# IMPEDANCE CALCULATIONS OF INDUCTION MACHINE ROTOR CONDUCTORS.

Dr.-Ing. O. I. OKORO
Department of Electrical Engineering
University of Nigeria, Nsukka
Enugu State.
Email: ogbonnayaokoro@hotmail.com

**ABSTRACT**

The exact calculation of the impedance of induction machine rotor conductors at several operating frequencies are necessary if the dynamic behaviour of the machine is to give a good correlation between the simulated starting torque and current and the experimental results. This paper describes a method of' calculating the impedance of Rectangular and Trapezoidal rotor bars. An R-L parallel network is used to model each of the Rotor bars. A computer optimisation Algorithm is developed and from which the Rotor circuit parameters at several frequencies are estimated. The model solutions may then be used to find any desired steady-state or transient performances of induction machines.

## INTRODUCTION

For the past 30 years, the dynamic behaviour of induction machines has received a considerable attention in most researched works [1.2, 3.4.5.6]. However, the analysis is based on the simple idealised machine model without deep-bar effect and with the assumption that the rotor resistance remains constant at the d.c. value. Consequently, this method of analysis usually leads to a very large error in the computation of the starting torque and current at several frequencies for squirrel-cage machines with deep rotor bars.

Pertinently, the need for the development of accurate models for induction machines becomes highly necessary. This is necessitated by the fact that in an induction machine, the lower portions of the bars of the rotor cage are linked by more slot leakage flux than the upper portions of the bar. Under dynamic conditions, the lower section of the rotor bar experiences a higher inductance than the upper section of the rotor bar due to non-uniform flux distribution, thereby causing the current to flow primarily in the upper portion of the bar [7] Also, the re-distribution of the current flowing in the rotor bar effectively increases the resistance of the bar. This phenomenon of decrease in inductance and increase in resistance of rotor conductors is known as the deep-bar effect or the skin -effect [7.8.9,]. This effect is predominant in machines operated from non-sinusoidal power sources and large induction machines with rotor bars that have large bar depth to bar width ratio The paper describes a method of calculating the impedance of rectangular and trapezoidal rotor conductors. An R-L parallel network used to represent the Rotor bar types is

also discussed as well as the optimisation technique applied.

## 2 The Rectangular Rotor-Bar

Figure 1 shows n sections of rectangular rotor bar. In order to derive the impedance equation for the rectangular rotor bar, the rotor bar conductor is divided into n-equal number of sections [7]. By so doing, the current distribution within each section can be considered uniform.
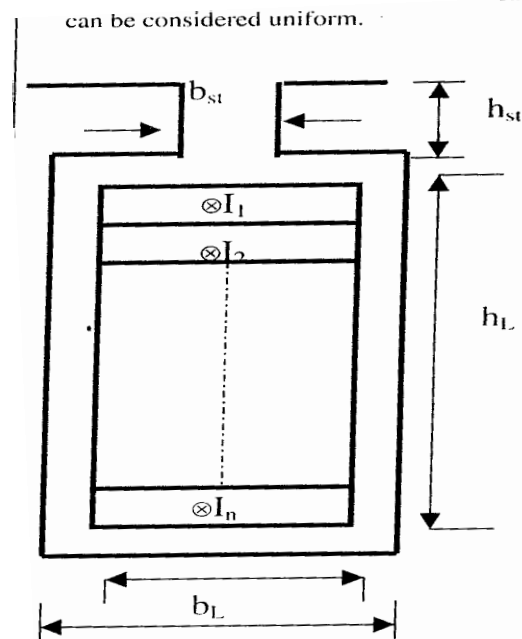


**Figure 1:** n sections of Rectangular Bar

Total resistance of the rectangular bar is,

$$R_T = \frac{L_s}{(\chi_{cu} h_L b_L)} \qquad (1)$$

The resistance of each section is therefore given by

$$R_L = (L_s)/(\chi_{cu} h_{sec} b_L) \qquad (2)$$

Where

$$h_{sec} = \frac{h_L}{N} \qquad (3)$$

The magnetic flux density in section one of the bar is

$$\underline{B}_1 = \mu O \underline{I}_1 / b_{Nut} \qquad (4)$$

Therefore, the magnetic flux between the first and ficitions section becomes,

$$\underline{\phi}_1 = B_1 L_s h_{sec} = \underline{B}_1 L_s h_{sec}$$
$$= \mu o \, h_L \, L_s \, \underline{I}_1 / N b_{Nut} \qquad (5)$$

In analogy, the magnetic flux between sections 2 and section 3, becomes

$$\underline{\phi}_2 = \left( \mu_o L_s h_L \big/ N b_{Nut} \right) * (\underline{I}_1 + \underline{I}_2) \qquad (6)$$

In general, the magnetic flux between the section n and (n+1) of the rotor bar is,

$$\underline{\phi}_n = \left( \mu_o L_s h_L \big/ N b_{Nut} \right) * \sum_{v=1-v}^{n} I \qquad (7)$$

It can be seen from figure 1 that a loop results between the nth and (n+1)th section. Therefore kirchhoff's voltage equation could be applied.

$$U = 0 = iR + \frac{d\Phi}{dt} \qquad (8)$$

Between the nth and (n+1)th section, we have

$$0 = \underline{I}_n R_L = \underline{I}_{n+1} R_L + j\omega \underline{\phi}_n \qquad (9)$$

Combining equations (7)and (9), and with little manipulation, the total bar current is

$$\underline{I}_{n+1} =$$
$$\underline{I}_n + j \left( \omega \mu_o h_L^2 b_L \chi_{cu} \big/ N^2 b_{Nut} \right) * \sum_{v=1-v}^{n} I \quad (10)$$

The total voltage is

$$\underline{U} = R_W \underline{I}_n + j\omega L_{wi} \underline{I}_n \qquad (11)$$

The total bar impedance of the rotor is calculated by adding the bar outside inductance as shown in figure 1,

$$L_a = \left( \mu_o h_{st} L_s \big/ B_{st} \right) \qquad (12)$$

$$\underline{Z}_{total} = R_{Wi} + j\omega(L_a + L_{wi}) \qquad (13)$$
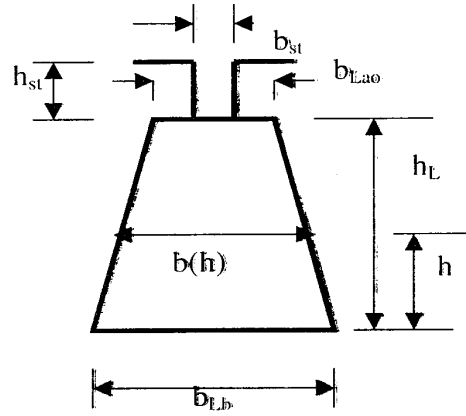
## 3. The Trapezoidal Rotor Bar



**Figure 2: Trapezoidal Rotor – Bar**

Here also, the trapezoidal rotor-bar of figure 2 is divided into n-equal sections. Each section width being a function of h and is given by [9]:

$$b(h) = b_{1b} \left( 1 - h_{(n)} \big/ h_1 \right) + \frac{h_{(n)}}{h_1} b_{La} \quad (14)$$

And

$$h(n) = \frac{h_{sec}}{2} + h_{ec}(n - 1) \qquad (15)$$

$$h_{sec} = \frac{h_1}{N} \qquad (16)$$

The resistance of each section is,

$$R_L = \frac{(N L_s)}{\chi_{cu} h(n) b(n)} \qquad (17)$$

$$\underline{U} = R_L \underline{I}_n \qquad (18)$$

The current is as given in equation (10). With equations (12), (10),and (18), the total impedance of the trapezoidal rotor conductor can be computed.
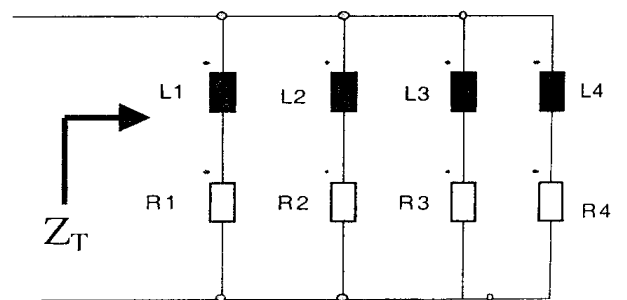
### 4 R-L Rotor Model



**Figure 3:** Parallel R-L network

The parallel R-L network shown in figure 3 is used in the modeling of the rotor bars. The network total impedance is given by,

$$Z_T = \frac{(Z_1 Z_2 Z_3 Z_4)}{(Z_2 Z_3 Z_4 + Z_1 Z_3 Z_4 + Z_1 Z_2 Z_4 + Z_1 Z_2 Z_3)} \qquad (19)$$

Where,

$$Z_1 = R_1 + j\omega L_1$$
$$Z_2 = R_2 + j\omega L_2$$
$$Z_3 = R_3 + j\omega L_3$$
$$Z_4 = R_4 + j\omega L_4$$

5. simulation Results

MATLAB m-file for the calculation of the total impedance of the rectangular and trapezoidal rotor bars is developed [10]. The parameters of the bars used in the simulation are as shown in Appendix

A. the frequency- impedance plots are presented in figures 4 and 5 for rectangular and trapezoidal rotor bars respectively.
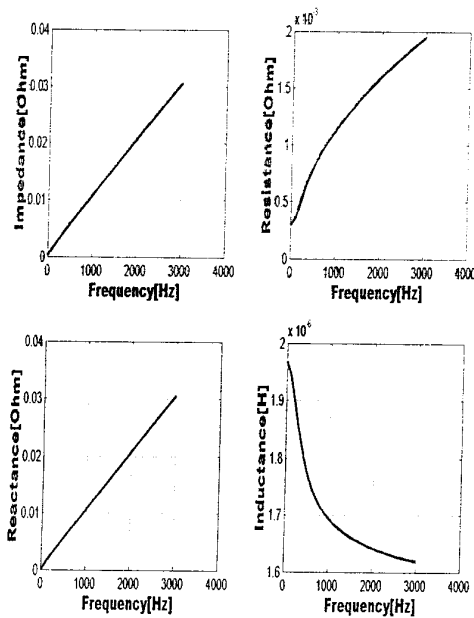


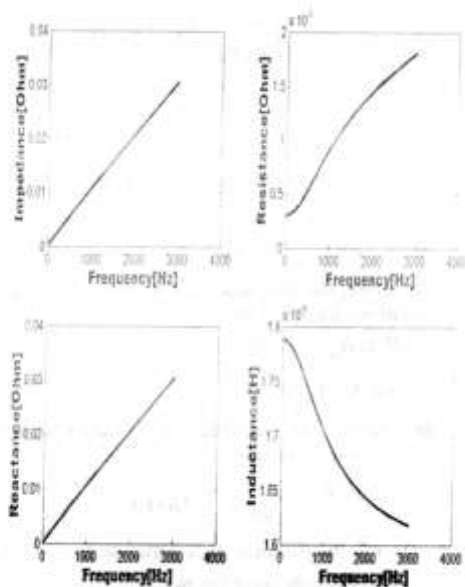**Figure 4:** Rectangular Rotor-bar plot



**Figure 5:** Trapezoidal Rotor-bar plot

To obtain the parameter of the rotor circuit model of figure 3, a simple constrained optimisation algorithm that attempts to give the best agreement between the actual impedance of the rotor bar and the model impedance is developed. In the optimisation algorithm, the cost function, F being minimized is defined by,

$$F = \sum_{i=1}^{N} \Big[ \big((real(Z_{opt}) - real(Z_T))^2\big) + imag(Z_{opt}) - imag(Z_T)\big)^2 \Big]$$

(20)

$Z_T$ give the actual rotor bar impedance at frequency, f and the optimization technique provides the values for the rotor circuit, $Z_1$, $Z_2$, $Z_3$, and $Z_4$, such that the effective rotor impedance in model is $Z_{opt}$. the frequency-impedance optimisation plots are shown in figure 6and Figure 7 while the results that give the estimated rotor circuit parameter are shown in Appendix B
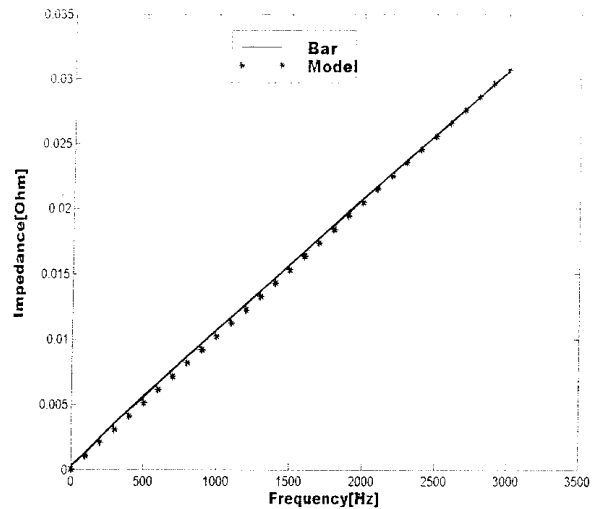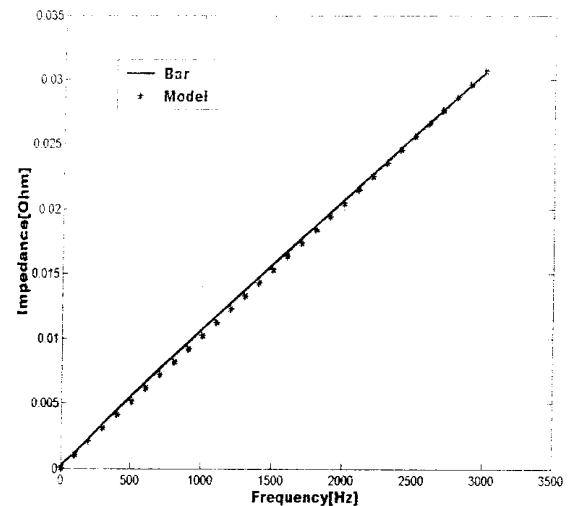


**Figure 6:** Rectangular Rotor-bar optimisation plot



**Figure.7:** Trapezoidal Rotor-bar optimisation plot

## 6 CONCLUSIONS

A simple method for calculating the impedance of rectangular and trapezoidal rotor conductors has been presented in this paper. The optimised solution of the R-L-net work rotor model as shown in Figures 6 and 7 are in good agreement with the actual rotor bar characteristics of the induction machines. The optimised solutions may then be used to find any desired steady-state or transient performances of induction machines operating over a wide range of frequencies where the deep-bar effects are noticeable.

## 7 Nomenclature

| | | |
|---|---|---|
| $L_s$ | = | length of the rotor bar |
| $\chi_{cu}$ | = | conductivity of copper conductor |
| $h_L$ | = | height of rotor conductor |
| $b_L$ | = | width or rotor conductor for rectangular bar |
| $b_{Lao}$ | = | Upper width or the trapezoidal bar |
| $b_{Lbo}$ | = | lower width or the trapezoidal bar |
| $b_{st}$ | = | width of the end ring |
| $h_{st}$ | = | height of the end ring |
| N | = | total number of sections |
| $\mu_o$ | = | permeability of free space |
| La | = | Bar outside inductance |
| $R_{wi}$ | = | the Bar resistance |
| $b_{Nut}$ | = | Width of the rotor bar |
| $\omega$ | = | angular frequency |
| $L_1, L_2, L_3, L_4$ | = | inductance of the R-L Rotor model |
| $R_1, R_2, R_3, R_4,$ | = | resistance of the R-L Rotor model |
| $R_L$ | = | resistance of each section of the bar |
| $Z_T$ | = | total mode I impedance |
| j | = | complex operator |
| F | = | Cost function |
| f | = | frequency |

## 8 REFERENCES

1. Krause, P.C; Thomas, C. H. simulation of symmetrical Induction machinery Transaction IEEE. PAS-84, Vol.11, 1965,pp .1038-1053.

2. Jordan, H. E.: Digital computer Analysis of induction machines in Dynamic systems. IEEE Transactions on power Apparatus and systems, Vol. PAS-86, No.6, June 1967.722-728.

3. Chahopadhyay, A.K.: Digital Computer Simulation of an Adjustable- Speed induction motor Drive with a cycloconverter-type thyristor- commutator in the Rotor. IEEE Transaction on industrial Electronics and control instrumentation, Feb; 1976, PP.86-92.

4. Krause.P.C: Lipo, T.A.: Analysis and Simplified Representations of a Rectifier-inverter indution motor drive. IEEE Transactions on power Apparatus and systems. Vol. PAS-88. No.5. May 1969. PP. 588-596.

5. Jordan. H.E: Analysis of induction machines in Dynamic systems. I EEE Transactions on power Apparatus and systems. Vol. PAS- 84, No. 11, Nov: 1965, PP. 1050-1088.

6. Krause, P.C.: Simulation of unsymmetrical 2- phrase induction machines. IEEE Transactions on power Apparatus and systems, Vol. PAS-84. No .11, Nov: 1965,PP.1025-1 037.

7. Cochran. P.L.: Polyphase induction motors: Analysis, Design, and Application-Marcel Dekker. Inc. Nell York.1959.

8. Alger, P.L.: lnduction machines: Their behaviour and uses: Gordon and Breach Science Publishers SA. USA, 1995

9. . Weidemann, B.:Hilfsblatter Zur Vorlesung Elektrische Maschinen, Universitat GHKassel, 1991.

10. MATLAB User's Guide, The Mathworks, Inc. Cochituatc Place, 24 Prime Parkway. Natick. 1991.

### Appendix A: Machine Data

| Description | Rectangular bar | trapezoidal |
|---|---|---|
| Type of rotor cage | Steel | Steel |
| Conductivity of rotors bars | $56 Sm/mm^2$ | $56 Sm/mm^2$ |
| Number of pole pairs | 3 | 3 |
| Number of rotor slots | 24 | 24 |
| Number of stator slots | 36 | 36 |
| Stator bore diameter | 200mm | 200mm |
| Width of end ring | 2mm | 2mm |
| Height of end ring | 4mm | 4mm |
| Insulation thickness | 0.5mm | 0.5mm |
| Bar length | 0.6m | 0.6m |
| Stator outside diameter | 195mm | 195mm |

| | | |
|---|---|---|
| Stator inside diameter | 185mm | 185mm |
| Thickness of the circular element | 10mm | 10mm |
| Permeability of free space | $4\pi * 10^{-7}$ H/m | $4\pi * 10^{-7}$ H/m |
| Number of rotor bar divisions | 100 | 100 |
| Height of rotor bar | 10mm | 10mm |
| Width of rotor bar | 5mm | - |
| Bottom width of bar | - | 8mm |
| Upper width of bar | - | 5mm |

**Appendix B: estimated rotor circuit parameters**

| Bar types | $L_1$ | $L_2$ | $L_3$ | $L_4$ | $R_1$ | $R_2$ | $R_3$ | $R_4$ |
|---|---|---|---|---|---|---|---|---|
| rectangular | 4.12µH | 8.2µH | 4.0µH | 2.5mH | 110.250µΩ | 50µΩ | 23µΩ | 5.2mΩ |
| Trapezoidal | 2.1µH | 15.0µH | 41.0µH | 21.0µH | 111.300µΩ | 0.10 mΩ | 5.2µΩ | 20µΩ |