PROCESS SETTING MODELS FOR THE MINIMIZATION OF COSTS DEFECTIVES

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Abstract
*The economy of production controls all manufacturing activities. In the engineering component manufacture; limits are always specified, either one-sided or two-sided. This paper attempts to determine the mean setting so as to minimise the total loss through under-limit complaints and loss of sales and goodwill as well as over-limit losses through excess materials and rework cos*ts.

Models are developed for the two types of setting of the mean so that the *minimum costs of losses are achieved. Also, a model is developed for inspection frequency sample size and discovery of defective products which result in minimum cost*

1. Introduction

Quality control aims at minimizing defectives during and after production. During production, the objective of the quality control is to evaluate the processes so as to decide when adjustment to the process must be undertaken to prevent out of control situation. It is necessary to carry out tests at intervals and from the results, decide on the process condition

At the end of the production, samples are taken for inspection to determine whether the end product satisfies the conditions required of it. If inspection is done at the end of the production all items which fail the inspection test are reworked or scrapped. This will involve costs which would have been avoided if the process was controlled during production.

In most engineering production, the parameters to be controlled are limits of acceptance which are averaged around the nominal dimension. In most consumable production, such as packaging the limit is on one side only, that is, not less than or not more than, as it is in the composition requirement of drugs.

In some processes, there is generally no way of knowing what percentage are above the limit stated. With the use of optimal, setting methods, the process can be so set that only a decided percentage will fall below or above the required limit and that the total costs of below and above limits will be minimum.

There is also the inspection interval which will result in minimum costs of inspection and scraps production.

2. Optimal Setting Process Models

2.1 Optimal setting of process mean in the case of one-sided limit

In filling operation, the process average net weight must be set. The standards prescribe the minimum weight which is printed on the packet. This set of quality control problems has one-sided limit (the minimum net weight). Underweight packets will lead to costs of complaints. Overweight means loss of, material without reward. Given the minimum weight of a packet as W, and a standard deviation of a normal distribution of the filling process as σ, it is necessary to determine the process mean μ so that the total cost C_T of overweight cost C_I, and underweight complaints cost per packetC_2 will be minimum. Let the material cost be C_1per gramme and the complaint cost beC_2 per underweight packet.

The distribution of· the net weights can be described as:

$$f(\times) = \frac{1}{\sigma}\sqrt{2\pi}\exp\left[-\frac{1}{2}(\frac{x-\mu}{\sigma})^2\right] \qquad (1)$$

and the total cost C_T per packer

$$C_T = C_2\int_{-\infty}^{w} f(\times)d\times + C_1(\mu - w) \qquad (2)$$

What is to be determined is μ.what minimizes $C_{T.}$ this process setting is the work of quality control department.

C_{Tmin} is obtained by differentiating with respect to μ_* the expression for C_T and equating it to zero

$$\frac{dC_T}{d\mu} = C_2\int_{-\infty}^{\times=w} f(\times)d\times$$

$= C_2 \int_{-\infty}^{\times=W} \frac{1}{{}^{\sigma}\sqrt{2\pi}} \exp\left[-\frac{1}{2}(\frac{\times-\mu}{\sigma})^2\right] + C_1$

(3)

$= \frac{-C_2}{{}^{\sigma}\sqrt{2\pi}} exp\left[-\frac{1}{2}(\frac{\times-\mu}{\sigma})^2\right] + C_1$

$\frac{DC_T}{d\mu} = 0$

$\rightarrow C_1 - \frac{C_2}{{}^{\sigma}\sqrt{2\pi}} exp\left[-\frac{1}{2}(\frac{w-\mu}{\sigma})^2\right] = 0$

$exp\left[-\frac{1}{2}(\frac{w-\mu}{\sigma})^2\right] = \frac{C_1}{C_2}{}^{\sigma}\sqrt{2\pi}$ (4)

μ=μ for minimum total deviation costs.

$\mu. = w + 2\sqrt{(-21n\frac{C_1}{C_2}{}^{\sigma}\sqrt{2\pi})}$ (5)

From equation (5), there could be no solution

$if\ in\ \left[\frac{C_1}{C_2}{}^{\sigma}\sqrt{2\pi}\right] > 0. this\ means\ that$

$\frac{C_1}{C_2}\ \sigma\sqrt{2\pi} > 1\ or\ C_2 <\ C_1\ \sigma\sqrt{2\pi}$

This means that no overweight is given if penalty for underweight $C_2$is relatively small with respect to the cost of overweight and variation in the process. This implies .a degeneration because minimum cost would be achieved if the packets were sold empty

. Example of the application of the process setting:

Suppose that w=, 250gms.

Standard Variation,σ - 2gm,

Cost of Unit Overweight, C_l = ₦ 0.36/gm, and cost of complaint (underweight),C_2= ₦10.00,then

$\mu.$

$= 250$

$+ \sqrt{-2\ In\left(\frac{0.36 \times 2\sqrt{2\pi}}{10}\right)} = 253.70gm$

If the mean is put at 253.70gm, the minimum loss and cost will be achieved with the setting.

2.2 Optimum setting of process mean for two-side limits

This type of quality requirement is frequently met in machine operations. In turning operations, two-sided limits are required for the diameter of a shaft. If the costs of rejects are equal on both limits, it can easily be seen that the best setting for the process is the mid-value between the two limits. In most cases, the costs are not equal and the problem of setting for the minimum average total costs arises. If the diameter of the shaft is too small nothing can be done about it and the product is lost. If it is too large, then rework is possible. There are two modes for this type of setting problems.

1 Cost independent on setting and the amount of over diameter.

In developing the model, let

C_1 = cost for items with too small diameter, i.e. scrap value .

C_2= cost of item with too large diameter, i.e. rework cost

T_l = lowest tolerance and

T_2 = the highest tolerance

C_T = the average total cost

$C_T = C_1\int_{-\infty}^{T_1} f(x)dx + C_2\int_{T_2}^{\infty} f(x)dx$ (7)

f(T) is normally distributed with a mean o μ and standard deviation σ.

For minimum C_T,

$\frac{dc_T}{d\mu} = -C_1 f(T_1) + C_2 f(T_2) = 0$

$\frac{f(T_1)}{f(T_2)} = \frac{C_2}{C_1}$

$= \exp\left[\frac{1}{2}(\frac{T_2-\mu}{\sigma})^2 - \frac{1}{2}(\frac{T_1-\mu}{\sigma})^2\right]$

$\mu. = \frac{T_1+T_2}{2} - \frac{\sigma^2}{T_2-T_1}. \text{In}\frac{C_2}{C_1}$ (8)

$\mu. = \frac{T_1+T_2}{2} if C_2 = C_1$

$\mu. < \frac{T_1+T_2}{2}, if C_2 > C_1 and In\frac{C_2}{C_1} > 0$

This means that the setting is shifted away from most expensive tolerance limits.

2. *In this model, the cost of turning the shaft is dependent on the setting.*

The cost of too small diameter. per item is constant and is equal to scrap cost. Cost of too large diameter is dependent on the amount of over diameter (more repair work).

The total average cost is given by

$$C_T = C_1\int_{-\infty}^{T_1} f(x)dx + \int_{T_2}^{\infty} C_2(x)f(x)dx + C_3(\mu)$$

It is assumed that both C_2(x) and $C_3(u)$ are second degree functions, that is, extra material removal is quadratically dependent on diameter.

$putting\ C_2(\times) = \alpha(\times - T_2)^2 + \beta$

$C_3(\mu) = y\mu^2$

$$C_T = C_1 \int_{-\infty}^{T_1} f(\times) d\times + (\beta - \alpha {T_2}^2) \int_{T_2}^{\infty} f(\times) d\times$$
$$-2\alpha T_2 \int_{T_2}^{\infty} \times f(\times) d\times + \alpha \int \times^2 f(\times) d\times + y\mu^2 \quad (9)$$
$$\frac{dC_T}{d\mu} = C_{1.} \frac{1}{\sigma\sqrt{2\pi}} . exp\left[-\frac{1}{2}(\frac{T_1-\mu}{\sigma})^2\right]$$
$$+(\beta - \alpha {T_2}^2).(\frac{1}{\sigma\sqrt{2\pi}}) exp\left[-\frac{1}{2}(\frac{T_2-\mu}{\sigma})^2\right]$$
$$-\left(2\alpha {T_2}^2 \frac{1}{\sigma\sqrt{2\pi}}.\right) exp.\left[-\frac{1}{2}(\frac{T_2-\mu}{\sigma})^2\right]$$
$$+2\,y\mu = 0$$
$$\therefore -(\beta - \alpha T_{22}) exp\left[-\frac{1}{2}(\frac{T_2-\mu}{\sigma})^2\right]$$
$$+C_1 exp\left[-\frac{1}{2}(\frac{T_1-\mu}{\sigma})^2\right] = 2\gamma\mu\sigma\sqrt{2\pi}$$

This equation can be solved by iteration with computer. It can also be solved if numerical values are given, for example, if a = 0. r = o and β= C_2, then equation (11) is transformed to the solution of equation (7).

2.3 Optimum Process Inspection Model

Process inspection is designed to discover, as quickly as possible, deviations in the process by means of regularly taking samples and inspecting them. The factors which determine the effectiveness of the inspection are:

- The sample size: The probability to discover a certain deviation depends on the sample. Size.
- Sample frequency: This determines the damage caused by deviation until it is discovered.
- Control limits: They determine the probabilities of both discovering a deviation and of taking necessary action because an observation falls outside these limits.
- Action policy: In the case of observation outside the limits, the policy determines the damage caused by the .deviation.

The model for this type of problem is so complex that it has not yet been possible to formulate a general model for all types of problems in this area. A simplified model is, discussed. Assume that in a process sudden deviation of the average occurs, which is persistent in times- and on the average is n units of measurements. Let the process be normally distributed with standards average, μ and standard deviation σ

The process is inspected by taking. a successive sampling of items at intervals of k between samples. Assuming inspection cost to be C_1 per sample and C_2 per item inspected and average damage caused by deviation C_3 per item, then under normal distribution, the control limits are fixed at μ ± 2σ. =μ ± 2σ/√n.
The probability to discover the disturbances through the sample can be expressed as (for positive)

$$p = (\overline{\times} > \mu) + \ {}^{2\sigma}/_{\sqrt{n}} / \ \mu = \mu + \Delta)$$
$$p = \int_{\mu+2\sigma/\sqrt{n}}^{\infty} (\frac{\sqrt{n}}{\sigma\sqrt{2\pi}})\, exp\left[-\frac{1}{2}(\frac{\times - \mu - \Delta}{\sigma/\sqrt{n}})\right] d\mu$$

The average number of intervals during, which the disturbances lasted without discovery is then given by (1/p-1/2), so that the average damage caused by the deviation is (1/p-1/2) KC_3

If such disturbances occur on the average once in items produced, then the average damage cost per item produced is (1/P-1/2) KC_3/λ.

The inspection cost per sample is C_1 + nC_2.A sample of n items taken from K items produced leads to inspection cost per item produced as

$$\frac{C_1 + nC_2}{K}$$

The total relevant cost now is the sum of the costs per item.

$$C_T = \left(\frac{1}{P} - \frac{1}{2}\right)\frac{K}{\lambda} C_3 + \frac{(C_1 + nC_2)}{K} \qquad (13)$$

Where C_1, C_2, C_3 are assumed known.
One is now to optimize the inspection system by choosing n and K so that C_T is a minimum for equation (12). The solution finally leads to

$$\mathrm{p} - \frac{1}{2\mathrm{p}^2} = \frac{\mathrm{C_1}}{\mathrm{C_2}} + \mathrm{n}\frac{\mathrm{dp}}{\mathrm{d\mu}} \qquad (14)$$
$$and\ K^2 = \frac{2p}{2-p} \lambda\, (\frac{C_1}{C_2} + \frac{nC_2}{C_3}) \qquad (15)$$

K can be obtained if equation (15) is solved

If w3e define $\mu. = 2 - \frac{\Delta\sqrt{n}}{\sigma}$

the expression in equation (12) will be transformed to

$$p = \int_{\mu}^{\infty} \frac{1}{\sqrt{2\pi}} exp - \frac{1}{2}\mu * 2_{d\mu};\ \frac{dp}{d\mu} = \frac{\Delta}{2\pi\sqrt{(2\pi n)}}$$
(12)

3. Conclusion

It is shown that the setting of process mean should take into account the cost of deviation in either side of the mean so that the mean is adjusted towards the limit whose violation creates more losses in terms of money. It is therefore necessary that the mean of two-sided limits be determined so as to represent the economic .setting of the process mean. For one sided limits, the process mean should be set far above the lowest allowable limit so that only a specified percentage of items will fall below the lowest allowable limit. In doing so, recognition should be given to the extra cost of the products above the limit stated.

Finally, during production, sample size and sample frequency should be determined with respect to minimizing costs of inspection and costs of scraps and complaints. This means that the size and frequency must be so determined that out of control conditions could be detected as quickly as .possible.

Nomenclature

C_T = Total Cost of defectives

C_1 = Cost of underweight (under size)

C_2 = Cost of over weight (over size)

C_3 = Cost of inspection

K = No of items to- be produced before inspection

n = Sample size

P = Probability of occurrence

T_1 = Lower tolerance

T_2 = Upper tolerance

W = Minimum weight

α, β, Υ = Proportionality constants

λ = No. of items produced

Δ = Small increase of mean

σ = Standard deviation

μ = Mean

μ = Mean setting resulting in minimum cost.