SMALL SIGNAL ANALYSIS OF LOAD ANGLE GOVERNING AND EXCITATION CONTROL OF AC GENERATORS

E. S. OBE, AMIEE
Department of Electrical Engineering,
University of Nigeria, Nsukka, NIGERIA
obeunn@yahoo.com

ABSTRACT

A method of obtaining direct pictorial information on the behaviour of synchronous machines under different conditions of control using a single variable related to its load angle is presented. The technique employs a simple control scheme to the linearized model of synchronous machine equations. The scheme is simple and can be manipulated on a PC if adequate computational data is provided. Simulation results obtained using MATLAB ® were revealed by angle-time plots. These results are very similar to results obtained by frequency response analysis. Results of other related schemes modeled with laboratory micro-machines also attest to the validity of this technique. Four control modes were simulated and it is shown that in all but one, there is a remarkable improvement in system damping than all the conventional schemes considered in the study. More significantly, responses of the new scheme display a better performance in terms of speed of response, control co-ordination and simplicity than the well-known techniques of employing power system stabilizers (PSS) or using terminal voltage for control of exciter and speed signal for governor. Above all, there is no need for supplementary signals that could further complicate the proposed control loop.

Keywords: *load angle control; coordinated control;stability*

List of symbols and abbreviations

Φ_d= direct axis flux
α_i = real part(s) of complex pole(s)
β_I = imaginary part(s) of pole(s)
Φ_q = quadrature axis flux
A_i = real part(s) of complex residue(s)
B_i = imaginary part(s) of complex residue(s)
a_i, = constant(s) of pole(s)
G = gain
G_s = product G(s)g(s)
H = inertia constant
I_d = direct axis current
I_q = quadrature axis current
K_a = constant
K_D= system damping coefficient, p.u. torque/ p.u. slip
K_i, = constant(s) of residue(s)
M = H/nf
N = turns per phase (main winding).
Np = turns/pole/phase - main winding
p = number of pole pairs.
P = permeance or power
p=jw or the s operator
P_e, = electrical power
t = time (sec)
T_e = output electrical torque
T_i = turbine input torque
V_b, V = busbar voltage
V_d, V_q=voltages behind the direct and quadrature axes reactances
V_d'. V_q' = voltages behind the transient direct and quadrature axes reactances
V_{fd}= generator field voltage
X_d, X_q = direct and quadrature axis

reactances

$X_{d'}$,X_q' = transient direct and quadrature axis reactances

X_{dm} ,X_{qrn} = machine direct and quadrature axis reactances

X_s = synchronous reactance of the cylindrical rotor machine per phase,

X_t = transmission line reactance

α = delay angle

Δ = change in

δ= load angle

δ_0= operating load angle

ε = overall governor gain

θ = phase angle

μ = regulator gain

μ_o= Permeability of free space,

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τ = time constant

τ_{do} = generator field time constant

τ_g = governor feedback time constant

τ_I = overall time constant of the governor control loop

τ_n = secondary oil relay time constant

τ_o = primary oil relay time constant

τ_r = reheater storage time constant

τ_s = servomotor time constant

τ_t = entrained steam time constant

ω = speed

ω_o = synchronous speed.

1. INTRODUCTION

Steinmetz [1] first observed the problem of power system stability and related it to the synchronous machine feeding the system in 1920. For ease of understanding and convenience, the stability problems and the methods of obtaining the solutions were divided into two main categories: small signal disturbance stability [5] and large signal disturbance stability. Large signal disturbance stability is related to short duration or transient period, which is usually associated with a few seconds and high prefault power. It is concerned with the response of the system to different kinds of faults. For these studies, well established and powerful software and computer programs (ETAP®, power Station®, to mention just a few) are readily available for the study of the transient stability. Small signal stability studies relate to small and frequent switching changes in power systems and it is the basic requirement for fundamental operation of power systems [2,3,10], whichever type of the power system disturbance occurs, they are always manifest on the synchronous machines feeding it and cannot be dissociated from each other in the frequency domain, and therefore frequency response analysis adopted by other researchers [2-4] could be misleading.

Generally, in any case, the major requirement is that of ensuring sufficient damping of the system oscillations and subsequent extension of stability limits. This uniquely permits the consideration of the system as a linear one about an operating point and hence the application of well established linear control theory to the analysis of the machine behaviour. The process of linearization by small departures is already a well-established phenomenon [2-4] and it is advantageous because by restricting the changes under investigation to a narrow band, there is little variation in flux levels, and the inductances can be regarded as constants; and the terms involving the product of two (or more) deviations can be neglected, because it will be small with respect to other terms.

Of all the authors mentioned in the application of small oscillation theory to ac generator stability studies [2-7], none explored the use of angle time plots for the observation of the system behaviour primarily because of the complexity of the control loop due to the many and varied control strategies employing different networks signals derived from rotor speed, rotor acceleration, power, armature current, time error, tie-line, frequency, airgap flux, terminal voltage feedback control and different combinations of these. However, when these are properly applied, they provide good damping of rotor oscillations after a disturbance and improve system stability [8], but its assessment is only possible in a laboratory as it is cumbersome to use analytical techniques when different combinations of the signals are used.

Rotor angle has been suggested in [9] as a very vital control signal and this work aims at evaluating the efficacy of the use of such signals for the control of the same machine. Load variations in a synchronous power system produce changes in the load angles of the synchronous generators feeding it. If these load angle variations are known, then their influence on the generator terminal voltage can be predicted and the variations needed in generator excitation and steam flow to counteract the disturbance and maintain the terminal voltage at its desired level can be effected. The control signal (load angle) is derived from an auxiliary tacho-synchronous machine [9] attached to the shaft of the synchronous generator, thus making the signal the rotor angular position type, which is a more direct way of measuring and synthesizing δ. Use will be made of simple feedback control techniques employing time response analysis in analyzing such machine dependent signals in the control of both the exciter and the governor. The overall idea is to employ proportional control to strengthen or weaken the field and at the same time bias the governor in order to prevent the alternator from becoming unstable.

2. STUDY OF SYNCHRONOUS MACHINE BEHA VIOR

The linearized equation of a synchronous machine reveals a transfer function in operational form as:

$$\frac{\Delta T_1}{\Delta \delta} = MP^2 + K_D P + \frac{{V^2}_b(X_d - X_q)}{X_d - X_q}\cos 2\delta_0$$

$$+ \frac{G(P)g(p)V_b \sin\delta_0 \left(X_{dm}X_qV_b \sin\delta 0_0 {V'}_q - X_{qm}X_dV_b\cos\delta 0_0 {V_d}'\right) + V_b^2\sin^2\delta_0 X_q(X_d(P) - X_d)}{X_d - X_qG(P)g(p){V_0}'X_1 - X_d(P))} \quad (1)$$

The derivation of equation (1) is shown in the Section 6 (Appendix). In the frequency domain, the condition necessary for continued sinusoidal oscillation may be obtained by substituting p = jω in equation (1). For this type of solution, no disturbing force is necessary i.e. $\Delta T_i = 0$. When this substitution has been made, it is clear that equation (1) will reduce to the sum of a real and an imaginary part, both of which must be zero for the equation to be satisfied. Equating the real part to zero gives an equation, which may be solved to determine the natural frequency of oscillation of the system. The imaginary part reveals the amount of damping (either positive or negative) required to cancel that inherent in the system. However, this entails enormous computational aid and even after that, stability assessment will not be in the form of angle-time plots. To obtain such analytical plots, the machine response to a unit step mechanical load input will be used to assess system stability. The machine chosen for this study is a three -phase salient pole alternator delivering power to an infinite bus through a double circuit transmission line.

2.1 THE UNCONTROLLED MACHINE

Firstly, only the damper windings will act as the stabilizer for the machine. Because of the absence of a voltage regulator, the effect of damper windings in this case is represented as K_D as in equation (2) below.

For an unloaded synchronous machine,

$$G(P) = \frac{1}{1+\tau_{do}p}, X_d(P) = \frac{X_d + X_d{}'\tau_{do}P}{1+\tau_{do}p}, g(p) = 0;\ K_D = \frac{\tau_{do}(X_d - X_d{}')V_b^2 \sin^2\delta_0}{X_d^2 + \omega^2\tau_{do}X'{}_d^2} \qquad (2)$$

When the equation set (2) are substituted into equation (1), the transfer function of a synchronous machine, which has neither a regulator nor a governor, is obtained as:

$$\frac{\Delta_{T1}}{\Delta\delta}(P) = MP^2 + K_DP + \frac{V_bV_{fd}}{X_d}\cos\delta_0 + \frac{V^2{}_b(X_d - X_q)}{(X_dX_q)}\cos 2\delta_0 + \frac{V^2{}_b\sin^2\delta_0(X_d - X_q)t_{do}P}{X_d(X_q + X_dt_{do}P)} \qquad (3)$$

$$Or\ \frac{\Delta T_t}{\Delta\delta}(P) = MP^2 + f(\delta_0) + f(P) \quad (4)$$

Where

$$f(\delta_o) = \frac{V_bV_{fd}}{X_d}\cos\delta_o + \frac{V^2b\,(X_d - X_q)}{X_dX_q}\cos 2\delta_o,$$

$$and\ f(P) = \frac{V_b^2\sin^2\delta_o(X_d - X'_{d'})\tau_{do}P}{X_d(X_q - X_d\tau_{do}P)} + K_DP \qquad (5)$$

Equation (4) can also be rewritten as:

$$\frac{\Delta\delta}{\Delta T_1}(P) = \frac{1}{MP^2 + f(\delta_o) + f(P)} \qquad (6)$$

Dividing the numerator and the denominator by Mp^2, we have:

$$\frac{\Delta\delta}{\Delta_{T1}} = \frac{\frac{1}{MP^2}}{1 + (f(P) + f(\delta_o))\frac{1}{MP^2}} \qquad (7)$$

Equation (7) represents an output (load angle) and mechanical input (torque) ratio in the frequency domain. A simple block diagram that represents equation (7) is shown in Fig. 1 and this is the basic closed loop pattern of an uncontrolled synchronous generating system. The major difference between this and a conventional closed loop control system is that the feedback is inherent and cannot be broken; therefore, stability assessment techniques in the frequency domain like the use of Nyquist stability plots can only be applied analytically but not experimentally. It can be observed that in order to control the machine, the basic parameter to be modified is the function f(p). This is because it is the term containing the damping factor. The nature of equation (7) shows that the machine transfer function and hence the response to perturbations will be a function of only the rotor angle.

The parameters of the machine under study are as follows:

$$\times_d = 1.85, \times_d{}^! = \times_q = 1.755, \tau_{do} = 7.8s, H = 3.37, V_b = 1.0, V_f = 1.0 (all\ values\ in\ p.u)$$

After the substitution of these values, equation (7) can be expressed as:

$$\frac{\Delta\delta}{\Delta T_i}(s) = \frac{N_1s + N_O}{D_3s^3 + D_3s^2 + D_1s + D_O} \qquad (8)$$

where s = p = jw, and N_1,N_o, D_3 to D_o are constants based on the machine parameters and operating load angle.

The application of the residue theorem to equation (8) shows that it can be reduced to

$$\frac{\Delta\delta}{\Delta T_i}(s) = \frac{k_1}{s+a} + \frac{A+jB}{s+\alpha-j\beta} + \frac{A-jB}{s+\alpha+j\beta} \qquad (9)$$

For a step input of unit amplitude A, the response becomes:

$$\Delta\delta(s) = \frac{1}{s} + \frac{K_1}{s+a} + \frac{A+jB}{s+\alpha-j\beta} + \frac{A-jB}{s+\alpha+j\beta} \quad (10)$$

The time domain expression of this function may then be obtained as:

$\Delta\delta(t) = 1 + K_1 e^{-at} + 2e^{-\alpha t}[A\cos\beta t - B\sin\beta t]$ (11)

To assess the stability of the machine under different loading conditions, plots of equation (11) against time for different values of 0_0 is obtained and are as shown in Fig. 2. It is seen that at different loading conditions, the overall response is a damped sinusoid described about an operating point. The plots are made on a per unit of operating load angle. This shows that the damping decreases as 0_0 is increased. This continues up to $\delta_o=90^0$ when the machine loses synchronism implying that the exponential term of equation (11) will be a non-decaying lime function.

2.2 Control of excitation alone

If the machine of Fig 1 is now subjected to the control of AVR with terminal voltage feedback, the complete transfer function can be obtained by substituting:

$G(P) = \frac{1}{1+\tau_{do}}$; $X_d(P) = \frac{X_d + X_d'\tau_{do}P}{1+\tau_{do}P}$; $g(P) = \frac{-\mu}{1+\tau_e P}$' (12)

and

$$K_D \frac{\frac{V^2 b\sin\delta_o}{\times q}\begin{bmatrix}\mu\tau_{do}(V_d X_d X_{qm}\cos\delta_o - V_q X_{dm} X_q \sin\delta_o \\ -\mu\tau_e(V_d X_d X_{qm}\cos\delta_o - V_q X_{dm} X_q\sin\delta_o \\ +\tau_{do}(X_d - X_d)X_q\sin\delta_o\end{bmatrix}}{(\mu = qX_t + X_d - X_d\tau_e\tau_{do}\omega)^2 + \omega^2(\tau_e X_d + \tau_{do}X_d)^2}$$ (13)

into equation (1).This yields:

$$\frac{\Delta T_1}{\Delta\delta}(P) = MP^2 + K_D P + \frac{V_b V_{fd}}{X_d}\cos\delta_o + \frac{V^2{}_b(X_d - X_q)}{X_d X_q}\cos 2\delta_o$$
$$+\frac{V_b^2\sin^2\delta_o(X_d(p) - X_d)X_q + G(P)g(p)V_b\sin\delta_o\{\Psi(\delta_o)\}}{X_d X_q[G(p)g(p)V_q X_t - X_d(p)]}$$ (14)

or $\frac{\Delta T_1}{\Delta\delta}(P) = MP^2 + f(\delta_0) + f_R(P)$ (15)

$$f_R(P) = \frac{V_b^2\sin^2\delta_o(X_d(p) - X_d)X_q + G(P)g(p)V_b\sin\delta_o\{\Psi(\delta_o)\}}{X_d X_q[G(p)g(p)V_q X_t - X_d(p)]} + K_D P$$ (18)

Where

$\Psi(\delta_o) = (X_{dm}X_q V_b\sin\delta_o V'_q X_{qm} X_d V_b\cos\delta_o V'_d)$,

$f(\delta_0) = \frac{V_b V_{fd}}{X_d}\cos\delta_0 + \frac{V^2 b(X_d - X_q)}{X_d X_q}\cos 2\delta_0$

(17)

and

Rewriting equation (15), we have:

$$\frac{\Delta\delta}{\Delta T_i} = \frac{\frac{1}{MP^2}}{1 + (f(\delta_0) + f_R(P))\left(\frac{1}{MP^2}\right)}$$ (19)

Again, equation (19) represents an *output (load angle*-input (torque) ratio in frequency domain. In block diagram form, equation (19) is also as shown in Fig. 1 with the only difference that the feedback path, f (p) now modifies to f_R (p) according to equation (18). It can be seen that the regulator combines favorably with the basic closed loop pattern of the synchronous machine where it modifies the feedback element and therefore improves system damping. The forward path is however, not affected by this. For ease of transformation, equation (19) can be rewritten as a quotient of two polynomials:

$$\frac{\Delta\delta}{\Delta T_i}(s) = \frac{N_2 s^2 + N_1 s + N_O}{D^4 s_4 + D_3 s^3 + D_2 s^2 + D_1 s + D_o}$$ (20)

Where S=P=jw, N_2 to N_O and D_4 to D_o are constants depending on the per unit parameters of the machine and voltage regulator gain. When equation (20) is decomposed it becomes:

$$\frac{\Delta\delta}{\Delta T_i}(s) = \frac{K_1}{s+a} + \frac{K_2}{s+b} + \frac{A+jB}{s+\alpha-j\beta} + \frac{A-jB}{s+\alpha+j\beta}$$ (21)

For a step input of unit amplitude, the response becomes:

$$\Delta\delta(s) = \frac{1}{s} + \frac{K_1}{s+a} + \frac{K_2}{s+b} + \frac{A+jB}{s+\alpha-j\beta} + \frac{A-jB}{s+\alpha+j\beta}$$ (22)

The time domain expression of this function may then be obtained as:

$$\Delta\delta(t) = 1 + K_1 e^{-at} + K_2 e^{-bt} + 2e^{-at}[A\cos\beta t - B\sin\beta t] \quad (23)$$

This represents two decaying exponential terms and a damped sinusoidal term. Other additional machine and system parameters (in per unit) needed for this study are:

$$V_d = 0.974, V_q = 0.225, V_m = 1.0, V_d! = 0.79, V_q! = 0.60, X_t = 0.4$$

For regulator gain values ranging from 1.0 to 0.22, and operating load angle of δ_0=50°, the response is as shown in Fig. 3(a). Decrease in regulator gain produces increased damping and reduced undamped natural frequency of oscillation The reduction in the undamped natural frequency of oscillation is advantageous in that it prolongs the period of oscillation to allow enough time for speed governors to act. However, if the regulator gain is decreased below 0.24, damping is greatly improved but the regulator will begin to introduce sub-transient effects. Hence the optimal value of regulator gain for damping as far as this study is concerned will be some value above 0.25. This is similar to the findings of Aldred and Shackshaft [2] using frequency response methods.

Following as above for the same machine with excitation control only, but this time around, utilizing the load angle signal obtained in [10] for control, the response equations in the frequency domain will be:

$$\frac{\Delta\delta}{\Delta T_i}(s) = \frac{N_8 s^8 + N_7 S^7 + \cdots\cdots\cdots + Ns + N_O}{D_{10} S^{10} + D_9 S^9 + \cdots\cdots\cdots + Ds + D_O} \quad (24)$$

where N_8 to N_o and D_{10} to D_o are constants depending on the parameters of the machine system under study . Equation (24) can also be transformed to its time response form as:

$$\Delta\delta(t) = 1 + \sum_{i=1}^{4} A_1 e^{-at} + \sum_{i=1}^{4} 2e^{-at}(B_i \cos c_i t - C_i \sin c_i t) \quad (25)$$

Plots of equation (25) for values of regulator gain ranging from l00 to 0.1 and operating load angle of δ_0=50^0, are as shown in Fig 3(b). Comparing Figs. 3(a) and 3(b), it can be seen that the regulator provides good damping in a much shorter time even at very high values of regulator gain when terminal voltage is used. Decrease in regulator gain produces increased damping and reduced undamped natural frequency of oscillation thus prolonging the period of oscillation in both cases. However, if regulator gain values are reduced below 0.25, damping is greatly improved in Fig 3(a), but the regulator will begin to introduce sub-transient effects. This phenomenon is not observed in Fig 3(b) even though a wider range of loop gain values were used.

2.3 Control of steam flow alone

If only a speed governor now controls the machine, the control diagram will be as shown in Fig 4. The response times, τA, τB, τC τD, τE and τF are all delays associated with the governor pipework and oil servomechanism. The response equation in the frequency frame will be:

$$\frac{\Delta\delta}{\Delta T_O} = \frac{N_8 P^8 + N_7 P^7 + \cdots + N_2 P^2 + NP + N_C}{D_{10} P^{10} + D_9 D^9 + \cdots + D_2 P^2 + DP + D_c} \quad (26)$$

Equation (26) transforms to:

$$\Delta\delta(t) = 1 + \sum_{i=1}^{4} Ae^{-at} + \sum_{i=1}^{3} 2e^{-at}(B_i \cos c_i t - C_i \sin c_i t$$

(27)

Plots of equation (27) at an operating load angle of 50_0 and loop gain values ranging from 0.1 to 1 are shown in Fig. 5(a). At a governor gain value of 0.8, the response is a sustained oscillation (hunting) indicating that at this value of governor gain, the machine is marginally stable. Any further increase in gain value results in unbounded response.

If on the other hand, the steam input of the machine under consideration is now controlled by load angle signals, the control diagram will again be similar to that of Fig. 4 with the sole exception that the feedback term is $\frac{K}{(1+\tau_1 P)(1+\tau_M P)(1+\tau_N P)}$ where τ_I., τ_M and τN are delays in the load angle sensing equipment, relays and solenoid respectively that models the system. The values of these response times range from 2ms to 10ms [1, 12]

The response equation in this case then becomes:

$$\frac{\Delta\delta}{\Delta T_o} = \frac{N_9 p^9 + \cdots . + N_8 P^8 + \cdots + N_2 P^2 + PN + N_o}{D_{11} P^{11} + D_{10} p^{10} + \cdots + D_2 P^2 + DP + D_o} \quad (28)$$

This equation transforms to:

$$\Delta\delta(t) = 1 + \sum_{i=1}^{3} Ae^{-at} + \sum_{i=1}^{4} 2e^{-at} (B_i \cos c_i t - C_i \sin c_i t \quad (29)$$

If the analysis is performed as above, a plot of equation (29) for various values of governor gain is shown in Fig. 5(b). It is seen from both Fig. 5(a) and (b) that the responses are perfectly described about the operating points 0_0 In terms of damping and speed of response, Fig. 5(b) is undoubtedly more superior to Fig. 5(a). This can be easily attributed to delays in the speed governor loop. Suffice it to mention here that the responses of Fig. 5(b) do not vary appreciably over the wide range of gain values considered. This implies that the system is "stiffer" when controlled by δ signals.

2.4 Simultaneous control of both the excitation and steam flow

In most AC generators, excitation and steam flow control are expected to run concurrently for best results. Excitation control is expected to maintain terminal voltage within some specified limits and dictate the flow of reactive power, while steam flow control (governing) looks after the load angle and real power flows. In order to investigate the performance of this combined operation, we first consider the concurrent use of terminal voltage controlled excitation and speed governor on the system under study. The control loop will be of the form shown in Fig. 6(a), which for all intents and purposes can also be represented as Fig. 6(b) with no loss of generality.

The transfer function of Fig. 6(a) in operational form is:

$$\frac{\Delta\delta}{\Delta T_o} = \frac{N_{10} p^{10} + N_9 P^9 + \cdots + N_2 P^2 + NP + N_C}{D_{12} P^{12} + D_{11} p^{11} + \cdots + D_2 P^2 + DP + D_c} \quad (30)$$

The time response equivalent is:

$$\Delta\delta(t) = 1 + \sum_{i=1}^{3} Ae^{-at} + \sum_{i=1}^{4} 2e^{-at} (B_i \cos c_i t - C_i \sin c_i t \quad (31)$$

Plots of equation (31) at an operating load angle of 50^0 and voltage regulator and speed governor loop gain of 0.25 and 0.22 respectively is shown in Fig.7(a). Finally, when load angle signal is used for the control of both the exciter and governor, the control loop of Fig. 8 is obtained. The transfer function of Fig. 8 in operational form is:

$$\frac{\Delta S}{\Delta T_i}(s) = \frac{N_{12}p^{12}+N_{11}P^{11}+\cdots\cdots\cdots+NP+NP+N_O}{D_{14}P^{14}+D_{13}p^{13}+\cdots\cdots\cdots+DP+DP+D_o} \quad (32)$$

After due transformations, equation (32) becomes:

$$\Delta\delta(t) = 1 + \sum_{i=1}^{6} k_i e^{-at} + \sum_{i=1}^{4} 2e^{-at}(A_1 \cos c_1 t - B_1 \sin C_1 t) \quad (33)$$

For an operating load angle δ_O of 50^0 voltage regulator and governor gain values of 0.25 and 0.22 respectively, the plot of Fig. 7(b) is the response of the machine under study to unit step mechanical input power when both excitation and governor are controlled by load angle signals.

The distortions in the plot of Fig 7(a) can be attributed to the non- coordination of control i.e. the governor and exciter are not responding at the same time because of the delays in the governor loop. From Fig 7(b), it is seen that each controller provides its own contribution to system damping and effectively stabilizes the machine in a very much shorter time than obtained elsewhere. This plot reveals that a better damping is obtained when both the exciter and the governor are controlled by a single variable related to its load angle. Since both controllers aid each other, there is therefore a co-ordination of control. The level of this co- ordination is more than that reported in [13] and [14].

3. Features of the new control scheme

The following features make the use of load angle related signals for control of synchronous generators more attractive than the control of steam input using centrifugal watt governors and excitation using terminal voltage:

- It has been shown that a fast acting automatic voltage regulator may limit the initial rotor excursion, and a governor will reduce the rotor oscillation when a synchronous generator is subjected to severe system disturbance if the control signal is the load angle. With these control devices, the generator will resynchronize in a much shorter time than has been observed with speed governor controls and this suggests that ultimate stability limit may be higher than has been hitherto accepted by other researchers.
- The effective synchronizing stiffness of the machine is increased and as a result, the load-angle range for stable operation is extended. This increased stiffness will be very helpful when the machine is required to operate with reduced excitation e.g. on a leading power factor. The extension of steady-state operation does also imply an increased stability under transient conditions.
- The transmission network may vary considerably during system operation and because the control signal is non-linear, it is independent of the operating point on the system. This is considered to be advantageous because the tuning of linear controllers for a particular operating point requires additional apparatus and this in turn will require a high level of precision.
  - The control signal is sensitive to the actual parameter being modified (output power) which is directly proportional to $\sin\delta$.
  - Analysis performed on the use of load angle signal for excitation control shows that it is less effective in damping than

the results obtained when terminal voltage was used. There is, however, no sub-transient effect when load angle signal is used. More importantly, if the excitation control is used in conjunction with a governor, a superior damped response is obtained due to the co-ordination of control.

- The load angle control scheme in addition to its simplicity exhibits very good damping even at very high values of loop gain. In fact, the influence of loop gain values is very minimal.
- It is now obvious that in an interconnected system in which different generator sets are controlled differently, the set that is controlled by load angle will be faster in response than the other generating sets. This implies that it will always take a larger share of the busbar load than other sets and will continually be disturbed by those that have slower responses. It is therefore recommended that all sets of an interconnected system be governed in the same way.

4. Conclusions

The entire study has revealed how the various components of the synchronous machine control loop affect its damping. It has been demonstrated that by employing a load angle signal to the control of both the exciter and the prime mover, a very substantial improvement is effected both in the steady state and transient stability of a synchronous machine. The most attractive features of the new control tool are that responses are fast and the system works very well at a very wide variety of loop gain values and the stiffness of the system is increased.

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Appendix: Derivation of the small-oscillation equations for a synchronous generator: Park's d-q transformation

The small oscillation theory is bade on Park's [15] equations for an ideal synchronous machine [16], with no amortissuer windings, connected to an infinite system via a transmission line. The significant advantage of this procedure is that the inductance coefficients in the equations are independent of the rotor phase angle θ, which is not so if the phase equations are used. The procedure has two uses:

(a) By restricting the changes under investigation to a narrow band, there is little variation in flux levels, and the inductances can be regarded as constants.

(b) By restricting the changes to a narrow band, the terms involving the product of two (or more) deviations can be neglected, because it will be small with respect to other terms. The procedure of linearization by small departures is to set up subsidiary equations by replacing each variable (such as I_d) by a reference level plus a deviation (i_{do} + ΔI_d). The equations relating deviations from a reference situation have the same form as those for the original variables except where a product of variables occurs. For example consider an equation such as z = xy; this is written as

$$Z_O + \Delta Z = (X_0 + \Delta X)(y_0 + \Delta y)$$

$$= X_0 y_o + y_0 \Delta X + X_o \Delta y + \Delta X \Delta y \quad \text{(A1)}$$

Terms in the reference level only can be eliminated ($z_o = x_o y_o$), leaving equations between the deviations with the coefficients that can be considered constant over a limited region. Elimination of $z_o = x_o y_o$ yields

$$\Delta z = yo\Delta X + Xo\Delta y + \Delta X \Delta y \quad \text{(A2)}$$

Because the deviations are very small, the product ($\Delta x \Delta y$) tends to zero and the expression becomes:

$$\Delta z = yo\Delta X + Xo\Delta y \quad \text{(A3)}$$

The essence of Park's procedure is to establish an ideal model machine, which may be analyzed as a system of coupled circuits; to set up the differential equations of the mode; and to use a transformation of the variables to linearize the equations, which may then be expressed in operation form.

For a synchronous machine having an excitation winding on the rotor and a three-phase armature winding on the stator, Park's linearizing transformation replaces the armature phase quantities by two-axis quantities associated with the direct and quadrature axes on the rotor. Consider a three-phase salient pole synchronous machine shown in Fig.9.

$$V_a = V \sin\theta$$
$$V_b = V \sin(\theta - 2\pi/3)$$
$$V_c = V \sin(\theta - 4\pi/3)$$

The transformation for the voltage variable to d-q axis is

$$\begin{bmatrix} V_d \\ V_q \\ V_O \end{bmatrix} = \frac{2}{3} \begin{bmatrix} \cos\theta & \cos\left(\theta - \frac{2\pi}{3}\right) & \cos\left(\theta - \frac{4\pi}{3}\right) \\ \sin\theta & \sin\left(\theta - \frac{2\pi}{3}\right) & \sin\left(\theta - \frac{4\pi}{3}\right) \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix} \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix}$$

and similar equations apply to the phase currents and flux linkages. With these transformations, the equations of the synchronous machine may be written in the form:

$$D\ axis: V_{dm} = p\Phi_{dm} - \Phi_{qm} p\theta - R_m I_d$$
$$Q\ axis: V_{qm} = p\Phi_{qm} - \Phi_{dm} p\theta - R_m I_q$$
$$field: V_{fd} = p\Phi_{fd} + R_{fd} I_{fd}$$
$$where: \Phi_{fd} = X_{fd} I_{fd} - X_{ad} I_d; \Phi_{dm} = X_{ad} I_{fd} - X_{dm} I_d; and$$
$$\Phi_{qm} = X_{qm} I_d$$

The generated torque is:

$$T_u = \Phi_{dm} I_q - \Phi_{qm} I_d$$

The equations for a transmission line In Park's reference frame are as follows:

$V_d = p\Phi_{dt} - \Phi_{qt} p\theta - R_t I_d + V_{dm}$
$V_q = p\Phi_{qt} + \Phi_{dt} p\theta - R_t I_q + V_{qm}$
where $\Phi_{dt} = -X_t I_d$; and $V_{dt} = -X_t I_q$

The suffixes m and t indicate machine and transmission line quantities respectively. The *d* and *q* equations for the machine may be added to the *d* and *q* equations for the transmission line to give:

$V_d = p\Phi_d - \Phi_q p\theta - RI_d$
$V_q = p\Phi_q + \Phi_d p\theta - RI_q$
where $\Phi_d = \Phi_{dm} + \Phi_{dt}$; $\Phi_q = \Phi_{qm} + \Phi_{qt}$; and
$R = R_m + R_t$

The direct- and quadrature-axis components of the busbar voltages are as follows:

$V_d = V_b \sin\delta$
$V_q = V_b \cos\delta$

The equation of motion of the system is:

$$T_i = M\frac{d^2\delta}{dt^2} + K_D\frac{d\delta}{dt} + \Phi_d I_q - \Phi_q I_d$$

The following assumptions are also made:

(a) The speed change during the disturbance, which results in the machine remaining in synchronism, is a negligible fraction of fundamental speed, i.e. $p\theta$ is assumed constant and in the per-unit quantity is equal unity.
(b) Armature and transmission line resistance is negligible.
(c) The voltage induced in the armature by the rate of change of armature flux linkages, namely $p\Phi_d$ and $p\Phi_q$, are negligible when compared with the voltages $\Phi_q p\theta$ and $\Phi_d p\theta$, generated by the fluxes Φ_d and Φ_q rotating at fundamental speed.

With these assumptions, the equations of synchronous machines and transmission line reduce to:

$V_d = X_q I_q$ ....(A4)
$V_q = G(p)V_f - X_d(p)I_d$ ....(A5)
$V_d = V_b \sin\delta$ ....(A6)
$V_q = V_b \cos\delta$ ....(A7)
$T_i = Mp^2\delta + K_D p\delta + \Phi_d I_q - \Phi_q I_d$ ....(A8)

The operational expressions for G(p) and $X_d(p)$ are as follows:

$$G(p) = \frac{1}{1+\tau_{do}p} \quad \text{and} \quad X_d(p) = \frac{X_d + X_d'\tau_{do}p}{1+\tau_{do}p}$$

$V_f = g(p)(V_m - V_r)$ ....(A9)
$V_m^2 = V^2_{dm} + V^2_{qm}$ ....(A10)
$V_{dm} = V_d - X_t I_q$ ....(A11)
$V_{qm} = V_q + X_t I_d$ ....(A12)

Where $g(p) = \frac{-\mu}{1+\tau_e p}$

From equation (A4),
$\Delta V_d = X_q \Delta I_q$ ....(A13)
From equation (A5),
$\Delta V_q = G(p)\Delta V_f - X_d(p)\Delta I_d$ ....(A14)
From equation (A6),
$\Delta V_d = V_b \cos\delta_o \Delta\delta$ ....(A15)
From equation (A7),
$\Delta V_q = -V_b \sin\delta_o \Delta\delta$ ....(A16)
From equation (A8),
$\Delta T_i = \Phi\Delta I_q + I_{qo}\Delta\Phi_d - \Phi_{qo}\Delta I_d - I_{do}\Delta\Phi_q + Mp^2\Delta\delta + K_D p\Delta\delta$ ....(A17)

Now,
$\Delta V_d = -\Delta\Phi_q$ ....(A18)
$\Delta V_q = \Delta\Phi_d$ ....(A19)

Therefore,
$\Delta T_i = I_{qo}G(p)\Delta V_f - [I_{qo}X_d(p) + \Phi_{qo}]\Delta I_d + [I_{do}X_q + \Phi_{do}]\Delta I_q + Mp^2\Delta\delta + K_D p\Delta\delta$ ....(A20)

From equation (A9),
$\Delta V_f = g(p)\Delta V_m$ ....(A21)

Substituting for ΔV_m by using equs. (A11), (A12) and (A13), we have:

$$\Delta V_f = g(p)\{V_d'\Delta V_d - V_d' x_t \Delta I_q + V_q'\Delta V_q + V_q' x_t \Delta I_d\}$$ ....(A22)

where $V_d' = \frac{V_{dmo}}{V_{mo}}$ and $V_q' = \frac{V_{qmo}}{V_{mo}}$.

If equs. (A22) is substituted in eqns. (A11) and (A12), the complete set of small displacement equations for the machine and voltage regulator may be written as:

$\Delta T_i = \{I_{qo}G(p)g(p)\Delta V_d'\}\}\Delta V_d$
$+ \{I_{qo}G(p)g(p)\Delta V_d'\}\Delta V_q$
$+ \{G(p)g(p)I_{qo}x_t V_q' - \Phi_{qo} - X_d(p)I_{qo}'\}\Delta I_d$
$+ \{\Phi_{qo} + I_{do}X_q - G(p)g(p)\, I_{qo}x_t V_d'\}\Delta I_q$ ....(A23)
$+ \{Mp^2 + K_D p\}\Delta\delta$ ....(A24)
$0 = -\Delta V_d + X_q$
$0 = \{G(p)g(p)V_d'\}\Delta V_d + \{G(p)g(p)V_q' - 1\}\Delta V_q$
$+ \{G(p)g(p)V_q' X_t - X_d(p)\}\Delta I_d$ ....(A25)
$- \{G(p)g(p)V_d' x_t\}\Delta I$ ....(A26)
$0 = -\Delta V_d + \{V_b\cos\delta_o\}\Delta\delta$ ....(A27)
$0 = \Delta V_q + \{V_b\sin\delta_o\}\Delta\delta$

In matrix form, as equations (A23 – A27) can be written as:

$$\begin{bmatrix}\Delta T_i\\0\\0\\0\\0\end{bmatrix} = \begin{bmatrix} I_{qo}GV_d' & I_{qo}GV_q' & I_{qo}GX_tV_d' - \Phi_{qo} - X_d(p)I_{qo}' & \Phi_{do} + I_{do}X_q - GI_{qo}X_tV_d' & Mp^2 + K_d p \\ -1 & 0 & 0 & X_q & 0 \\ GV_d' & GV_q' & GV_q'X_t - X_d(p) & -GV_d'X_t & 0 \\ -1 & 0 & 0 & 0 & V_b\cos\delta_o \\ 0 & +1 & 0 & 0 & V_b\sin\delta_o \end{bmatrix} \cdot \begin{bmatrix}\Delta V_d\\ \Delta V_q\\ \Delta I_d\\ \Delta I_q\\ \Delta\delta\end{bmatrix} \quad \text{(A28)}$$

In equation A28, G represents the product G(p)g(p). When eqn. (A28) is solved for the ratio of $\frac{\Delta T_i}{\Delta\delta}$, it yields:

$$\frac{\Delta T_i}{\Delta\delta} = Mp^2 + K_D p + \frac{V_b^2(X_d - X_q)}{X_d X_q}\cos 2\delta_o$$ ....(A29)

$$+ \frac{G(p)g(p)V_b\sin\delta_o\left(X_{dm}X_qV_b\sin\delta o_o V_q' - X_{qm}X_dV_b\cos\delta o_o V_d'\right) + V_b^2\sin^2\delta_o X_q\left(X_d(p) - X_d\right)}{X_dX_q(G(p)g(p)V_q'X_t - X_d(p))}$$

This expression is same as equation (1) of section 1.

Diagrams

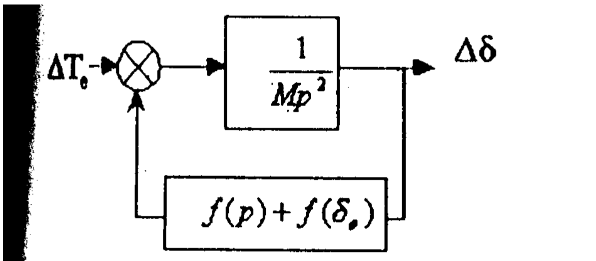


Fig. 1: The basic closed loop pattern of a synchronous machine

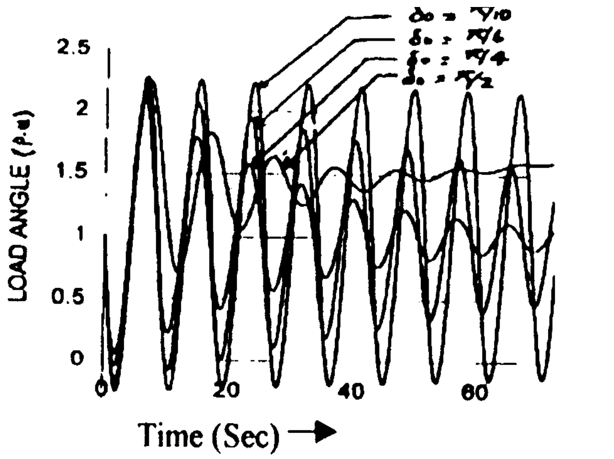
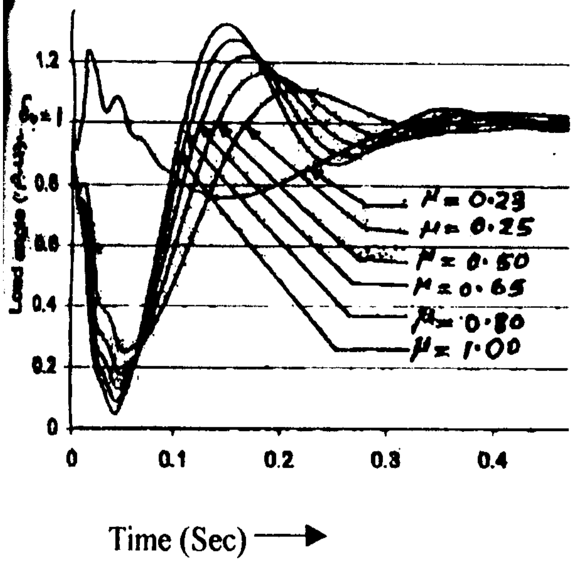
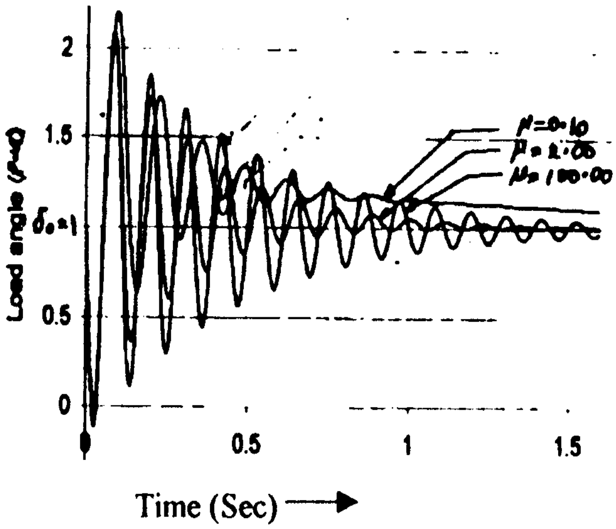


Fig. 2: Plots of load angle vs. time for the uncontrolled machine art various operating load angles.



(a) *Excitation control by terminal voltage signal.*



(b) *Excitation control by load angle signal.*

Fig. 3: Simulated angle-time plots of effect of voltage regulator gain only on stability.

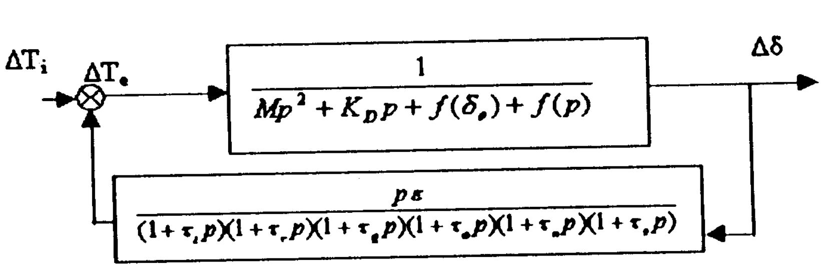
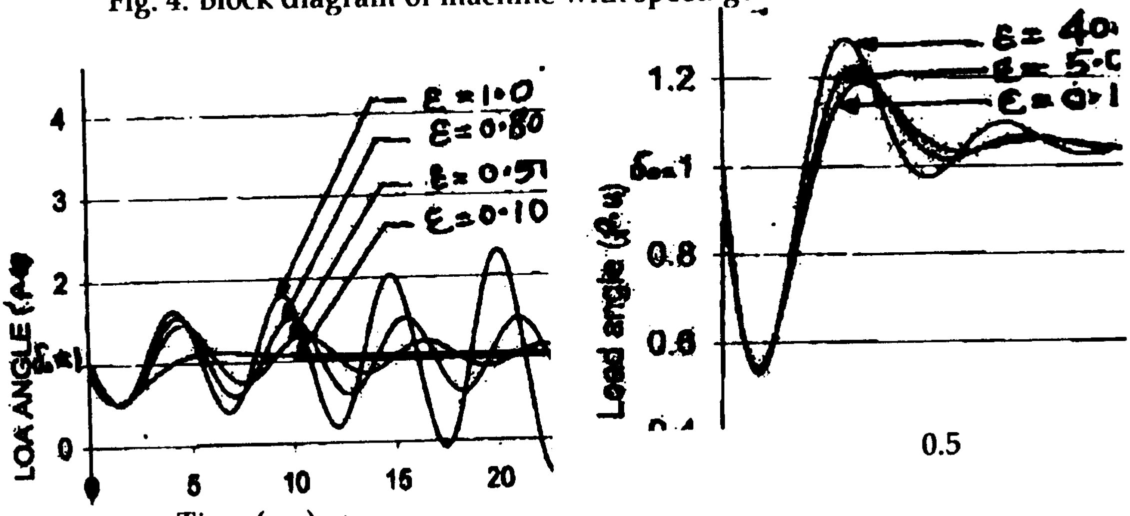


Fig. 4: Block diagram of machine with speed governor feedback

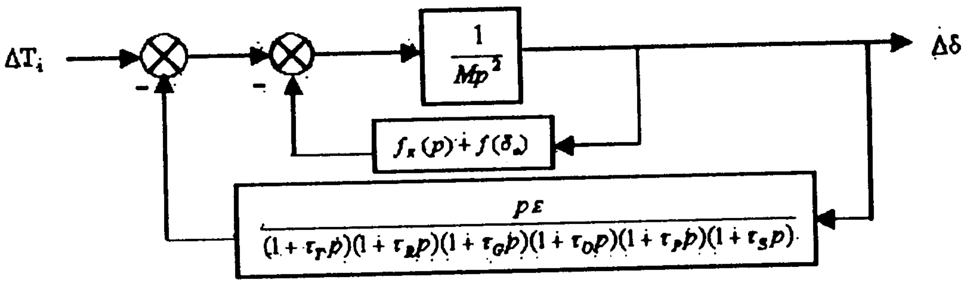


(a) *Control using speed signals*

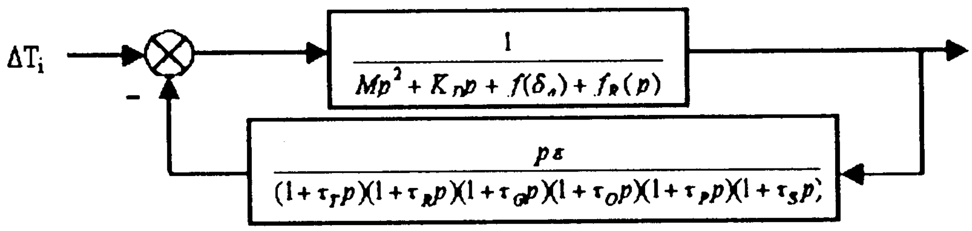
Time (sec) →

(b) *Control using load angle signals*

Fig. 5: Combined angle - time plots of effect of governor gain only on stability

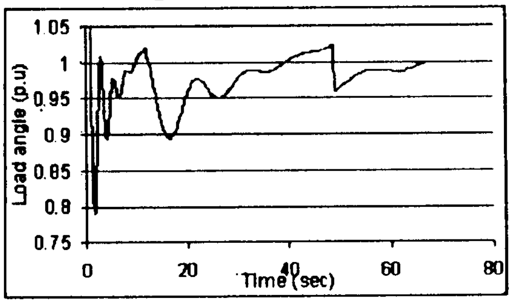


(a) *Control loop using voltage signal for excitation control and speed signal for governor*

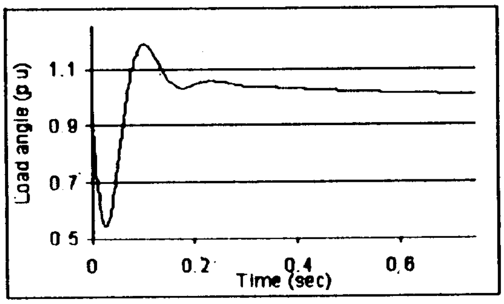


(b) *Control loop using load angle signal for control of both the exciter and governor*

Fig. 6. Combined excitation and steam flow control loops

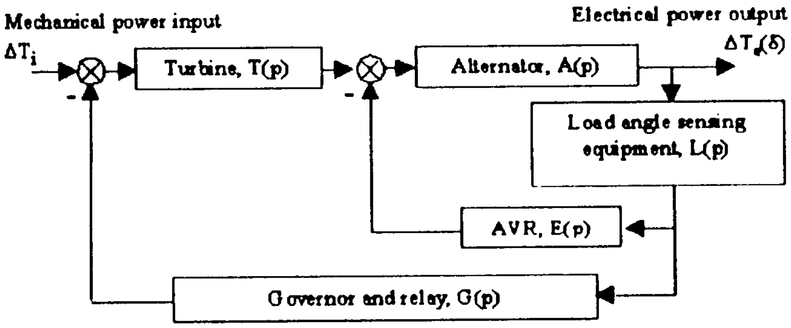


(a) Control using terminal voltage and speed signals



(b) Control using only load angle signal.

Fig. 7: Combined responses



Glossary

$$T(p) = \frac{K_2}{1+\tau_T p},$$

$$E(p) = \frac{\mu}{1+\tau_e p}$$

$$G(p) \frac{K}{(1+\tau_L p)(1+\tau_R p)}$$

and $L(p) \frac{K_1}{1+\tau_S p}$

Fig. 8: Combined control loop

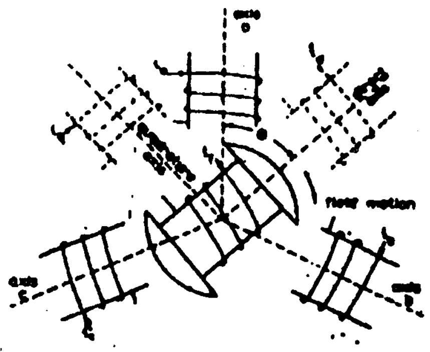


Fig. 9: d-q axes representation of a salient pole synchronous machine[17]