac{\partial N_i}{\partial x} & 0 \\ 0 & \frac{\partial N_i}{\partial y} \\ \frac{\partial N_i}{\partial x} & \frac{\partial N_i}{\partial y} \end{bmatrix} \quad (8)$$

In which

$$\begin{bmatrix} \frac{\partial N_i}{\partial x} \\ \frac{\partial N_i}{\partial y} \end{bmatrix} = [J]^{-1} \begin{bmatrix} \frac{\partial N_i}{\partial \zeta} \\ \frac{\partial N_i}{\partial \beta} \end{bmatrix} \quad (9)$$

And [J] is the Jacobian matrix given as

$$[J] = \begin{bmatrix} \frac{\partial x}{\partial \zeta} & \frac{\partial y}{\partial \zeta} \\ \frac{\partial x}{\partial \beta} & \frac{\partial y}{\partial \beta} \end{bmatrix} \quad (10)$$

In terms of the strain matrix of equation (8) the element strain vector is given as:

$$\{\varepsilon\} = [B]\begin{Bmatrix} u_i \\ v_i \end{Bmatrix} \quad (11)$$

The element strain energy of equation (2) can also be written in terms of material matrix [D] as:

$$U^e = \frac{1}{2}\iiint_v \{\varepsilon_{ij}\}^T [D]\{\varepsilon_{ij}\} dvol \quad (12)$$

Substitution of equation (11) into equation (12) and assuming a unit thickness for the element gives:

$$U^e = \frac{1}{2}\iint_A [B]^T \left\langle \begin{matrix} u_i \\ v_i \end{matrix} \right\rangle^T [D][B]\left\langle \begin{matrix} u_i \\ v_i \end{matrix} \right\rangle dA \quad (13)$$

$$= \frac{1}{2}\iint_A [B]^T \{\delta\}^T [D][B]\{\delta\} dA \quad (14)$$

For the present investigation the element shown in Fig. 2 is loaded with a uniform pressure q over the element boundary between nodes. The external work done by the load during displacement of the boundary is given as:

$$W^e = \int_b q\{\delta\}^T db \quad (15)$$

Substitution of equations (14) and (15) into equation (1) gives the total potential energy of the element as:

$$\pi^e = \frac{1}{2}\iint_A [B]^T \{\delta\}^T [D][B]\{\delta\} dA - \int_b q\{\delta\}^T db \quad \ldots\ldots (16)$$

$$= \frac{1}{2}\{\delta\}^T [K^e]\{\delta\} - Q^e\{\delta\} \quad (17)$$

In which $[K^e]$ is the element stiffness matrix

Q^e is the element boundary loads such that:

$$[K^e] = \int_{-1}^{1}\int_{-1}^{1} [B]^T [D][B] \det[J] d\zeta d\beta \quad (18)$$

$$Q^e = 2q \quad (19)$$

In practice the element stiffness matrix of equation (18) is obtained by performing a numerical integration using Gauss integration rule [6, 7]. The Rayleigh-Ritz process gives equilibrium equations as:

$$\frac{\partial \pi^e}{\partial\{\delta\}} = [K^e]\{\delta\} - Q^e = 0 \quad (20)$$

After assembly of element stiffness matrices and applied load into system stiffness matrix and system load vector, the system potential energy functional is given as:

$$\prod^s = \frac{1}{2}\{\Delta\}^T [K^s]\{\Delta\} - \{\Delta\}^T \{Q\} \quad (21)$$

In which {A} is system node displacement vector

$[K^s]$ is assembled system stiffness matrix

{Q}is assembled system load vector Again application of the Rayleigh-Ritz process lead to-the system equilibrium equations as:

$$\frac{\partial \prod^{s}}{\partial\{\Delta\}} = \left[K^{s}\right]\{\Delta\} - \{Q\} \qquad (22)$$

3.0 SOLUTION OF SYSTEM EQUILIBRIUM EQUATIONS

The system equilibrium equation for the present problem were solved after assembly and application of bound conditions using the Gaussian algorithm [5, 7]. The technique of sub structuring was used in the solution process so as to reduce the large core memo requirement in the banded form of storage system adopted. The gain in using this form of solution technique is that less core memory is used at each instance of the elimination and back substitution process. After the solution was carried out the results were presented element by element.

4.0 INVESTIGATION OF STRESS CONCENTRATIONS

Due to symmetry, only hall the ‘K’ joint was employed to investigate stress concentration in the present study as shown in Fig. 1. The finite element discretization and loading of the joint the purpose of stress analysis are shown in Fig. 1. In order to use the Gauss elimination process, element node numbering system was adopted [5] Thereafter, computer runs were made and both normal or direct stresses as well as shear stresses were obtained for element after a computer run was made Gauss points with highest element direct and shear stresses from the finite element analysis were marked as stress concentrations. Stress concentration were recorded both in the chord and the two braces of the *'K'* joint. The results for the chord are shown in Figs. 3 and 4 for Gauss points type 1 for direct stresses.

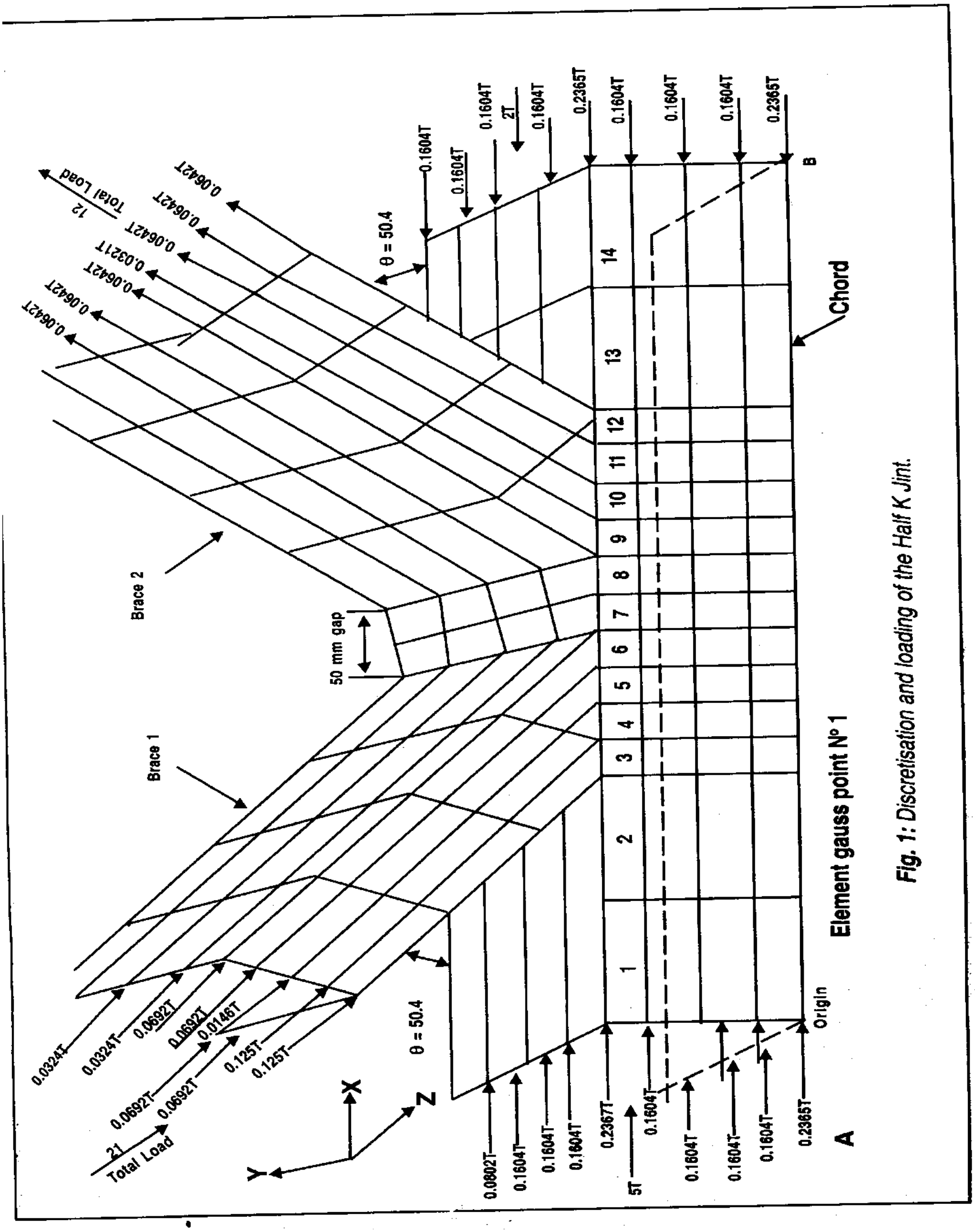

Fig. 1: Discretisation and loading of the Half K Jint.

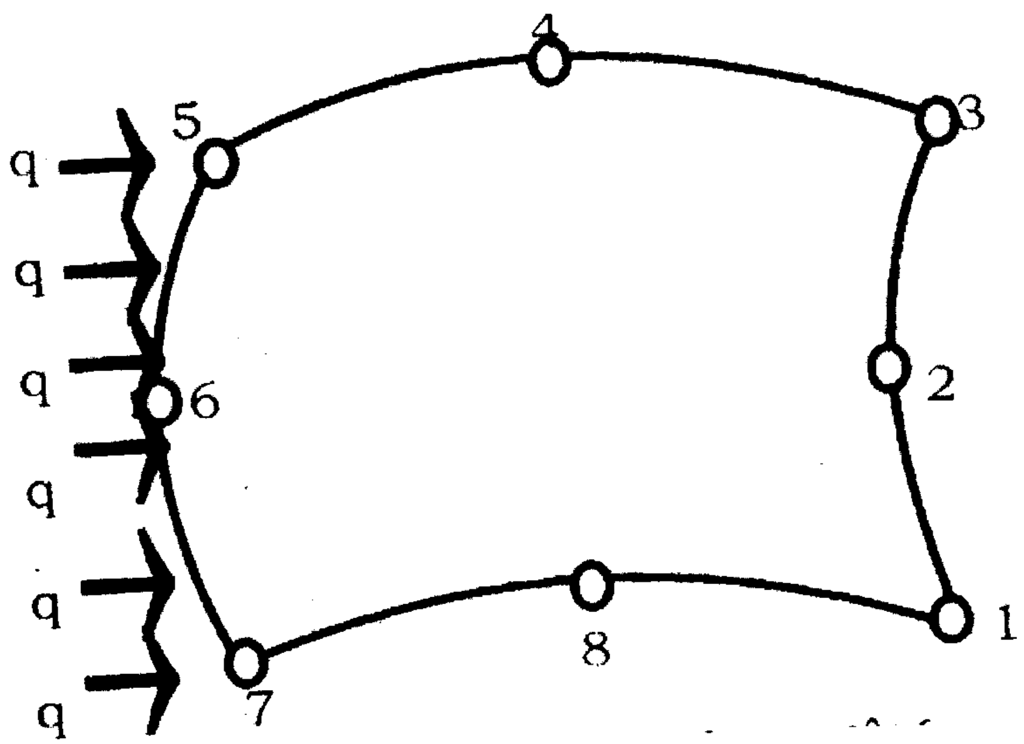

Fig. 2: An 8 Noded Isoparametric Element

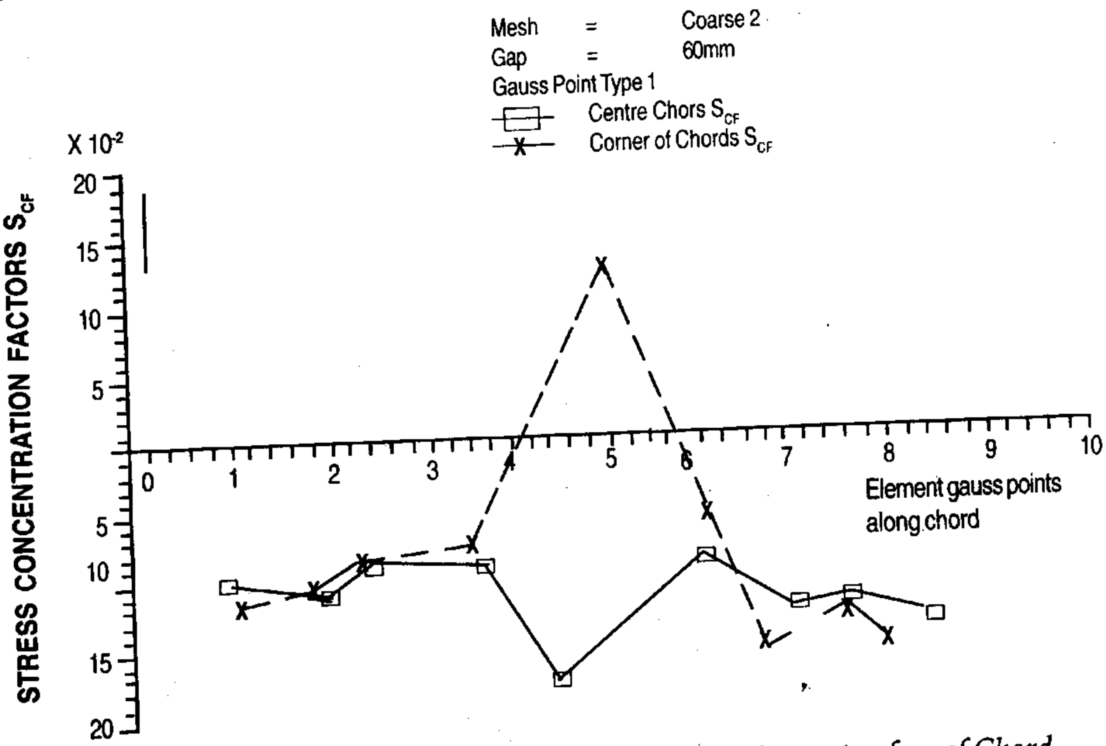

Fig. 3: Distribution of S_{CF} for A 60mm Gap K Joint on top face of Chord.

The gap between the two braces was kept constant at 50mm and Stress Concentration factor (S_{CF}) was obtained as:

$$SCF = \frac{\sigma_F}{\sigma_n} \qquad (23)$$

in which σ_F is finite element stress in the longitudinal axis of the chord and σ_N is average stress in the longitudinal axis given as:

$$\sigma_N = \frac{P}{A} + \frac{M}{Z} \qquad (24)$$

In which P and M are the applied axial load and moment respectively and A and Z are the area and elastic modulus of the section respectively. However, the result for the 50mm gap is not shown here because it is not critical. Critical cases are those for 60mm and 70mm gaps and are shown here in Figs. 3 to 7.

5.0 GAP STUDY

The effect of gap between braces (brace spacing) on the stress concentrations in the joint was investigated by sampling or marking some elements and studying the behaviour of the stress concentration factors in the elements as gap between the two braces was increased from no gap to 70mm gap. The results of the effect of brace spacing or gap on the chord are shown in Figs. 3 to 4 and those of the effect

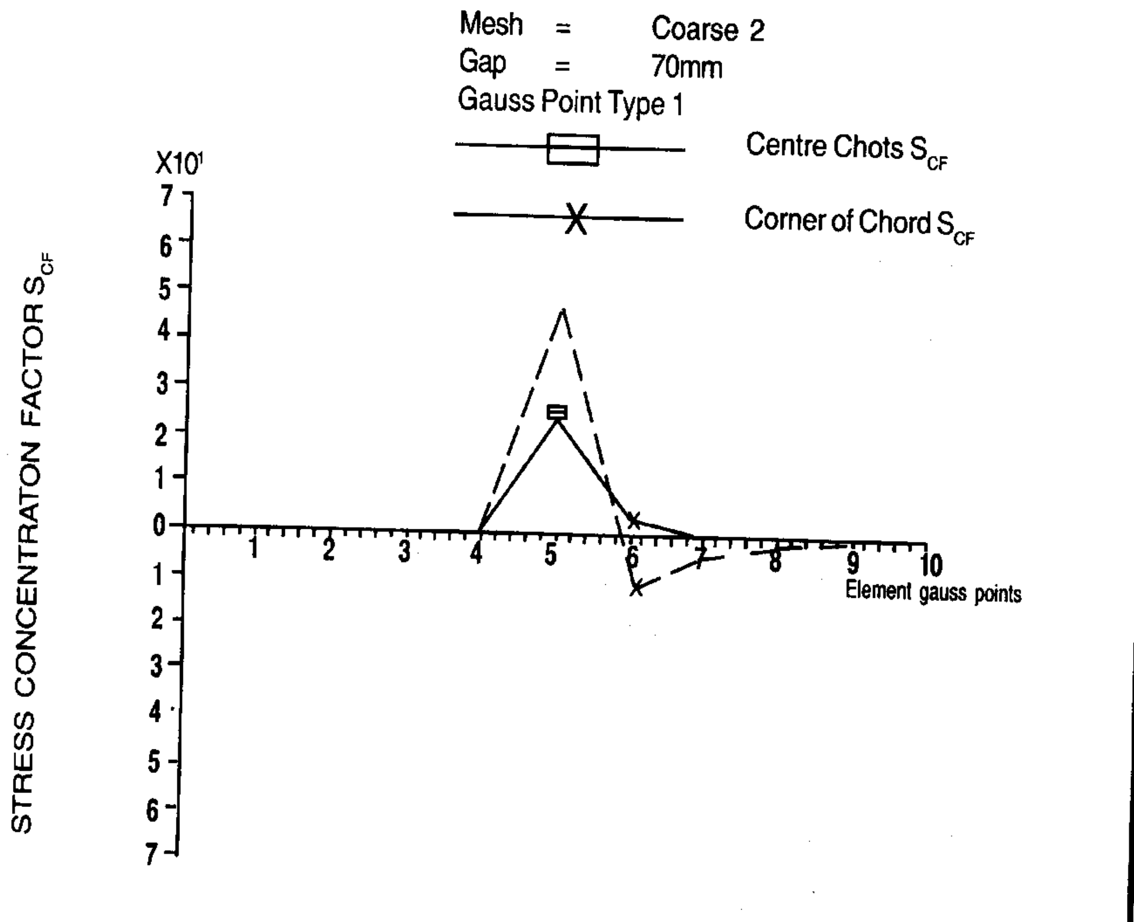

Fig. 4: Distribution of S_{CF} for A 70mm Gap K Joint on top face of Chord.

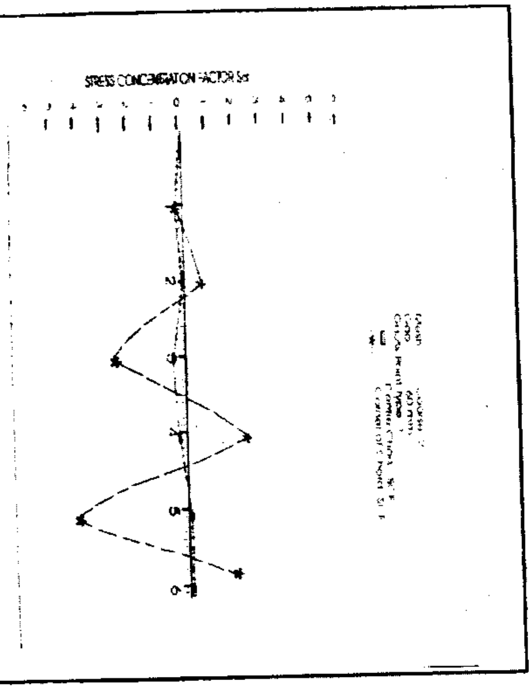

Fig. 5. DISTRIBUTION OF S_{cf} FOR BRACES 1 AND 2 OF 60 mm GAP K JOINT AT INTERSECTION WITH THE CHORD

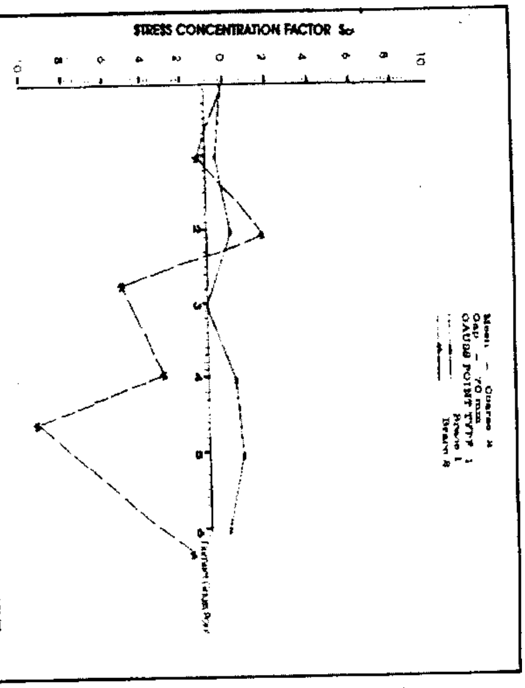

Fig. 6.DISTRIBUTION OF SCF FOR BRACE 1 AND 2 OF A 70MM GAP K JOINT AT INTERSECTION WITH THE CHORD

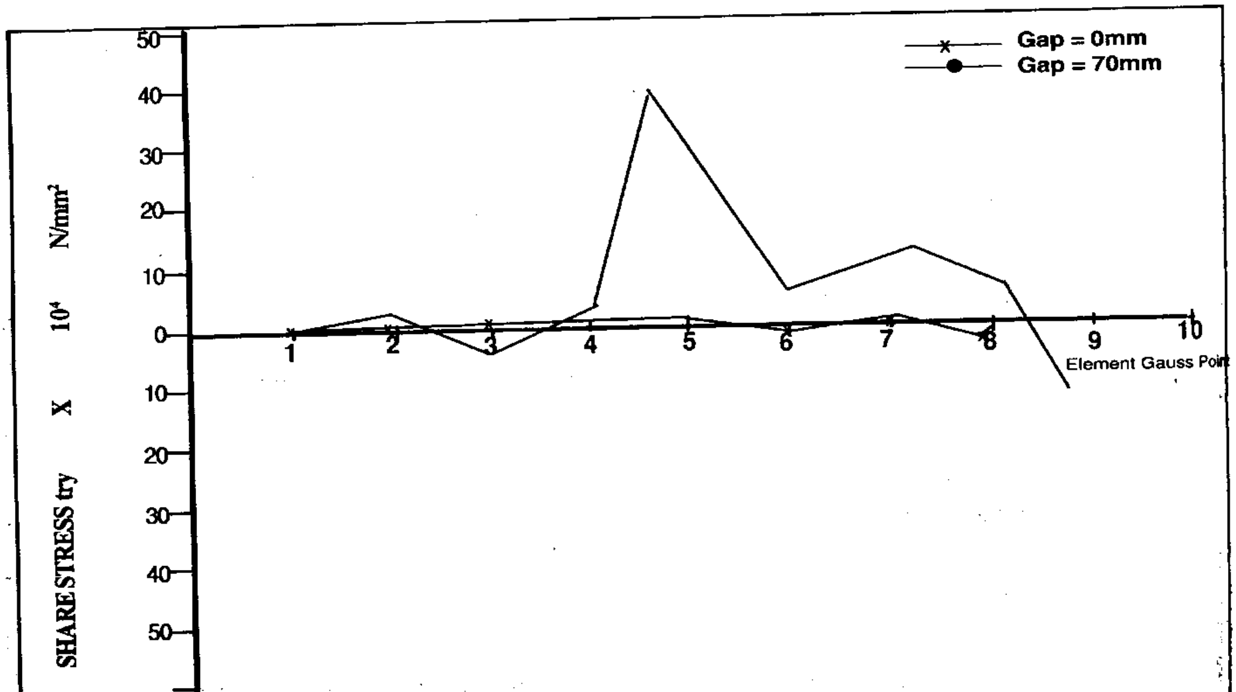

Fig. 7: *Share Stress Distribution along the Chord of an RHS K Joint.*

of brace spacing on the two braces are shown in Figs. 5 and 6. The effect of shear stresses on the chord for no gap and 70mm gap are compared in Fig. 7

6.0 DISCUSSION

Gauss point type 1 was used in the sample study. Plots of stress concentration factors (S_{CF}) against the element Gauss points for the center and corner of the chord were made and critical values are shown in Figs. 3 and 4. The results for no gap up to 30mm and even 50mm gap follow a similar pattern and the stress concentrations are not critical and are therefore not shown here. However, from the gap of 60mm up to 70mm, as shown in Figs. 3 and 4, the pattern has drastically changed and at the middle of the range of the Gauss points, the stress concentration factors, apart from having higher values, have increased from negative to positive values for corner of the chord while the center of the chord still displayed negative values. The change from negative to positive stresses indicates failure or overstress at the points or the elements where the change occurs.

The results for the braces are shown in Figs. 5 and 6. The pattern of stress concentration factors recorded for no gap up to 30mm gap is similar to the one for the chord but started to change from 60mm up to 70mm gap. Higher negative values are observed in brace two from 60mm to 70mm gap. The results of brace two tend to oscillate between negative and positive values from 60mm to 70mm gap.

The results of shear stresses for no gap and 70mm gap are shown in Fig. 7 and they indicate high shear stress concentration factors of about 45 which occur in element sample point 5 (Fig. 1). There is clearly a marked difference as far as no gap and 70mm gap are concerned. The result obtained in the present study using rectangular hollow K joint is very similar to that reported by Jiki [8] that lapped unstiffened tubular K joints are stronger than unlapped un-stiffened tubular K joints.

7.0 CONCLUSION

From the above results it can be said that lapped unstiffened rectangular hollow K joints are stronger than un- lapped un-stiffened rectangular hollow K joints. The results obtained in the present study using rectangular hollow (un- stiffened) K joints are in agreement with those reported earlier by Jiki [8] for un- lapped unstiffened tubular (circular) K joints.

In both studies, both high punching shear stresses from braces and bending of the chord seem to be critical when the spacing between braces exceed 50mm. Therefore, the braces in un-stiffened rectangular hollow section K joints should be spaced not more than 60mm apart with braced chord ratio of at least 1/2 for better results.

8.0 REFERENCES

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