APLLICATION OF LYAPUNOV'S SECOND METHOD IN THE STABILITY ANALYSIS OF OIL/GAS SEPARATION PROCESS

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ABSTRACT
In this paper, Lyapunov's method for determining the stability of non-linear systems under dynamic states is presented. The paper highlights a practical application of the method to investigate the stability of crude oil/natural gas separation process. Mathematical state models for the separation process, used in the application, are developed and presented in the paper. From the results obtained, some guidelines are recommended for a safe and efficient practice of separation of oil + water from oil + water + gas mixture in a crude oil process separator without getting into process instability. The recommended guidelines will be found useful in the oil and gas industry for safe and efficient oil and gas production business.

Keywords: Lyapunov's method; non-linear systems; dynamic states; stability, crude oil; real-life process.

1 Introduction

One of the ways of assessing the usability of a system (linear or non-linear) is to evaluate its dynamic stability. For non-linear systems, several methods have already been developed for this purpose. They are Nyquist's, Popov's and Lyapunov's methods [1]. Among those methods, Lyapunov's method (second method) is recorded to be the most general and powerful [1]. Its definition is presented in this paper.

Even though the method is recorded to be powerful, it has a major drawback, which seriously limits its practical applications. This drawback is the difficulty often associated with the determination of a function V(x), known as the Lyapunov function, that is required by the method and the development of a state model for a physical process [2]. It is so difficult that in most cases, the function V(x) is chosen arbitrarily and no guidelines are used in making this choice [1]. An unlucky and wrong choice of V(x) can give a wrong conclusion about the stability of a system.

It is based on these facts that Brogan, W. L., in his book, Modem Control Theory, wrote: "Unfortunately, the Lyapunov theorems give no indication of how a Lyapunov function V(x) might be found. There is no universally best method of searching for Lyapunov functions. The function V(x) can be assumed either by a pure guess or by intuition" [3].

This difficulty also made Ogata, K., to state in his book, Discrete Time Control Systems, that:

"Although the second method of Lyapunov is applicable in the stability analysis of any non-linear system, obtaining successful results may not be an easy task. Experience and imagination may be necessary in the application of Lyapunov theorems in stability analysis of most non-linear systems" [4].

The problem now is how to determine the Lyapunov function V(x), which presently is always assumed arbitrarily or chosen intuitively.

Fair enough, Barnett, S. [2], presented a procedure for determining the Lyapunov function, V(x), which is based extensively on variable gradient. Making use of that procedure, a practical application of the method (Lyapunov's second method) is illustrated in the paper. Before getting into the practical application of the method, it is necessary to give its complete scope, including the definition.

2 Lyapunov's Second Method and Non-linear Systems Stability

Lyapunov's second or direct method, as it is sometimes referred to, plays an important role in the stability analysis of both linear and non- linear systems. The concepts of this method are presented below

Lyapunov's second method is based on the generalization of the following fact:

"If a system is in an asymptotic stable equilibrium state, then the stored energy of the system displaced within the domain of attraction decays with increasing time, t, until it finally assumes minimum value at the equilibrium state" [4].

Lyapunov, in his applied mathematics work, introduced a function V(x), known as a Lyapunov function, and used it to evaluate the stability at equilibrium state mentioned above for both linear and non-linear systems. He defined the function V(x) as

$V(X) = X^{T}PX$ .... (1)[4]

where, x is a state vector (an n-dimensional vector), and P is a symmetric, positive-definite matrix expressed in an expanded and generalized form as

$$V(X) = [X_1 X_2 \cdots X_n]\begin{bmatrix} P_{11}\ P_{12}\ \ldots.P_{1n} \\ P_{21}\ P_{22}\ \ldots.P_{2n} \\ \vdots \quad \vdots \quad \vdots \\ P_{n1}\ P_{n2}\ \ldots.P_{nn} \end{bmatrix}\begin{bmatrix} X_1 \\ X_2 \\ \vdots \\ X_n \end{bmatrix} \quad (2)$$

Lyapunov then used the following conditions to investigate the stability of systems

(i) V(x) < 0 ∀ x ≠ 0

(ii) V(x) < 0 ∀ x ≠ 0

(iii) V(x) = 0, for x = 0

They generally provide sufficient conditions for the stability of systems. The method adopted by Lyapunov, as presented above, is known as Lyapunov's second method [4].The function V(x) is however difficult to determine [4]. However, as earlier mentioned, Barnett, S., [2] presented a method for determining it. The method is stated, without proof, in eqn. (3a):

$V(X) = [\nabla V(X)]^{T} X$ ....(3a)[2]

$$where\ \nabla V(X) = \begin{bmatrix} \frac{\delta V(X_1)}{\delta_{X1}} = \nabla V(X_1) \\ \frac{\delta V(X_2)}{\delta_{X2}} = \nabla V(X_2) \\ \vdots \qquad \vdots \\ \frac{\delta V(X_n)}{\delta_{Xn}} = \nabla V(X_n) \end{bmatrix} \quad (3b)$$

From which V(x) is given by

$V(X) = \int_0^X [\nabla V(X)]^T dX$ .... (4)

A general form of ∇V(x) is given by:

$$\nabla V(X) = \begin{bmatrix} \nabla V(X_1) \\ \nabla V(X_2) \\ \vdots \\ \nabla V(X_n) \end{bmatrix} = \begin{bmatrix} a_{11}X_1 + a_{12}X_2 + \cdots + a_{1n}X_n \\ a_{21}X_1 + a_{22}X_2 + \cdots + a_{2n}X_n \\ \vdots \qquad \vdots \\ a_{n1}X_1 + a_{n2}X_2 + \cdots + a_{nn}X_n \end{bmatrix} \ldots\ldots(5)$$

Eqns. (3a) and (4) can hence be used to determine V(x) and V(x) respectively for any non-linear system

It is pertinent at this point to mention that Lyapunov's first method, found in some mathematics texts, has no engineering or practical applications. In recent times, it is not much emphasized and is almost forgotten [5].

It should be noted that the $a_{ij's}$ are completely undetermined quantities and could be constants or functions of state variables x, and time, t. Also, it should be noted that the size of the matrix in eqn. (5) is determined by the order of the state model of a given system whose stability is to be investigated. An example below will demonstrate this point.

The models for V(x) and V(x) presented above have to be tested theoretically on a non- linear system in order to verify its validity.

2.1 An illustration

Given a system

$x_1 = x_2$

$X_2 = - X_2 - (X_1)^3$, the requirement is to discuss the stability of the system, making use of Lyapunov's direct method [6].

The models for V(x) and V(x) from eqns. (3a) and (4) respectively, will now be used to solve this problem. Now, with reference to eqn. (5), though the given system is a third order type, the entire system is in the second state order. A 2 x 2 matrix for ∇V(x) and V(x) will hence result.

Hence

$$\nabla V(X) = \begin{bmatrix} a_{11}X_1 + a_{12}X_2 \\ a_{21}X_1 + a_{22}X_2 \end{bmatrix} \ldots\ldots\ldots\ldots\ldots\ldots (6)$$

$$\nabla V(X)^T = \begin{bmatrix} a_{11}X_1 + a_{21}X_1 \\ a_{12}X_2 + a_{22}X_2 \end{bmatrix} \ldots\ldots\ldots\ldots\ldots\ldots (7)$$

recalling eqn.(3a)

$$\dot{V}(X) = [\nabla V(X)^T \dot{X}]$$

Substituting the state models for the given system in eqn. (3a),

$$V(X) = \begin{bmatrix} a_{11}X_1 + a_{21}X_1 \\ a_{12}X_2 + a_{22}X_2 \end{bmatrix} \ldots\ldots\ldots\ldots\ldots\ldots (8)$$

Substituting the values of X_1 and X_2 from the state models for the given system in eqn. 8,

$$\begin{bmatrix} \dot{V}_1(X) \\ V_2(X) \end{bmatrix} = \begin{bmatrix} a_{11}X_1 + a_{21}X_1 \\ a_{12}X_2 + a_{22}X_2 \end{bmatrix} \begin{bmatrix} 0 & X_2 \\ -(X_1)^3 & -X_2 \end{bmatrix}$$

$$= a_{11}X_1(0) - a_{21}X_1(X_1^3) + a_{12}X_2(0) - a_{22}X_2(X_1^3) + a_{11}(X_2) - a_{21}X_1X_2 + a_{12}X_2(X_2) - a_{22}X_2(X_2)$$

$$= 0 - a_{21}X_1^4 + 0 - a_{22}X_1X_1^3 + a_{11}X_1X_2 - a_{21}X_1X_2 + a_{12}X_2^2 - a_{22}X_2^2$$

$$= -a_{21}X_1^4 - a_{22}X_1^4 + a_{11}X_1X_2 - a_{21}X_1X_2 + a_{12}X_2^2 - a_{22}X_2^2$$

Let the $a_{ij's}$ =1……………………….. (9)[2]

$$\therefore \dot{V}(X) = V_1(x) + \dot{V}_2(x) = -2x_2^4 + X_1X_2 - X_1X_2 + X_2^2 - X_2^2$$

$$\dot{V}(X) = \dot{V}_1(X) + \dot{V}_2(X) = -2X_1^4, \textit{which is negative definite}$$

$$\dot{V}(X) = -2X_1^4 \ldots\ldots\ldots\ldots\ldots\ldots (10)$$

for x1≠0

It can be seen that eqn. (10) is negative-definite, hence satisfies one of Lyapunov's conditions for asymptotic stability; that V(x) must be negative- definite [4]

Now, solving for V(x), and recalling eqn. (4) where

$$V(X) \int_0^X [\nabla V(X)]^T dx$$

Substituting for [∇V(x) T from eqn. (7),

$$V(X) \int_0^X [a_{11}X_1 + a_{21}X_1 + a_{12}X_2 + a_{22}X_2] dx$$

Since the $a_{ij's}$ =1, recall eqn. (9)

$$\therefore V(X) = \int_0^X [2X_1 + 2X_2] dx$$

$$V\left(X = 2\int_0^{x1} X_1 dX_1 + 2\int_0^{X2} X_2 dx_2\right)$$

$$V(X) = X_1^2 + X_2^2 \ldots\ldots\ldots\ldots\ldots (11a)$$

For $x_{i's} \neq 0$

It can be seen that eqn. (11a) is positive-definite, hence satisfies one of Lyapunov's conditions for asymptotic stability; that V(x) must be positive- definite [4]

Also, from eqn. (11a), for $X_{i's} = 0$,

$$V(X) = 0 \ldots\ldots\ldots\ldots\ldots\ldots (11b)$$

This satisfies another Lyapunov's condition for asymptotic stability [4]. The given system is therefore asymptotically stable in-the-large since it satisfies all the Lyapunov's conditions for asymptotic stability.

3. A Practical Application

As earlier mentioned, among several existing methods, Lyapunov's method had been found to be the most powerful on the evaluation of the stability on non-linear systems. To demonstrate the practical application of the method, it is applied to the following real-life dynamic control engineering process in order to determine its stability.

The data recorded in table 1 represent the flow of a 3-phase crude oil into a separator (shown diagrammatically in fig. 1) and the separation of natural gas from the crude oil at Mobil Qua Iboe oil production flow station, Eket, Akwa Thorn State, Nigeria. It is desired to derive state equations governing the process and to determine (with justifications), whether the process is linear or non-linear. (For such a process to run smoothly, there should be no carry-over of the separated crude into the gas line. Such carry- over would signify instability). Also, it is desired to use Lyapunov's method to determine the stability of the process. (Valves "A" and "B" shown in fig. 1 are assumed to be open to some percentages to allow for the recorded data). Note: 3-phase crude oil, in the oil & gas industry, is a mixture of crude oil, water and natural gas; mixed in any proportion. Also, in the industry, uptill now, the unit of measurement is still in imperial, not metric. Hence, in the recorded data, and in other places in the paper, the units are in imperial.

On the date that the data were recorded, the temperature of the crude in the separator was 98° F while the separator pressure was at atmospheric, 14.7 Pounds per Square Inch (PSI), through the flare line. The practice now is that the valves 'A' and 'B' are opened to some percentages and the flare observed. The valves are continuously manipulated until a smokeless, or fairly smokeless, flare is obtained. This practice is not the best as it is unprecise, hence the need to carry out further research into the separation process.

Table 1: Production data from Mobil Qua Iboe Oil Production flow station, Eket, Akwa Thorn State,Nigeria, taken March 9-29, 1999 [7]

	3-Phase Crude Inlet		Oil + Water Produces (MMCFF/D) (Q_2)	Gas Produces (MMCFF/D) (Q_2)	$B = \frac{Crude\ Inlet\ (Ft^3/D)}{\text{Oil + Water Produces}\ (Ft^3/D)}$
9/3/99	715119	4.012	6503.48	42.30	616.95
10/3/99	724392	4.06	6527.21	40.05	622.01
11/3/99	748512	4.20	6602.82	41.32	636.09
12/3/99	741935	4.16	6522.02	43.11	637.84
13/3/99	731568	4.10	6498.44	46.32	630.92
14/3/99	740442	4.15	6501.44	42.28	638.32
15/3/99	722107	4.05	6500.81	44.81	62300
16/3/99	707725	3.97	6602.11	39.87	601.32
17/3/99	716701	4.02	6502.84	40.12	618.19
18/3/99	724499	4.06	6509.82	45.08	623.67
19/3/99	721859	4.05	6614.30	41.06	612.31
20/3/99	726291	4.07	6520.77	42.66	624.16
21/3/99	691546	3.88	6504.20	39.92	596.53
22/3/99	640155	3.60	6499.82	38.99	553.86
23/3/99	695654	3.90	6514.14	39.06	598.70
24/3/99	706868	3.96	6511.14	40.22	608.19
25/3/99	696010	3.90	6520.20	42.06	598.14
26/3/99	706868	3.97	6515.14	39.67	609.35
27/3/99	698633	3.92	6502.12	40.02	602.88
28/3/99	690438	3.87	6620.15	41.53	584.58
29/3/99	694634	3.90	6497.06	40.77	600.27

Note: BBls/D = Barrels per Day
MMCF/D = Millions Cubic Feet per ay
Ft^3/D = Cubic Feet Per Day
Ft/D = Feet Per Day

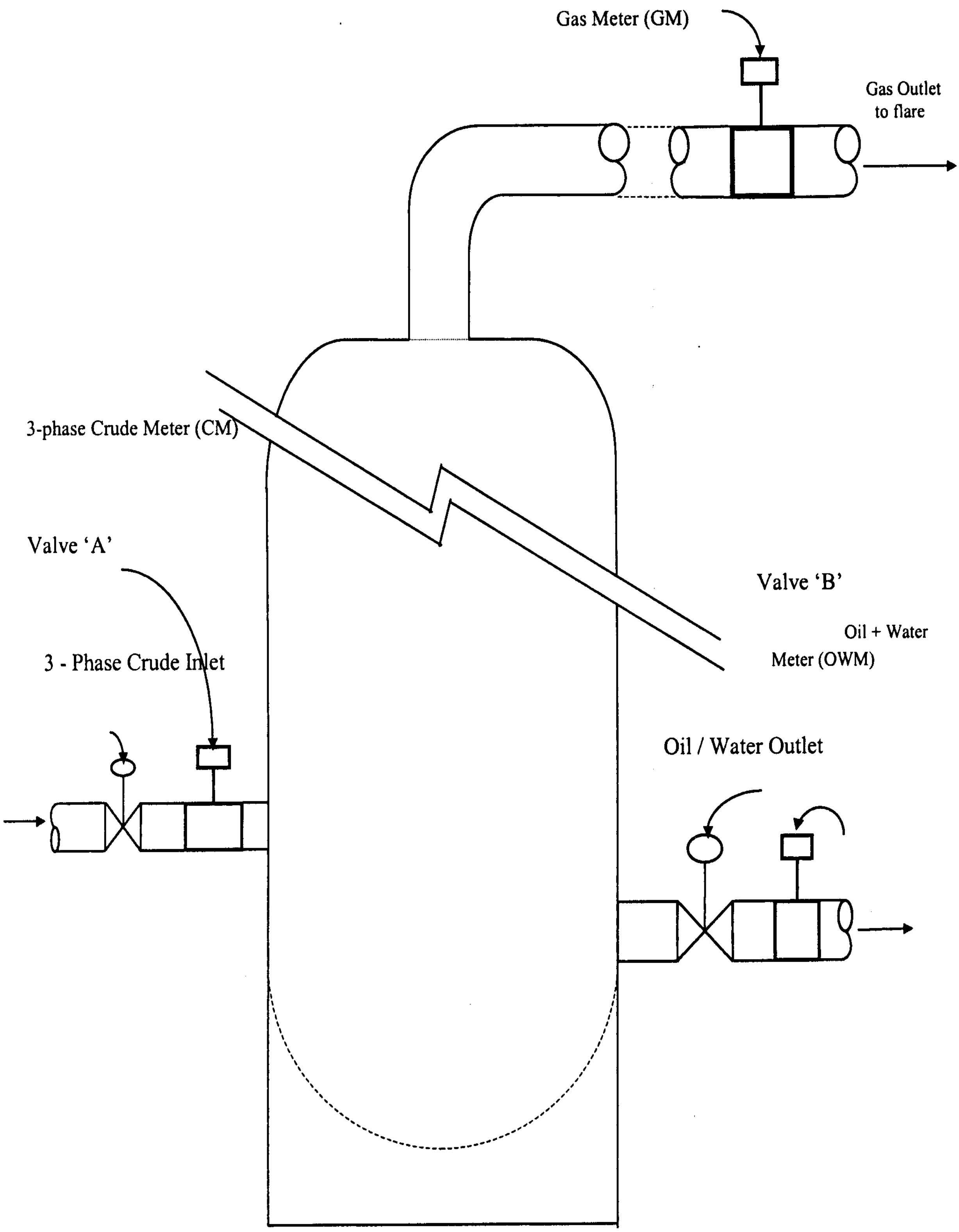

Fig. 1: Diagram of a Crude Oil/Gas process Separator Size of Separator:
12ft-diameter x 70ft-High

In order to apply the method, state equations governing the process must be developed. .
To develop state equations, first, equation of flow is derived as follows:

Q = AV (12) [8]

Where

Q = Quantity of oil + water + gas mixture flowing into the separator in ft^3/d.

A = Cross - sectional area of pipe in square inches.

V = The velocity of in-coming crude in feet per second (ft/sec.)

From eqn. (12),

$Q = A\frac{dx}{dt} = A\dot{x}$

$\dot{x} = Q/A =$ Q a (velocity of inlet crude into the separator)

Where a = $\frac{1}{A}$

$\dot{x} =$ Q a =………… (13a)

The 3 - phase fluid is separated to oil + water mixture and natural gas.
Let the volume of the separated fluid be characterized by X_2 in ft^3.
A time-derivative of X_2 gives the flow rate of separated Oil + Water and liberated gas.

Hence,

$\dot{x_2} = (-abx_2 - ax_2) - x_3)ft/d$……(l3b)

In Eqn. (l3b),

- abx_2 represents separated oil + water in ft^3,
- ax_2 represents flow line losses in ft^3, while x_3 represents the liberated gas also in ft^3.

The factor a (= 1/ A) is as defined above while b is as defined in Table 1. The coefficients of X_2 and x, in eqn. (l3b) both have negative senses because they are fluid flowing out of the separator. The quantity (-abx - aX_2) approximates to - abx, because the flow line losses, - ax_2, is by far less than the separated fluid, - abx. That is, since - ax_2<< -abx_2, eqn. (l3b) approximates to:

$\dot{x}_2 = (-abx_2 - x_3)ft/d$ ……….. (13c)

The rate ofc flow line losses is given by

$\dot{x}_2 = -ax_2 - ft/d$ …………… (13d)

Where x_1 = Volume of oil + water losses in ft^3.

It should be noted that he dimension, ft.d, in this case, is volumetric flow rate. That is, so much volume is "displaced, in feet," in so much "time, in day."
Now, looking at the recorded data in table 1, and taking a typical oil + water quantity and a corresponding quantity of gas produced, (that of 9/3/99) the following approach is used to determine the relation between the produced oil + water and gas.

Given:

Q_1 = 6,503.48 ft^3/d (Produced Oil + Water)

Q_2 = 42,300,000 ft^3/d (Produced Gas)

To establish a relationship between oil + water and gas produced, the following equation is postulated:

$(6503.48)^n$= 42,300,000……………(15)

$Log_{10}\ (6503.48)^n$ = Log_{10} 42,300,000

n Log_{10} 6503.48 = Log_{10} 42,300,000

n(3.813145809) = 7.626340367

n = 2.000012785

Other crude oil inlet readings from table 1, with their corresponding gas figures, gave the values of n to be between 1989 and 2014.
From here, it is established that the volumetric flow rate of gas out of the separator (illustrated in fig. 1) is approximately a square function of that of oil + water flowing out of the separator.

$\therefore$42,300.000 ≈ $(6503.48)^2 ft^3/d$ …… (16)

Eqn. (16) implies that

$x_3 = x_2^{2\ldots}$ …….(17)

Hence equation (l3c) becomes

$\dot{x}_2 = -abx_2 - x_2{}^2$ …… (18)

or $x_2 = -abx_2 - x_3$, (since $x_2^2 = x_3$, from eqn. (17))

Now, the liberated gas x_3 comprises natural gas such as methane (CH_4), ethane (C_2H_6), propane (C)Hg), butane (C_4H 10), etc, but cannot be separated to these components by the separator since it is a liquid/gas separator and not a gas fractionating column [9], hence, the flow rate of the separated products is zero.

That is, $\dot{x}_3 = 0\ ft/d$.

$\dot{x}_3 = 0\ ft/d$………………………..(19)

The state equations governing the process are hence given by eqns. (l3a), (13d), (18) and (19):

$\dot{x}_1 = ax^2$......... (Rate of flow line losses in ft/d)

$\dot{x}_2 = -abx^2{x_2}^2$...... …(Rate of flow of separated oil + water and gas in ft/d)

$\dot{x}_3 = 0$ ..... (Rate of flow of separated natural gas in ft/d).

Now, since one of the state equations representing the process contains an exponent higher than unity, (recall: $\dot{x}_2 = -abx_2 - {x_2}^2$, eqn. (18)), then the process is non-linear.

The value of X_1 (flow line losses, in ft^3 is obtained by integrating eqn. (13d) with respect to the time of flow as follows:

$\dot{x}_1 = ax_2$

$x_1 = ax_2 \int dt = ax_2 t$

$\therefore x_1 = ax_2 t, ft^3$ (20a)

where t is time in days.

Since a = $\frac{1}{A}$, (from eqn. (13)), for a 24 inch pipeline, (which is the diameter, d, of the pipeline in this application),

$a = \frac{1}{(\pi d^2)/4} = \frac{4}{3.142(24)^2} = 0.00221$

Hence for a time of 1 day

X_1 = 0.00221 (X_2) (1) = 0.00221 (X_2) ft^3..... (20b)

Since the state model representing the process is in the third state order, that is the highest state order being X_3, the system equation, from eqn. (5), (for this application) has to be a 3 x 3 matrix. Therefore to determine the stability of the process, eqn. (5) is recalled as follows:

$$\nabla V(x) = \begin{bmatrix} a_{11}x_1 + a_{12}x_2 + a_{13}x_3 \\ a_{21}x_1 + a_{22}x_2 + a_{32}x_3 \\ a_{31}x_3 + a_{32}x_2 + a_{33}x_3 \end{bmatrix}$$

For which

$$\nabla V(x)^T = \begin{bmatrix} a_{11}x_1 + a_{12}x_2 + a_{13}x_1 \\ a_{12}x_1 + a_{22}x_2 + a_{32}x_2 \\ a_{31}x_3 + a_{32}x_2 + a_{33}x_3 \end{bmatrix} \quad (21)$$

By eqn. (3a),

$\dot{V}(x) = [\nabla V(x)]^T \dot{x}$ (22)

Substituting for xn(s) from eqns. (l3c), (13d) and (19), eqn. (22) becomes:

$\dot{V}(x) = \nabla V(x) =$

$$\begin{bmatrix} a_{11}x_1 + a_{21}x_1 + a_{31}x_1 \\ a_{12}x_2 + a_{22}x_2 + a_{32}x_2 \\ a_{13}x_3 + a_{23}x_2 + a_{33}x_3 \end{bmatrix} \begin{bmatrix} 0 & ax_2 & 0 \\ 0 - & abx_2 - x_3 & \\ 0 & 0 & 0 \end{bmatrix}$$

$\dot{V}(x) = a_{11}x_1(0) + a_{21}x_1(0) + a_{31}x_1(0) + a_{22}x_2(0)$
$+a_{32}x_2(0) + a_{13}x_3(0) + a_{23}x_3(0) + a_{33}x_3(0)$
$+a_{11}x_1ax_2 + a_{21}x_1(-abx_2) + a_{31}x_1(0) + 12x_2ax_2$
$+a_{22}x_2(-abx_2) + a_{32}x_1(0) + a_{13}x_3ax_2 + a_{23}x_3(-abx_2)$
$+a_{33}x_3(0) + a_{11}x_1(0) + a_{21}x_1(-x_3) + a_{31}x_1(0)$
$+a_{12}x_2(0) + +a_{22}x_2(-x_3) + a_{32}x_2(0) + a_{13}x_3(0)$
$+a_{23}x_3(-x_3) + a_{33}x_3(0)$.

$\dot{V}(x) = 0 + 0 + 0 + 0 + 0 + 0 + 0 + 0 + 0 + a_{11}a_{x1}x_2$

$+(-a_{21}abx_1x_2) + 0 + a_{12}a{x_2}^2 + (-a_{22}ab{x_2}^2) + 0$
$+a_{13}ax_2x_3 + (-a_{23}abx_2x_3) + 0 + (-a_{21}x_1x_3)$
$+0 + 0 + (-a_{22}x_2x_3) + 0 + 0 + (-a_{23}x_1{x_3}^2) + 0$.

$\dot{V}(x) = a_{11}ax_1x_2 - a_{21}abx_1x_2 + a_{12}a{x_2}^2$
$a_{13}ax_2x_3 - a_{23}abx_2x_3 - a_{21}x_1x_3 - a_{22}x_2x_3 - a_{23}{x_3}^2$ …………… (23)

The aij's = are constant greater than zero [2]

Let the aij's = 1, (recoall eqn. (9)), hence eqn. (23) becomes

$\dot{V}(x) = ax_1x_2 - abx_1x_2 + a{x_2}^2ax_2x_3 - abx_2x_3 - x_1x_3 - x_2x_3 - {x_3}^2$
$= x_1x_2(a - ab) + {x_2}^2(a - ab)x_2x_3(a - ab) - x_1x_3 - x_2x_3 - {x_3}^2$

$\dot{V}(x) = (a - ab)(x_1x_2 + {x_2}^2 + x_2x_3) - x_3(x_1 + x_2) - {x_3}^2$ ………. (24)

Now, the factor "ab" in eqn. (24) must be greater than "a" (the coefficient of X2) for the following reason: The separated fluid, abx_2, is heavier than the in-coming 3 - phase crude represented by Q. This is largely because its gas content ${x_2}^2$ (which made it lighter), has been liberated. To reflect this increase in weight, the factor "ab" in eqn. (24) has to be higher than "a".

From that perspective, ab > a

$\Rightarrow$ b > 1

From here, it can be seen that the equation

$\dot{V}(x) = (a - ab)(x_1x_2 + x_2{}^2 + x_2x_3) - x_3(x_1 + x_2) - x_3{}^2$ ………. (24)

for X_i's $\neq 0$,

will always be negative, that is negative - definite, hence satisfying one of Lyapunov's conditions for stability. will always be negative, that is negative - definite, hence satisfying one of Lyapunov's conditions for stability.

The more negative the number for Vex) is, the more stable the process will be. To demonstrate this, one of the readings from table 1 is substituted below in eqn. (24), for verification of the result. The reading from table 1 that was taken on 9th March 1999 will be used.

Recalling eqn. (20b), $X_1 = 0.00221\ (X_2)$

on that date, $X_1 = 0.00221(6503.48) = 14.373$

$X_2 = 6503.48$ and

$X_3 = x_2{}^2 = 42{,}300{,}000$

Substituting these values in eqn. (24),

$\dot{V}(x) = (a - ab)\ [(14.373)(6503.48) + (6503.48)^2 + (6503.48)(42.3\ X\ 10^6)] - 42.3\ X\ 10^6(14.373 + 6503.48) - (42.3 \times 10^6)^{\ 2}$

$= (a - ab)[2.7513 \times 10^{11}] - 2.757 \times 10^{11} \times 10^{11} - 1.7899 \times 10^{15}$

$= (a - ab)(2.7513\ X\ 10^{11}] - 1.790 \times 10^{15}$

(a - ab)(2.7513 X 10"]-1.790xl

Since a = 0.00221, and on that date, b = 616.95

$\dot{V}(x) = (0.00221 - 1.363)(2.7513 \times 10^{11}) - 1.790 \times 10^{15}$

$\dot{V}(x) = 3.744 \times 0^{11} - 1.790 \times 10^{15} = 1.790 \times 10^{15}$

$\therefore \dot{V}(x) = -1.790 \times 10^{15}$ which is a negative number, hence satisfies one of Lyapunov's conditions for stability. Since eqn. (24) one of Lyapunov's conditions for stability process under examination is a stable one number, hence satisfies one of Lyapunov's conditions for stability. Since eqn. (24) one of Lyapunov's conditions for stability process under examination is a stable one.

To determine the positive-definiteness for another Lyapunov's condition eqn. (14) is recalled as follows:

$$V(x) = \int_0^x [\nabla V(x)]^T dx \qquad (25)$$

From eqn. (21),

$$\nabla V(x)^T = \begin{bmatrix} a_{11}x_1 + a_{21}x_1 + a_{31}x_1 \\ a_{12}x_2 + a_{22}x_2 + a_{32}x_2 \\ a_{13}x_3 + a_{23}x_2 + a_{33}x_3 \end{bmatrix} (26)$$

Since the aij's = 1, recall eqn. (9),

$$\nabla V(x)^T = \begin{bmatrix} 3x_1 \\ 3x_2 \\ 3x_3 \end{bmatrix} \qquad (27)$$

$$\nabla V(x)^T = 3x_1 + 3x_2 + 3x_3 \qquad (28)$$

$$\therefore V(x) = \int_0^x [3x_1 + 3x_2 + 3x_3] \qquad (29)$$

$$= 3\int_0^{x_1} x_1\, dx_1 + 3\int_0^{x2} x_2\, dx_2 + 3\int_0^{x3} x_3\, dx_3$$

$$= 3/2x_1{}^2 + 3/2x_2{}^2 + 3/2x_3{}^2$$

$$V(x)(= 3/2x_1{}^2 + 3/2x_2{}^2 + 3/2x_3{}^2)\ (30)$$

for x_i's $\neq 0$

Eqn. (30) will always be positive, that is positive-definite, hence satisfying another Lyapunov's condition for stability. Also, from eqn. (30), for for x_i's = 0, V(x) = 0 , thus satisfying another Lyapunov's condition for stability (4) .

Substituting values for X_1 X_2 and X_3, V(x) becomes

$V(x) = 3/2\ [(14.373)^2)^2 + (6503.48)^2 + (42{,}300{,}000)^2]$

$= 3/2\ [206.58 + 42295252.11 + 1.7893 \times 10^{15}]$

$V(x) = 2.6839\ X\ 10^{15}$, which is a positive number, hence confirming the positive definiteness for Vex) , another Lyapunov's condition for stability. Similarly, the more positive the number for Vex) is, the more stable the process will be. Since both eqns. (24) and (30) satisfy Lyapunov's conditions for stability, the process under investigation is a stable one.

4 Discussion

From the practical application of the Lyapunov's method just illustrated, it can be established that for the process in which the data are recorded in table 1, to run smoothly, within stability limits, the following conditions must hold:

(i) $0 < a << 1$ (31)

(ii) $1+ \leq b \leq x_2$ (32)

If condition (i) (eqn. (31) is violated, this would imply that the cross sectional area (A) of the inlet and outlet flow pipes are very small which would further imply that crude oil production will be very low. If production is that low, then the business will not worth the trouble; not a good business practice: With reference to table 1, if condition (ii) (eqn(32)) is violated, say b = 0, this would mean that there is no crude oil inlet to the separator, that is, no production is going on . If b < 1, this would imply that more oil + water is produced than the inlet flow into the separator, which is not possible. Hence the region $b \leq 1$, can be classified as a forbidden region. If b = 1, this would mean that the crude that is getting into the separator is what is getting out as oil + water produced; that is, no gas is produced, implying that $x_3 = 0$ (which cannot be true). If $b = 1^+$,this would imply that oil + water and gas separations are beginning to take place; oil + water is going out through valve 'B' (fig. 1) and gas is going out-through the gas line to flare. The meaning of this is that the process is running smoothly. For

$b = x2^+$,this would imply that part of the oil + water is beginning to get into the gas line (beginning of instability). For $b > x_2$, this would mean that more oil + water is getting into the gas line which would be a clear case of process instability. What these analyses tell us is that to operate efficiently within safe and comfortable stability limits, some conditions have to be met relative to the factor "b" and produced quantity of oil + water mixture, X_2.

A graphical illustration in fig. 2 provides a guide for selecting the factor "b" relative to the quantity of oil + water (X_2) to be produced in order to stay within production stability limits. This will ensure that the model for V(x) (eqn. (24), remains negative thus satisfying Lyapunov's condition for stability. The conditions that allow for safe production, within stability limits, are expressed graphically in fig. 2 .

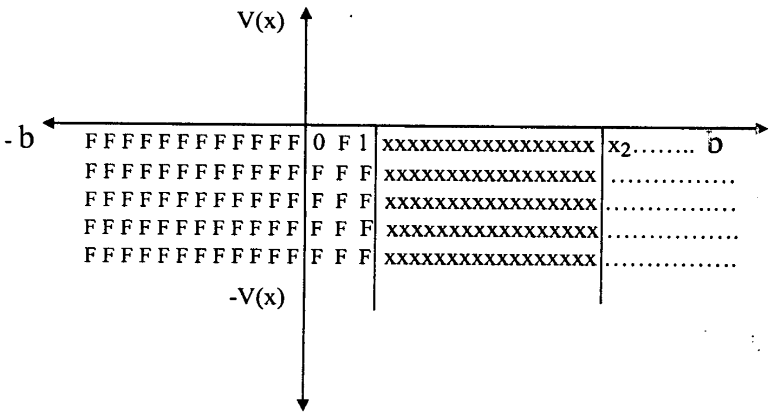

Fig. 2: A graph showing stability, instability and forbidden regions for the process recorded in Table 1.

Legend:
xxxxxxx Region of process stability
.... Region of process instability
FFFFFF Forbidden region

Physical interpretation of the graph is that one should choose the quantity of oil + water to be produced, X_2, (in ft^3) in a day. Thereafter, the person should select a factor for "b" slightly greater than I but not more than X_2 then get to eqn. (24) and carry out necessary substitutions to solve for V(x), which should give a negative number, signifying process stability.

The factor "a" (= $\frac{1}{A}$ in eqn. (13)) is determined by the cross sectional area (A) of the inlet and outlet pipes.

The diameter, d, of the pipes should not be less than 4 inches, otherwise the production rate of oil + water would be too low which would not be a good business practice. What this implies is that the factor "a" should be greater than "0" but less than $\frac{1}{A}$, where

$A = \pi d^2 = \frac{\pi}{4}(4)^2 \frac{square}{4}\ inches$

$A = 12.568\ square\ inces$

and $a = \frac{1}{A} = \frac{1}{12.569}$

a = 0.0796

a ≅ 0.08

Hence eqn. (31) can be modified to read 0 < a ≤ 0.08 …………(33)

The pipe size for this application is 24 inches in diameter hence,

a = 1/(π $(24)^2$/4) = 4/(3.142) $(24)^2$ = 0.00221.

With those limits for "a" and "b", the production will remain stable for the process in which the data are recorded in Table 1.

5 Conclusion

In the paper, Lyapunov's second method has been applied to a real-life oil production process to determine safe production limits for process stability. The data for the process (shown in table 1), were recorded at Mobil Oil Production flow station, Eket, Akwa Thorn State, Nigeria, where one of the authors (U. T. Itaketo) is working as an instrumentation engineer. It has been mentioned that the major factors that usually hinder the practical application of the method are the development of mathematical state models for the physical process under stability study and the determination of a Lyapunov function for the process. Practical guidelines have been offered on how to apply the Lyapunov method to a real-life process; an oil production process, in order to operate safely within stability limits.

Oil industry engineers and operators will generally find the recommended guidelines useful in their oil production business.

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