THE SENSITIVITY OF EVAPOTRANSPIRATION MODELS TO ERRORS IN MODEL PARAMETERS

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ABSTRACT

Five evapotranspiration (Et) model-the penman, Blaney - Criddel, Thornthwaite, the Blaney – Morin-Nigeria, and the Jensen and Haise models – were analyzed for parameter sensitivity under Nigerian Climatic conditions. The sensitivity of each model to errors in any of its measured parameters (variables) was based on the relative error introduced by the parameter in the predicted Et at various perturbations of the parameter. Three levels of sensitivity, herein termed sensitivity, ratings, were established, namely: Highly Sensitive (Rating:1); Moderately sensitive' (Rating:2); and 'not too sensitive'(Rating: 3). The ratings were based on the amount of error in the measured parameter to introduce + 10% relative error in the predicted Et. The level of importance and the care required in the measurement of each parameter with respect to the model in which it occurs are therefore established.

Keyword: sensitivity; evapotraspiration models; importance; predicted; measured.

INTRODUCTION

Indirect methods of determining evapotranspiration (Et) consist generally of the use of evapotranspiration models. These models determine evapotranspiration by relating measured climatic variables to measured evapotranspiration, Such climatic variables include air temperature, wind speed, relative humidity, solar radiation, sunshine hours, vapour pressure, etc. Two major causes have been identified as to why indirect methods do not predict or determine evapotranspiration as well as do the direct methods. The first is the inherent inability of certain models to effectively predict evapotranspiration, even under the conditions in which they were formulated. Evapotranspiration prediction capability varies from model to model. The second cause is the error introduced into model prediction due to error in the measured climatic variables used in the model. Such errors are capable of greatly undermining model prediction. Some models, however, have the ability to ideally determine evapotranspiration with an accuracy comparable to that of measured evapotranspiration. For such models, therefore, the only source of error in the measured parameters, provided, in generally they are used in the same or similar climatic conditions under which they were formulated.

A given model responds differently to error in its parameters. Error in some parameters may have serious adverse effect on performance, while error in other parameters have no appreciable effect. It is believed that, for those models known to be capable of predicting evapotranspiration acceptably well, when those "very sensitive" parameters are identified and measured accurately, the models will perform at their optimum capacity.

All evapotranspiration prediction models are mathematical expressions of the evapotranspiration phenomenon. Also, all mathematical expressions of a process involve some element of stochasticism. Hence, mathematical *models formulated* to represent a process or phenomenon will be conceptual to some extent. The reliability of such models therefore depends on the extent to which they can be verified. According to Overton and Meadows [1], no model verification is complete without proper sensitivity analysis. Work has been done in Nigeria to test the reliability of some of the evapotranspiration models in use in Nigeria [2, 3], but there has been no reported work known to the author on their sensitivity analysis. Therefore, for such evapotranspiration models to be reliably used in Nigeria, their sensitivity analysis is important.

Again, Jensen [4], writing on the use of empirical formulae for the determination of evapotranspiration rates, said that such formulae (models) can be used when the "absolute accuracy" of the measured climatic variables have been found adequate. Through sensitivity analysis, it can be ascertained what accuracy of measurement of a given parameter is needed in a model. It is only after such analysis that both the model and the measured data can be used with confidence.

The objective of this work is to determine the sensitivity (to error in model parameters) of some Et models found to determine evapotranspiration fairly accurately under Nigerian climatic conditions. Such analysis will expose those parameters in the analyzed model that will cause the model to predict poorly due to small errors in the measured parameters. When such parameters have been observed, emphasis can then be laid as to the degree of accuracy required in their measurement or estimation.

2. THE EVAPOTRANSPIRATION MODELS ANALYZED

The evapotranspiration models analyzed in this work are the Penman, Blaney and Criddle, and the Blaney-Morin-Nigeria models. Others are the Thornthwaite and the Jensen and Haise models. These models are as given *below:*

1 The Penman Model (1948)

This is given as [7]:

$$Etp = \frac{\Delta Qn + \gamma Ea'}{\Delta + \gamma}$$

Which, after expansion and substituting the necessary terms, becomes

$$Etp = [\Delta/\gamma\{(1-r)RA(0.18+0.55n/N) - \sigma Ta^4(0.55-0.092ve_d)(0.10+0.90\,n/N) +0.35\,(e_a-e_d)(1+0.0098w_2)]/(1+\Delta/\gamma) \quad (2)$$

Where

Etp = Potential evapotranspiration in mm/ day
r = Surface reflectivity = 0.05 for open water.
RA = incident radiation outside atmosphere in mm of evaporable water per day.
n = duration of sunshine hrs for the interval.
N = maximum sunshine duration for the same interval
σ = Stefan-Boltzman constant
Ta = temperature in degrees Absolute
e_d = actual vapour pressure (mm Hg)
e_a = Saturated Vapour pressure (mm Hg)
w_2 = wind speed at a height of 2 meters, in miles per day.
γ = Psychrometric constant (mb/°C)
Δ = rate of change with temperature of the saturation vapour pressure (mb/°C)
Qn = net radiation (mm of water)

2.The Blaney-Morin-Nigeria model

$$Etp = r_f(0.45T+8)(520-R^{1.31})$$

Where

Etp = Potential Evapotranspiration, mm/day
R_r= Ratio of monthly maximum possible radiation outside the atmosphere to the annual maximum radiation
T = Temperature in ^{0}C
R = Relative humidity in percent.

3. The Blaney-Criddle Model

E_{crop} = KP (0.46 t + 8.13), mm/month (4)

Where,

E_{crop} = Crop evapotranspiration, mm/ month
K = Crop factor
P = Ratio of maximum

expected sunshine hours for the month (or season) to the maximum expected hours for the year.

T_c = Temperature in ^{0}C.

4. The Jensen and Haise Model

Etp $=0.7C_t(t_c + t_\times Ra$ (5)

where,

Etp = Potential evapotranspiration, mm/day

Ct = Temperature coefficient

t_c = Mean air temperature, in oC

t_x = Intercept of Etp/Rs vs. temperature regression line with the temperature axis.

Ra = Incident radiation as equivalent depth of evaporation in mm/ day.

5. The Thornthwaite Model

Etp = 1.6Ld $(10T/I)^a$, cm/month (6)

Where

Etp = a 30-day estimate of evapotranspiration in cm.

Ld = day time hours in units of 2 hours per day

T = mean monthly air temperature in °C

I = Seasonal or annual heat index

a = an empirical exponent computed by the relation: a = 0.00000067513 - 0.000077112 - 0.017921 - 0.49239

The first three models (Penman, Blaney- Morin- Nigeria, and the Blaney-Criddle models) have been tested and found applicable in Nigeria [3,5]. The Thornthwaite model has been tested in Ibadan, Nigeria, and found fairly applicable in that part of the country [2]. The Jensen and Haise model has potential for good performance in the arid regions of Northern Nigeria, having been developed in similar arid conditions of the United States. Based therefore on what has been said about each of the models above, they were selected for sensitivity analysis.

3. METHODOLOGY

Sensitivity analysis of model parameters is carried out by keeping all parameters constant but one, and peturbating the last such that variations in the objective function can be examined [1]. These authors further stated that" if small perturbations of the parameter produce large changes in the objective function, the system is said to be sensitive to that parameter". In this study, therefore, all parameters in a given model were kept constant while the one the sensitivity of which was desired was varied from + 1 % to + 20%. It was assumed that in practice measurement errors normally would not fall outside this range. The value of the parameter at each variation was used, with other parameters at their original values, to calculate the day's potential evapotranspiration. Thereafter, the error in prediction, termed relative error (in per cent), was computed using the relationship:

$$percentage\ Relative\ Error = \frac{(Approximate\ etp - base - condition\ Ept) \times 100}{base - condition\ Etp}$$

Approximate Etp is the potential evapotranspiration computed using the approximated (varied) climatic variables, while based-condition etp is that computed from error-free (original) climatic variables.

The climatic variables used in all the analysis were collected from four locations in Nigeria judged by the author as approximately representing the various climatic conditions of Nigeria. The locations are Enugu (Lat. 6°28'N), Ilorin (Lat. 8^o 28'N), Jos (Lat. 9^{0}52' N), and Samaru, Zaria (Lat. 10^{0}11'N). Tables 1 and 2 show sample data collected and used in some of the models.

The sensitivity ratings of the parameters in a given model were arrived at as follows: if at small variations of a given parameter, form ± 1 % to ± 10%, a relative error of up to ±10% was introduced into the prediction, the model is said to be very sensitive to the parameter. (The ± 10% limit for prediction error is fixed here because beyond this limit prediction by the model may not be judged satisfactory [6]. The parameter itself is then classified as "highly sensitive", and is said to be of first order of importance in the model. It is then given a sensitivity rating of 1. If the ±10% relative error was introduced when the parameter was being varied between ±10% and ± 20%, the parameter is classified as "moderately sensitive", and is said to be of second order of importance. It is then given a rating of 2. If at±20% variation a parameter does not introduce a relative error up to ± 10%., the parameter is classified as "not too sensitive". It is of low order of importance, and is given a rating

4. RESULTS AND DISCUSSIONS

Table 3 is an example of the results obtained in this study. It is given for the Penman model, others being omitted here for the sake of brevity. Similar results were obtained for other months of the year and for the three other locations in this study.

From the columns for ratio of error to predicted Et, it can be seen those parameters that are highly sensitive, and not too sensitive moderately sensitive. A ratio of 1and above means that parameter is highly sensitive
and above means that parameter is highly sensitive'. A ratio of 0.5 and above but less than 1 means the parameter is moderately sensitive; while a ratio of less than 0.5 means the parameter is too sensitive. Fig. 1 represents the same result. Table 4 is the summary of the results for all the models analyzed.

What these sensitivity results have revealed are that for each of the models to predict Et as best as it can, all the parameters in it rated 1 must be regarded with utmost care and must be measured very accurately so that the Et predicted there from does not appreciably deviate from observed Et. Those rated 2 should, as much as possible, be measured with accuracy comparable to those rated 1. One may not bother much about the accuracy of measurement of those parameters rated 3 for use in the models in which they are so rated. Approximate values of such parameters can always be used without significantly affecting the results.

5 CONCLUSION

This work has identified those parameters in the selected models that must be measured very accurately in order that evapotranspiration be predicted with great accuracy. Information is therefore now available for all those who may be engaged in the measurement of those parameters especially for evapotranspiration purposes. It is believed that when these climatic variables are measured in accordance with the standards here (by use of suitable equipment), evapotranspiration prediction in Nigeria will be greatly improved. This, in turn, will improve the numerous designs that are usually involved in water resources planning, irrigation and hydro-geological works, etc., that require evapotranspiration rates as input. Nigeria will benefit immensely from such improved design.

Table 1: Sample data input for BMN ET model

Parameter	Error-free Parameter Value	Varied parameter values				
		1%	5%	10%	15%	20%
Radiation ration, r,	0.0854	0.0863	0.0897	0.0930	0.0982	0.1025
Temperature, T, (0C)	25.90	26.16	27.20	28.49	29.70	31.08
Relative Humidity, R, (%)	78.00	78.78	81.90	85.80	89.70	93.60

Table 2: Sample data input for Jensen and Haise ET model

Parameter	Error-free Parameter Value	Varied parameter values					
		1%	-1%	10%	-10%	20%	-20%
Temperature, t_c (^0C)	30.40	30.70	30.10	33.44	27.36	36.48	24.32
Incident Radiation, Ra, (mm/ day)	14.09	14.23	13.95	15.50	12.68	16.91	11.27

Table 3: Parameter Sensitivity in Penman Model for Samaru, Zaria

	Variations in Error-free Parameter Value											
	1%		-1%		10%		-10%		20%		-20%	
Parameter*	Error in pre-dicted Etp (%)	Ratio of error to per-cent varia tion	Error in pre-dicted Etp (%)	Ratio of error to per-cent variatio n	Error in pre-dictd Etp (%)	Ratio of error to per-cent varia tion	Error in pre-dicted Et (%)	Ratio of error to per-cent varia-tion	Error in pre-dicted Etp (%)	Ratio of error to per-cent varia- tion	Error in pre-dicted Etp (%)	Ratio of error to per-cent varia-tion
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)
Incident Radiation, Ra	1.00	1.00	-1.00	1.00	10.00	1.00	-10.00	1.00	20.00	1.00	20.00	1.00
Actual vapour pressure, e.,	0.55	0.55	-.057	0.57	5.40	0.54	-054	0.55	10.68	0.53	11.20	0.56
Actual Sunshine hours, n	0.51	0.51	-.0.32	0.32	4.24	0.22	0.4.5	0.40	08.58	0.52	-8.19	0.41
Max. Sunshine hours, N.	-0.33	0.33	0.52	0.52	-3.66	0.37	4.68	.047	-6.82	0.34	10.45	0.52
Wind Speed, W	0.12	0.12	-0.12	0.12	0.58	0.16	0.53	0.13	0.13	0.16	3.08	0.15
Air Temperature Ta	-0.09	0.09	0.09	0.09	-0.09	0.09	0.92	0.09	-1.85	0.09	1.80	0.09

*Parameters are arranged in order of decreasing sensitivity.

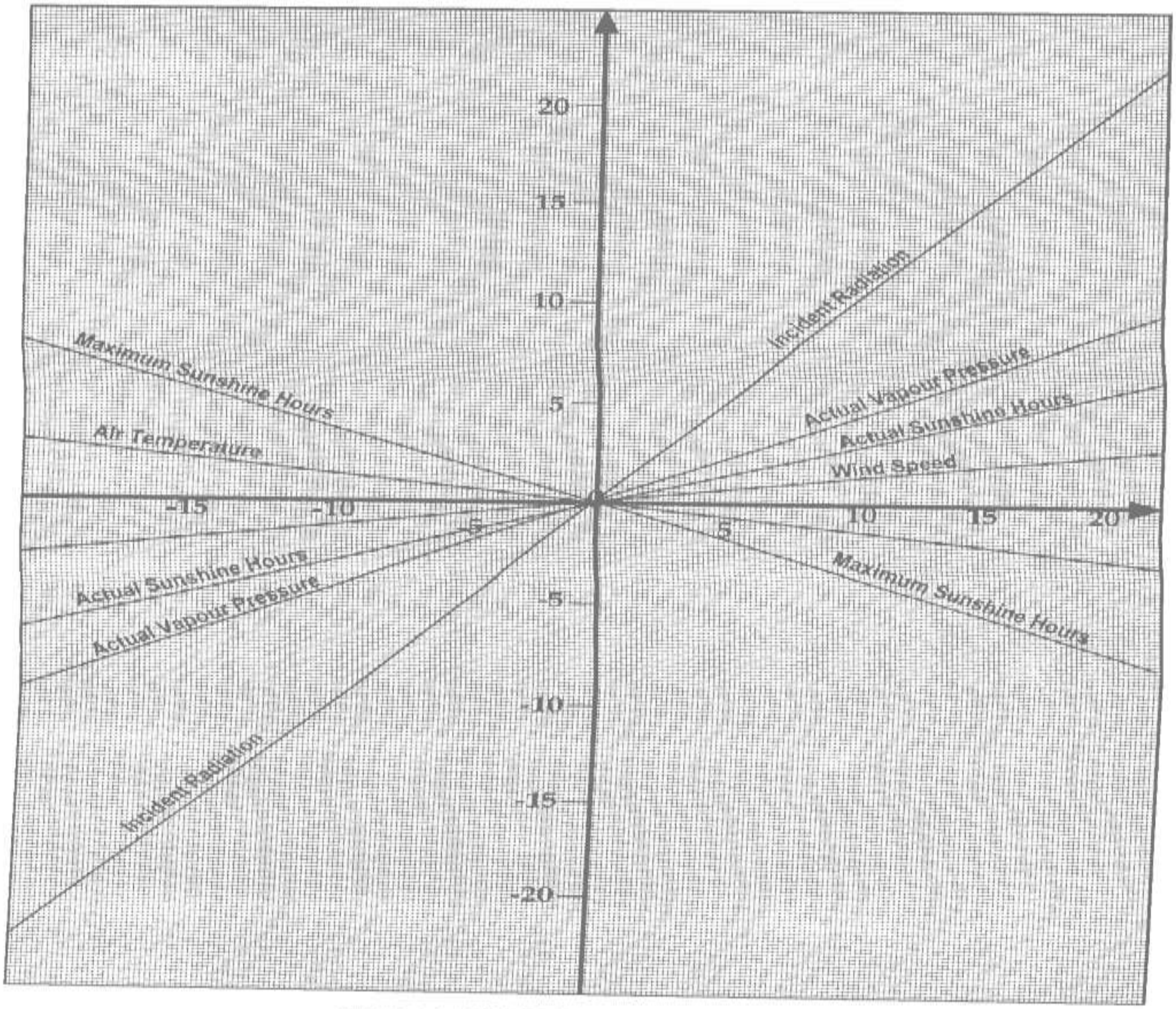

Fig. 1: *Relative error in predicted Etp by the enman model due to variations in model parameter values.*

Table 4: Summary of Parameter Sensitivity Ratings for the Selected Evapotranspiration models

Model	Parameter Sensitivity Rating		
	1(Highly Sensitive)	2(Moderately Sensitive	3(Not too Sensitive)
Penman	Incident Radiation	Vapour Pressure Actual Sunshine Hours Maximum Possible Sunshine Hours	Temperature Wind Speed
BMN	Radiation Ratio Incident Radiation	Temperature	-
Jensen and Haise	Relative Humidity Incident Radiation	Temperature	-
Blaney-Criddle	Percentage of Annual Sunshine Hour	Temperature	-
Thornthwaite	Temperature Sunshine Coefficient	-	-

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