SIMPLIFIED ELASTIC DESIGN OF UNCRACTED REINFORCED CONCEPT SECTION

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ABSTRACT

Due to the limitations of the traditional elastic design approach of uncracked section as provided by BS 5337, an alternative design approach is proposed by which the precise depth of the neutral axis and hence the actual area of steel reinforcement are determined for any stipulated value of permissible tensile stress of concrete. The obtained result finds application in elastic flexural design of uncracked section such as required for water-retaining structures.

NOTATION

X	=	Depth of neutral axis (mm)
A_{st}	=	Area of steel (mm^2)
h	=	Depth of section (mm)
d	=	Effective depth of section (mm)
b	=	Width of section (mm)
p_c	=	Maximum compressive stress in concrete N/mm^2
p^*_c	=	Permissible Compressure Stress in concrete N/mm^2
p_t	=	Maximum tensile stress in concrete N/mm^2
P^*_t	=	Permissible stress of steel N/mm^2
p_s	=	Stress in Steel N/mm^2
P^*_s	=	Permissible stress of steel N/mm^2
E_s	==	Modulus of elasticity of steel N/mm^2
E_c	=	Modulus of elasticity of concrete N/mm^2
M	=	Modular ratio
Σ_c	=	Maximum elastic (compressive) strain in concrete
Σ_t	=	Maximum elastic (tensile) strain in concrete
Σ_s	=	tensile strain in steel

INTRODUCTION

Elastic design of reinforced concrete structures in comparison with Limit State design has been shown to be uneconomical [1]. However, for water-retaining structures and structures subjected to repeated loading, elastic design is safer and more reliable.

The widely used approach to elastic design of uncracked section is provided by BS5337 "The Structural Use of Concrete for Retaining Aqueous Liquids" [2]. In this approach the required area of steel reinforcement, A_{St} is determined using the formula

$$A_{st} = \frac{M}{p_s^*(d - \frac{X}{3})} \qquad (1)$$

Where

$$X = d[(15R)^2 + 30R - 15R] \qquad (2)$$

$$R = R_1 \frac{h}{d} \qquad (3)$$

And

$$R_1 = \frac{A_S}{bh} \qquad (4)$$

In the above equations M is the applied moment, d is the effective depth, x is the depth of neutral axis, P^*_s is the permissible steel stress, R_I is the ratio of steel area to that of the section, b is the width of the section, and h is the depth of the section. The adequacy of the section in resisting crack is ensured if

$$h \geq \frac{M}{kb} \qquad (5)$$

where

$$K = \frac{P_t}{1-\beta}\,\frac{1}{3} - (1-\beta)\beta \quad + 14R_1\left(\frac{d}{h} - \beta\right)^2$$

(6)

And

$$\beta = \frac{0.5 + 14R_1(\frac{d}{h})}{1 + 14R_1} \qquad (7)$$

The above design procedure lacks consistency. For instance, the determination of A_{St} (Eq (1)) requires the knowledge of x the depth of neutral axis which in turn demands the knowledge of R_1 (see Eq. (4)). In essence, to determine A_{St} requires a trial value of A_{st} to be assumed in order to evaluate the depth of neutral axis. The assumed value of ASI is hardly the optimum and in some cases may not even satisfy Eqs. (l) and (5). If this happens another trial value of A_{St} is chosen and calculations are carried out all over again until strength condition, (Eq. (1)) and crack-resistance condition, (Eq. (5)) are satisfied and this can make the entire design exercise clumsy.

Due to the above limitation this paper suggests an alternative design approach which enables the depth of neutral axis x as well as adequate area of steel (A_{st}) to be directly and accurately determined for any stipulated permissible value of the tensile stress of concrete without any additional assumptions order than the ones underlying the elastic method of design [1,3]. The section thus designed inherently satisfies the requirement for both strength and crack resistance.

DIRECT DETERMINATION OF NEUTRAL AXIS AND AREA OF REINFORCEMENT

All basic assumptions underlying the elastic design method [1,3] are valid. Besides, the depth of neutral axis and the area of steel are regarded as unknown quantities and the tensile stress in concrete is assumed to have reached its permissible value i.e. $P_t = P^*_t$ The force of resistance of concrete in the compression zone F_c is (Fig. 1)

$$F_c = {}^1\!/_2\, bXp_c \qquad (8)$$

where P_1 is the maximum normal stress in concrete in the compression zone and b is the width of the section. By similar triangles (Fig. 1).

$$p_c = \frac{x}{h-x} p_t \qquad (9)$$

where P_c, is the maximum tensile stress of concrete in the tension zone.

Consequently,

$$F_c = \frac{bx^2}{2(h-x)} p_t \qquad (10)$$

The force of resistance of concrete in the tension zone is

$$F_t = \frac{b(h-x)}{2} P_t \qquad (11)$$

The force of resistance of steel reinforcement is

$$F_S = P_S A_S \qquad (12)$$

where P_s is the stress in steel reinforcement given as

$$P_s = E_s \epsilon_s \qquad (13)$$

In Eq. (13) E_s, and ϵ_s, are modulus of elasticity of steel and strain in steel respectively.

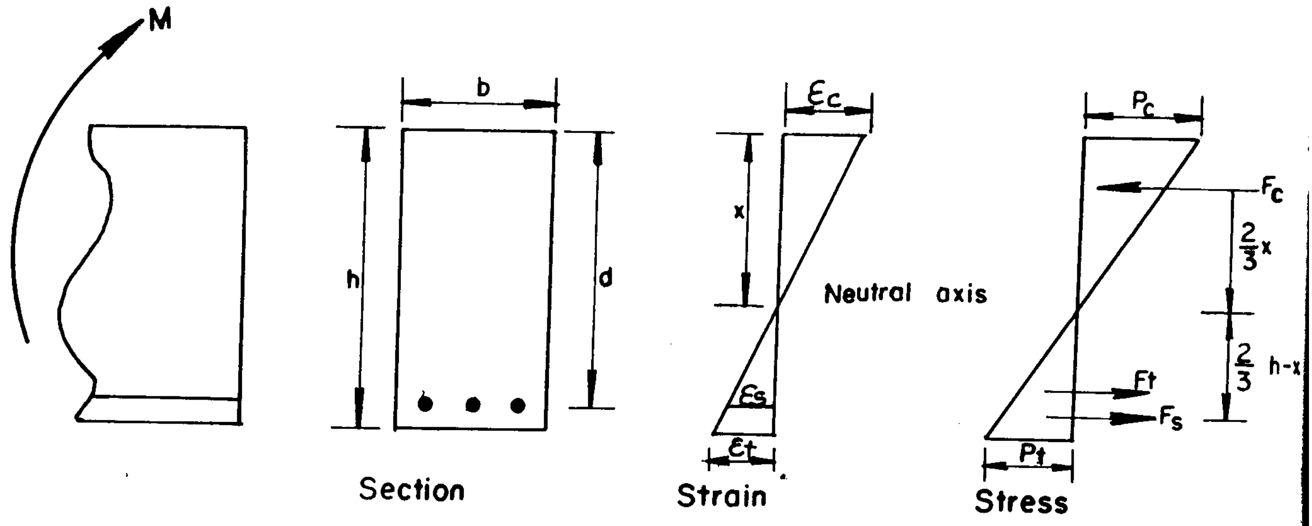

Figure I. Elastic Strain and Stress Blocks of Uncracked Reinforced Concrete Section

From Fig. 1

$$\epsilon_s = \frac{d-x}{h-x}\epsilon_c \qquad (14)$$

where d is the effective depth and $\in_c$ is the maximum strain of concrete in the compression zone. Consequently, Eq. (13) takes the form

$$P_s = \frac{mx(d-x)}{(h-x)^2}p_t \qquad (15)$$

where m = E_s/E_c is the modular ratio.

Taking moment about the line of action of force in steel (F_s), gives the following equation of equilibrium:

$$M + \left[\frac{(3d-2h)-x}{3}\right]F_t - \left(d - \frac{x}{3}\right)F_c = 0$$

(16)

Taking note of Eqs. (10) and (11), Eq (16) can be written as follows:

$$6M\,(h-r) + b\,(h-x)^2\,(3d-2h-x)p - - b\,(3d-X)X^2\,P_t = 0 \qquad (17)$$

After expanding Eq. (17), the cubic and quadratic terms cancel out yielding the following expression for the depth of neutral axis x.

$$x = \frac{h[6M+(3d-2h)bhp_t]}{3[2M+(2d-h)bhP_t]} \qquad (18)$$

The above expression for x which is dependent upon loading magnitude differs significantly from the one presently used (see Eq. (2)) in elastic design of uncracked section as provided by design handbook [4]. It is worth noting that all the quantities featuring in the right hand side of Eq. (18) are all known unlike in the case of Eq. (2) which assumes a value for the ratio R_1· Once the depth of neutral axis x is known the adequate area of steel is determined by considering the equilibrium condition in the horizontal direction

$$F_c = F_s + F_t \qquad (19)$$

Simplifying the above expression taking note of Eqs. (10), (11), (12) and (15) leads to the following expression for A_{St}

$$A_{st} = \frac{bh(h-x)(2x-h)}{2mx(d-x)} \qquad (20)$$

Again A_{St} obtained in Eq. (20) differs from the one given by Eq. (13). Eqs (20) implies that when

$2x - h \leq 0 \quad i.e. \quad x \leq 0.5\text{h} \qquad (21)$

there is theoretically no need for reinforcement. However, for this range of values of x, nominal reinforcement is used in actual practice. For the values of x in the range

$h/2 < x < d \qquad (22)$

area of reinforcement is determined using Eq. (20). The following equation of equilibrium [3]

$$\mathrm{P_t} = \frac{\mathrm{M}}{\mathrm{bh}\left[\frac{\mathrm{h-x}}{3} + \frac{\mathrm{mA_{st}.x}}{\mathrm{bh}}\frac{(\mathrm{d-x})}{(\mathrm{h-x})^2}\frac{\mathrm{d-x})}{3}\right]} \qquad (23)$$

obtained by taking moment about the line of action of F_c is used in verifying the adequacy of the section in resisting crack. The section is said to be adequate if the calculated values of x and A_{st} ensure that P_t does not exceed its permissible value. The values of x (Eq. (18)) and A_{St} (Eq. (20)) obtained in this work satisfy Eq. (23) in exact manner as will be shown in the illustration that follows.

NUMERICAL ILLUSTRATION

Let us consider as an example a section of water retaining structure subjected to a bending moment of 15 KNm due to hydrostatic force. The physical and geometric properties of concrete and steel are:

$$P_C^* = 11N/mm^2; P_t^* = 2.02N/mm^2; m = 15: P_s^* = 115/mm^2: h = 200mm: b = 1000mm; concrete\ cover\ = 50mm: d = h - 50 = 150mm\ (fig. 2)$$

Using Eq. (18) the depth of neutral axis gives

$$x = \frac{200[6(15)10^6 + (3(150-2)(20)(1000)(200)(2.02))]}{3[(2)(15)10^6 + (2)(150-2000)(1000)(200)(2.02)]} = 104.357mm$$

Using Eq. (20), the area of steel A_{st} gives

$$A_{st} = \frac{100x200\ (2x104.357 - 200)\ (200\ -\ 104.357)}{2x15(150\ -\ 104.357)104.357} = 1166.204mm^2$$

From Eq. (23), the maximum tensile stress in concrete P_t gives

$$P_t = \frac{15(10^6)}{1000(200)\left[\frac{200-104.357}{3} + \frac{1{,}825{,}522.64}{1000(200)}\left(\frac{150-104.357}{200-104.357^2}\right)\left(\frac{150-104.357}{3}\right)\right]}$$

This value is exactly equal to the given permissible value.

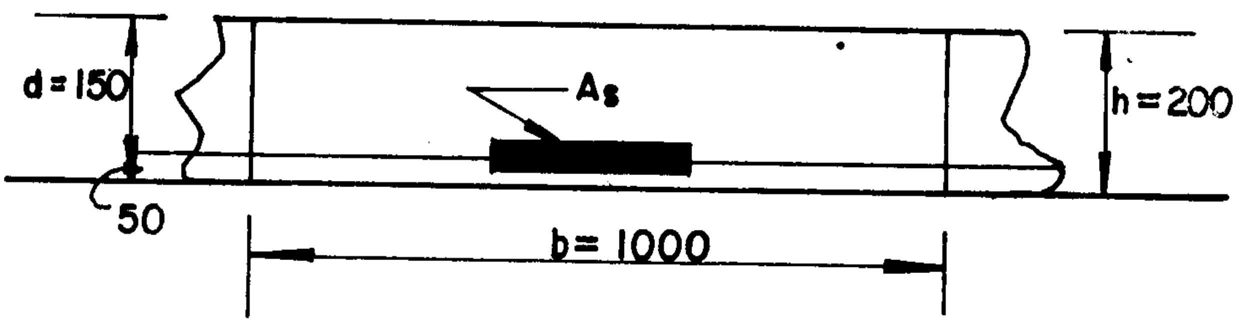

Figure 2. One Meter Length of the Section Under Design.

DESIGN CHART

The use of the new formulae (Eqs. (18) and (20)) obtained in this work in design can further be simplified by expressing the relationship between A_{St} and x in graphical form (Chart). This can be achieved if Eq. (2) is written as follows:

$$Y = \frac{(1-Z)(2Z-1)}{2Z(q-z)} \qquad (24)$$

$$where\ Y = \frac{mA_{st}}{bh}\ Z = \frac{x}{h}\ and\ q = \frac{d}{h}3$$

The relationship between y and z can be plotted for various values of q as shown in Fig. 3.

CONCLUSION

The new design formulae obtained in this work are shown to be accurate and do not impose any further limitation except the ones underlying elastic method of design. A major advantage of the new design formulae is that their use ensures exact determination of the depth of neutral axis as well as area of steel unlike the presently used formulae which involve some element of trial and error. It is the author's view that Eqs. (18) and (20) will provide a shorter alternative for elastic design of uncracked section.

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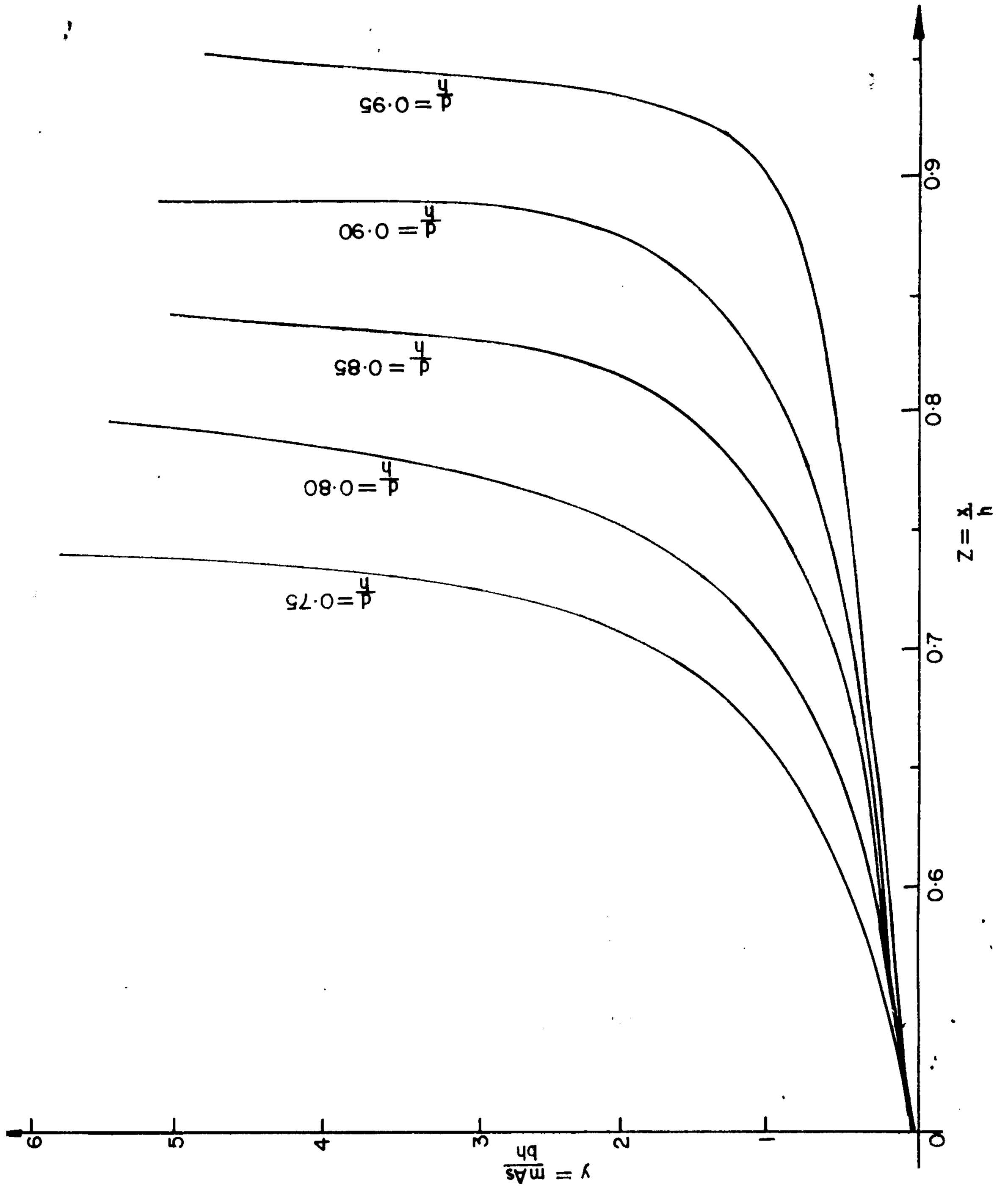

Figure 3. Chart For Elastic Design of Uncracked Reinforced Concrete Section.