DYNAMIC ANAYSIS OF TALL BUILDINGS

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ABSTRACT

The dynamic response (natural frequency) of tall building frame models which permit joint ration and the inclusion of the columns axial load were investigated.

The stiffness co-efficient of axial members were modified by the use of relevant stability functions through which the contributions of the axial forces were reflected. Comparisons of the responses of models with flexible horizontal members with the exclusion and /or inclusion of the columns axial loads to those of identical frames modeled as shear frame s and considering the buildings as multi degree of freedom systems, based on numerical results were made. The results showed that (a) model with no restriction on joint rotations gave improved results, (b) the inclusion of the axial loads further improved the result and hence, the models, (c) The axial load effect became more significant as the building got ″taller″ and (d) at beam to column rigidity (EI) rations below 12 the sear frame gave result s that different by more than 5% from those obtained using the improved models. It is, therefore, recommended that for tall buildings with low horizontal to vertical member rigidity ratios models which permit flexibility of members and joint rotation should be used. For very tall frames, the axial loads should not be ignored.

1.0 INTRODUCTION.

Tall frame constitute the main load bearing component of tall building [1].

Apart from static loads, structural frames are also subjected to dynamic forces which have their possible sources from earthquakes, explosion, impact loads, moving loads, wind and other disturbances. The common feature of all dynamic disturbances is that they generate vibrations in the structure upon which they act. Consequently, prerequisite to the design of such a structures is a good insight into its vibration motions and, in particular, the natural frequency, w.

The knowledge of the natural frequencies and the associated modes of vibration enable the engineer not only to evaluate some dynamic parameters necessary for the design of the structure but to also predict the likelihood of resonance. As far as possible, natural frequencies should, therefore, be determined with some fairly good accuracy.

Tall frames are essentially systems with infinite degrees of freedom. However, some simplifications are made in their dynamic analyses by considering them as multi degree of freedom system using the shear frame model in which the horizontal members are assumed to be infinitely rigid in comparison to the vertical members. Consequent upon excitation the shear frame only sways in its plane. The hear frame known to give results which may differ greatly from the actual. Also, it does not include the effect of axial loads over the years the form and properties of tall buildings have changed substantially. Lighter floor systems and curtain wall constructions have are now very common. Modern designs have also led to buildings that are more susceptible to dynamic excitation [2]. In view of these, it becomes increasingly difficult to justify the continued use of the shear frame model for tall frames.

In this work an attempt is made to (1) investigate the effect of axial loads on the natural frequencies of tall rigid building frame with flexible horizontal members as multi degrees of freedom systems and (2) establish a critical value of beam-column EI ratio, if any below which the shear frame gives natural frequencies that differ by more than 5% from those obtained when (i) no restrictions are placed on joint rotations and (ii) axial loads and joint rotations are considered. A related work has been done on (1) above for a shear frame [3].

2.0 EQUATION OF FREE MOTION

Using the principle of superposition, the equation of free vibration can be written as: [K] [Y] + [M] [ÿ] = [0] where [K] is the lateral stiffness matrix of the building, [M] is the diagonal matrix of the lumped masses at the floors. [γ] is a column matrix of the lateral displacements at the floor levels. [y] is a matrix that contains the second derivatives of the displacements at the floors.

Assuming a periodic solution to equation 1 results to the generalized eigen problem:

$$[K]\,[\varphi]=\omega^2\,[M]\,[\varphi] \qquad (2)$$

Where, [φ] is the amplitude array. The n (n being the degree of freedom of the structure) solutions to equation 1 can be written as:

$$[K]\,[\varphi] = [M]\,[\psi]\,[\Omega] \qquad (3)$$

where, [ψ] is an n x n matrix of eigenvectors, [Ω] is a diagonal matrix listing the eigenvalues (square of the free vibration frequencies).

Many methods of solving the eigenvalue problem are available such as the sturm sequence, subspace iteration and the Jacobi method [4-7].in this work, the Jacobi method will be adopted.

3.0 DYNAMIC ANALYSIS

3.1 Description of models.

The basic assumptions for each of the models to be considered are as given below.

Model 1: the shear frame or vertical pole

The shear frame model assumes that (i) the total mass of the structure is concentrated at the floor levels; (ii) the floor slab action integrally with the beams makes the beams infinitely rigid when compared to the columns, (iii) the deformation of the structure is independent of the axial forces in the column. Thus a building frame can then be modeled as a vertical pole with the masses concentrated at the floor levels and the rigidities of the vertical members of the original frame, say at the i^{th} floor level summed up to give the rigidity o the pole at the i^{th} floor [8].

Model 2: model with no restriction on joint radiation

This, like the shear frame model, assumes that the masses are lumped at the floor levels but the girders are not assumed to be of infinite rigidity when compared to the columns. The effect of vertical inertia is negligible and the axial deformation of the structure is independent of the axial force in the columns.

Model 3; model with joint rotations and column axial loads included

This is similar to model 2 except that the columns axial loads are include. The effects of the axial load are taken into account by treating the columns as beam-column elements.

2.3 Evaluations of the lateral stiffness matrix elements.

Tall buildings are generally three-dimensional structure. Although a three dimensional analysis is possible, the computations are often quite involving. A two- dimensional analysis is used except in some special cases. The major difference in the models lies in their lateral stiffness matrices, [K] of equation 3. The element K_u of the lateral stiffness matrix is the restoring force at floor level i when floor level j is given a unit sway. Stiffness matrix elements for model 1 if floor unit sway then the relevant stiffness matrix elements are given as:

$$K_{1j} = 12\left(\frac{EL_{i-1}}{L_1^3} + \frac{EL_1}{I^3,}\right)$$

$$K_{1j} = \frac{12EI}{I^3}, K_{1j} = \frac{12EI}{I^3} \qquad (4)$$

$$K_{1j} = 0, \text{for } j \neq i-1, i, i+1$$

Stiffness matrix of models 2 and 3

In addition to the assumptions already stated the following are also made in the analysis of tall buildings: contribution from the out of plane stiffness of floor slabs are neglected, tensional stiffness of beams, columns and planar walls are neglected and the effect of axial load on beams and girders is usually negligible.

The stiffness matrix elements can only be obtained after a complete static analysis of the structure, using, say, the classical displacement method as described below:

i. The joints of the frame are assumed fixed and unit rotations applied at each, one at a time and the resulting end moment diagrams, M_i (i=1 to v the total number of joints) are then drawn using the appropriate element bending stiffness.

ii. The structure is then assumed to be given a lateral displacement of at floor level k, other lateral displacements of other floor being assumed to be zero. The resulting fixed end moment diagram M_k is drawn.

iii. The compatibility equations are then set up and are given as

$$\gamma_{LI}\Theta_I + \gamma_{L2}\Theta_2 + \cdots\cdots\cdots\cdots + \gamma_{Lv}\Theta_v = R_1X_k$$
$$\gamma_{2I}\Theta_I + \gamma_{22}\Theta_2 + \cdots\cdots\cdots\cdots + \gamma_{2v}\Theta_v = R_2X_k$$
$$.. \quad .. \quad . \quad .. \quad . \quad . \quad . \quad . \quad . \quad .$$
$$\gamma_{vI}\Theta_I + \gamma_{v2}\Theta_2 + \cdots\cdots\cdots\cdots + \gamma_{vv}\Theta_v = R_1X_k \qquad (5)$$

Where r„ (i-1to *v*,j=1 to *v*) is the moment generated at join t i due to a unit rotation at i (i=1 to u, the degree of freedom) is given a

joint j, R, X_k is the moment generated at joint i due to a unit sway at floor level k, = (i=1 to *v*) is the unknown rotation at joint 1.

iv. Eq. 4 is solved and the unknown values of the joint rotation determined. The final bending moment diagram is then determined using the relationship.

a. $M = M_k + \sum_{i=1}^{1} M\Theta$ (6)

M_k is the final bending moment diagram when the structure is given a unit lateral sway at floor level k.

V The shear force and hence the stiffness matrix elements, K_{ik} (i=1=n) at each floor level can now be calculated using the local equilibrium principle.

Vi steps (ii) to (v) are repeated for the other floors. It should be noted that only the right hand side of equation changes each time the process is repeated.

The formulation of the stiffness matrix of tall frames as described above is not only complex but also time consuming and prone to errors as equation 2 has to be formulated and solved *v* times.

3.2 Computer program

A computer program stiffeig, was developed by anya [9] using the stiffness method described above for (a) calculating the stiffness matrix elements of regular frames for various beam/ column EI ration for models 1,2,and 3, previously described

and (b) solving the resulting eigenvalue problem to determine the natural frequencies and modes of vibration using the Jacobi method. The input data are the number of storey's and bays, the floor and roof loads, the member elements' El values and lengths. Others are the columns axial loads, the lumped masses at the floor levels, the Jacobi tolerance value the model type to be analysed for and the Jacobi iteration limit *stiffeig* first calculates the left hand side coefficients of the compatibility equation (equation 5) using the relevant element stiffness for the model type and stores the in a banded form. For each degree of freedom it generates the right hand side of equation 5 and solves the resulting simultaneous equation to determine the joint rotations using the Gaussian elimination method. The final bending moments are then obtained using equation 6 from which the lateral stiffness elements for that degree of freedom are calculated. When all the lateral stiffness elements have been evaluated and stored in a banded form, the program starts the solving of the eigen problem (equation 3) using the Jacobi iteration method. The mass matrix is decomposed and the generalized eigen problem is transformed to the standard form. A unit eigenvector array is then set up and the iteration value is set to zero, its initial value. The program then searches for the largest off diagonal stiffness matrix element and calculates the effective zero value, The updating of the stiffness and eigen vector matrices is done until the largest off diagonal stiffness matrix element is less than the effective zero value or the iteration limit is reached. The eigenvector array is then normalized and the natural frequencies are calculated. A check is then carried out to test the accuracy of the results obtained.

3.3 Test problems.

Stiffeig was used to determine the natural frequencies of some tall building frames.

The first building is the 15 storey-2 bay office building, the plan and section of which are as shown in Fig 3a and 3b respectively. The other building is a 10 storey whose plan is the same as that of the 15 storey building but whose elevation was obtained from that of the 15 storey building by removing the appropriate number of floors from the top. All the beams are 585 x 250mm. The material of construction is concrete of unit weight of $24kN/m^2$ and modulus of elasticity, E of 20 x 10^6 kN/m^2.

The structural properties of the buildings are assumed uniform along the length of the building and an analysis of an interior frame is assumed to yield the response of the entire building.

3.3.1 Calculation of design loads

The design loads were calculated using BS 811O design code [9] and are as shown in Table 3.1

Column axial loads.

The column axial loads were calculated using the design loads in Table 3.1 and assuming that the columns carry half of the loads on the beams spanning into them as shown in Fig 3c. Thus at each floor level a column, say, column C1 carries the portion of the floor load demarcated by adcd while that for column C2 is cdef and that by column C3 is efgh, The loads on the various panels are then as in Table 3.2.

Lumped masses

The lumped masses at the floors are: At roof = 300 x 10 x 3.6 = 10800kg Other floors = 360 x 10 x 3.6 13000kg

4.0 DISCUSSION OF RESULTS

The plots of natural frequencies of the 5 and 10 storey buildings against beam to column E.I ratios for the models are as given in Figs. 4.1and Fig 4.2.

From fig. 4.1 and 4.2 the following are discernable; (i) the natural frequency curves for model 1 (ii) the inclusion of the axial loads did improve the results. The frequencies obtained for model 3(when the axial loads were included) were constantly less than those for models 1and 2. However the values did not differ very much from those of obtained for model 2 where only joint rotations are allowed. In fact the two curves could be plotted as one. It is expected that as the number of storeys increases the differences between the two models will increases (iv) The beam to column EI ration below which model 1(the shear frame) gave results that differ from those of models 2 and 3 by 5% are respectively 12.5 for the 10 storey building approximately. The ratio increases with increase in the numbers of storeys.

4.0 RECOMMENDATIONS/CONCLUSION

The following recommendations are made from the study;

(i)The effect of axial forces should not be ignored in the dynamic analysis of very tall building especially when light flooring systems are used. however for moderately tall building where the axial loads are not considerable much, the effect of the axial loads may be ignored and the structure depending on the beam-column EI ratio, be analysed as one with flexible horizontal members or as a shear frame (ii) Tall building whose floor slabs are constructed with lightweight material or those whose beam to column EI ratios are less than 20 should not be analysed as shear frames. Such buildings should be modeled as one with flexible members with/without the inclusion of the column's axial loads

NOTATIONS

[K]	Building lateral stiffness matrix.
[M]	Mass matrix.
E	Modulus of elasticity
I	Second moment of area.
W	Natural frequency
n	dynamic degrees of freedom
v	number of joints.

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TABLES AND FIGURES

Table 3.1: Design load calculation to BS 8110, 1997

Parameter	Roof (125mm thick with access)	Upper floor (150mm thick)
Self weight	0.125 * 24 = 3kN/m² = 300kg/m²	0.15 * 24 = 3.6kN/m² =360kg/m²
Finishes, say,	0.5kN/m²	1.2kN/m²
Dead load	3 + 0.5 = 3.5kN/m²	3.6 + 1.2 = 4.8kN/m²
Imposed load	1.5kN/m²	3kN/m²
Design load	1.4 x 3.5 + 1.6 x 1.5 = 7.3kN/m²	1.4 * 4.8 + 1.6*3 = 11.52kN/m²

Table 3.2: Load distribution to panels.

Panel	Location	
	Roof	Upper floor
Abcd	7.3 * 3.6 * 3 = 79kN	11.52 * 3.6 * 3 = 125kN
Cdef	7.3 * 3.6 * 5 = 132kN	11.52 * 3.6 * 5 = 208kN
Efgh	7.3 * 3.6 * 2 = 53kN	11.52 * 3.6 * 2 = 83kN

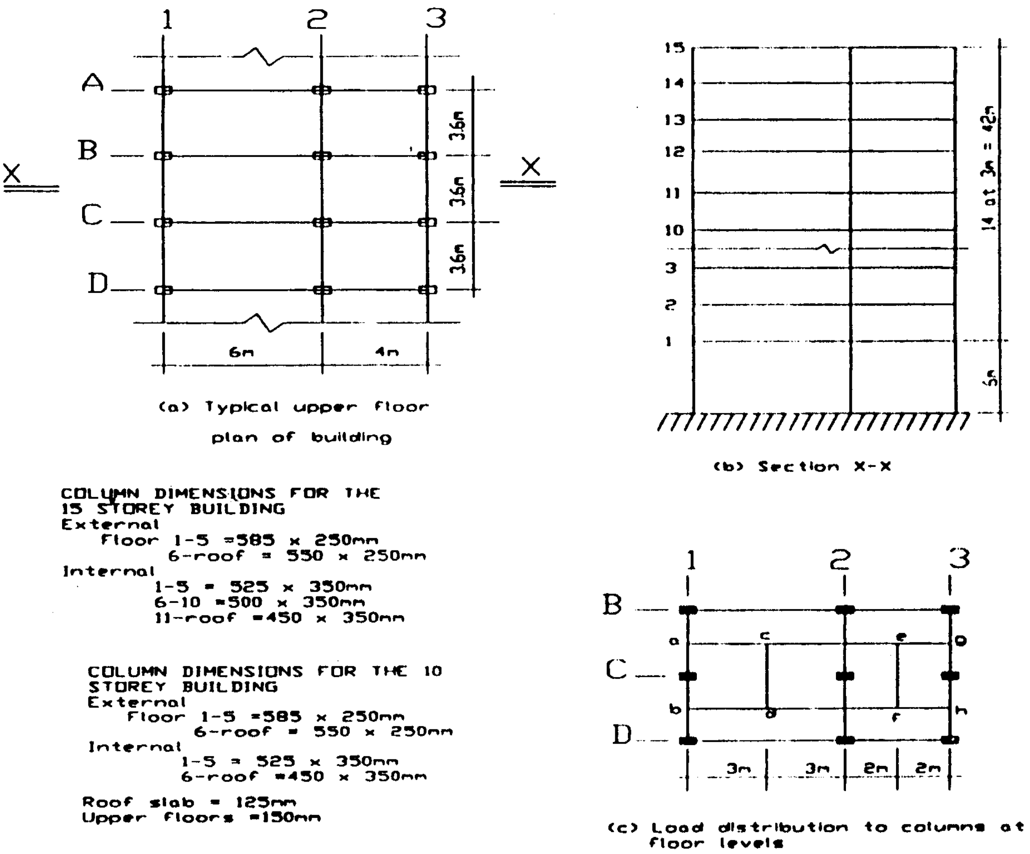

Fig. 3.1 Typical upper floor plan and section of the 15 storey building

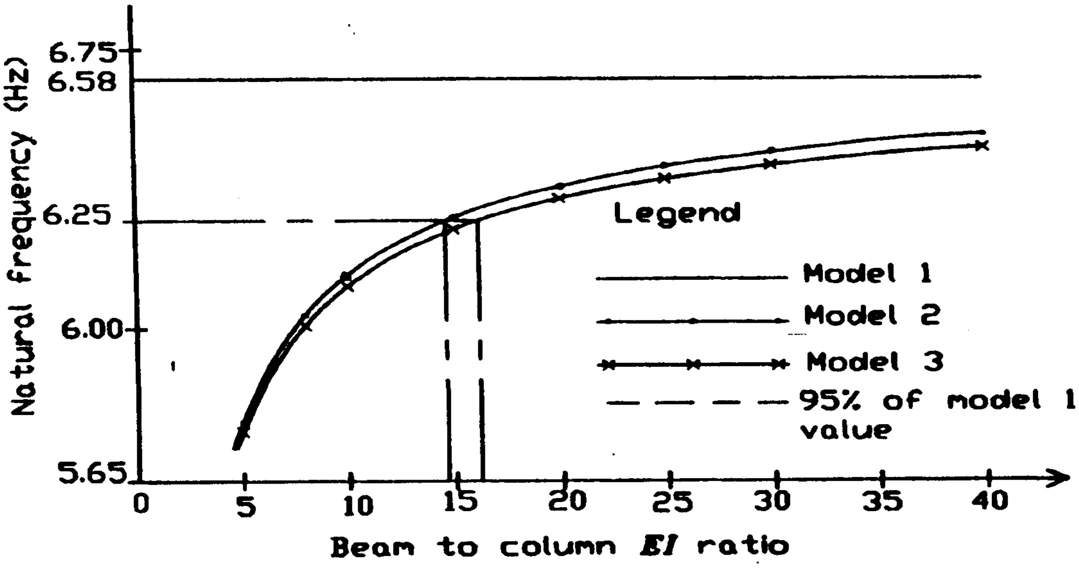

Fig. 4.1 Natural frequencies of the 15 storey building for various beam to column *EI* ratios.

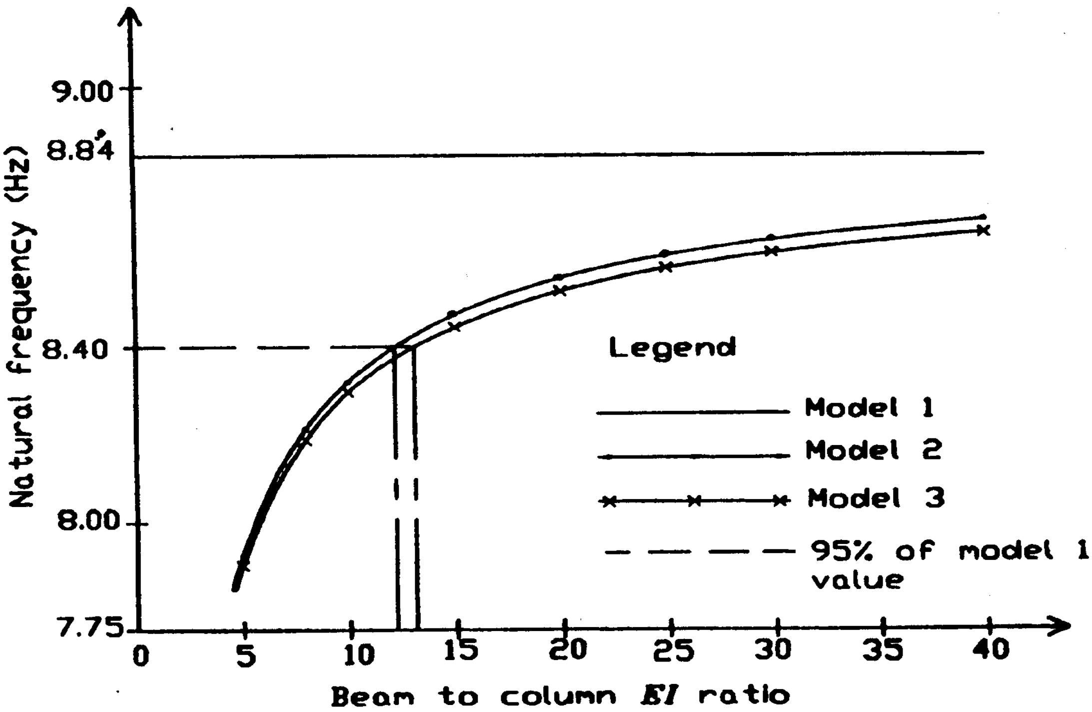

Fig. 4.2 Natural frequencies of the 10 storey building for various beam to column *EI* ratios.