MODELLING FOR OPTIMAL NUMBER OF LINE STORAGE RESERVOIRS IN A WATER DISTRIBUTION SYSTEM

By

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Abstract
A mathematical model by which one can derive the most appropriate number of line storage reservoirs (which satisfy stated criteria) was developed and solved using an algorithm developed specially for the purpose.
A hypothetical example was solved using a 16-node network on a flat, plane where the flow to each demand, centre is 946 m^3/day. In a developing country with a consumption of about 150 litres per capita per day (lped), this is approximately 6.300 consumers per node or a total of 100,000 consumers for the network. Distances between the line storage reservoirs are 3.2 kilometres.
The results indicate that the least costly solution required two (2) line reservoirs for a system cost of ₦236, 190. In addition, as the supply duration to each demand centre increases, the system costs initially decrease, attain a minimum at 21 hours of supply and thereafter increases.

1. Introduction

Elevated storage reservoirs haphazardly placed in the distribution system may not only create environmental blight, but also prove to be expensive. While the use of an elevated reservoir system can be advantageous it is imperative that the reservoir system location be integrated into the water distribution management plan. Under the continually changing consumption rate , that water systems experience (figure 1), storage facilities within the system permit more uniform pumping rates and hence more efficient operation, in addition to providing reserves for fires and, other emergencies.
In the U.S.A. and other developed countries, the normal continuous flow operating condition is to run pumps at constant, steady rate even during off-peak hours, and to store the water in reservoirs for periods of peak demand [l]. In Nigeria, the same procedure is followed.

1.1Types of Line Storage

The predominant types of distribution (or line) storage facilities are:

(i) pumped or gravity ground storage, and
(ii) gravity elevated storage.

System designs generally utilise elevated or ground storage reservoirs or a combination of both. Groat [3] indicated that the combined use of elevated and ground level storage is desirable in water distribution systems, in order to balance the considerations of reliability with those of cost and community acceptance.
One major design problem with, line storage reservoirs is that the economic number of reservoirs in a system is not obvious. Several simulation studies [4, 5, 6] have included the existing locations in their analyses. However, no attempt bas been made to derive the optimal number of line storage reservoirs even for the simplest cases.

1.2 Optimum Number of Reservoirs

Two approaches often used in water distribution line storage systems to meet peak demands at demand centres are:

(a) One central storage facility and
(b) Several storage facilities where local consumptions are satisfied by on-site line storage systems, at each demand centre.

The economies-of-scale in, storage system construction and operation and maintenance (O & M) favour one storage reservoir, whereas the consideration of pipeline distribution costs favours multiple storage reservoirs. For a particular distribution system this represents a trade- off and the approach which satisfies overall consumption for water at demand centres at least cost will be a compromise between one and many line reservoirs.

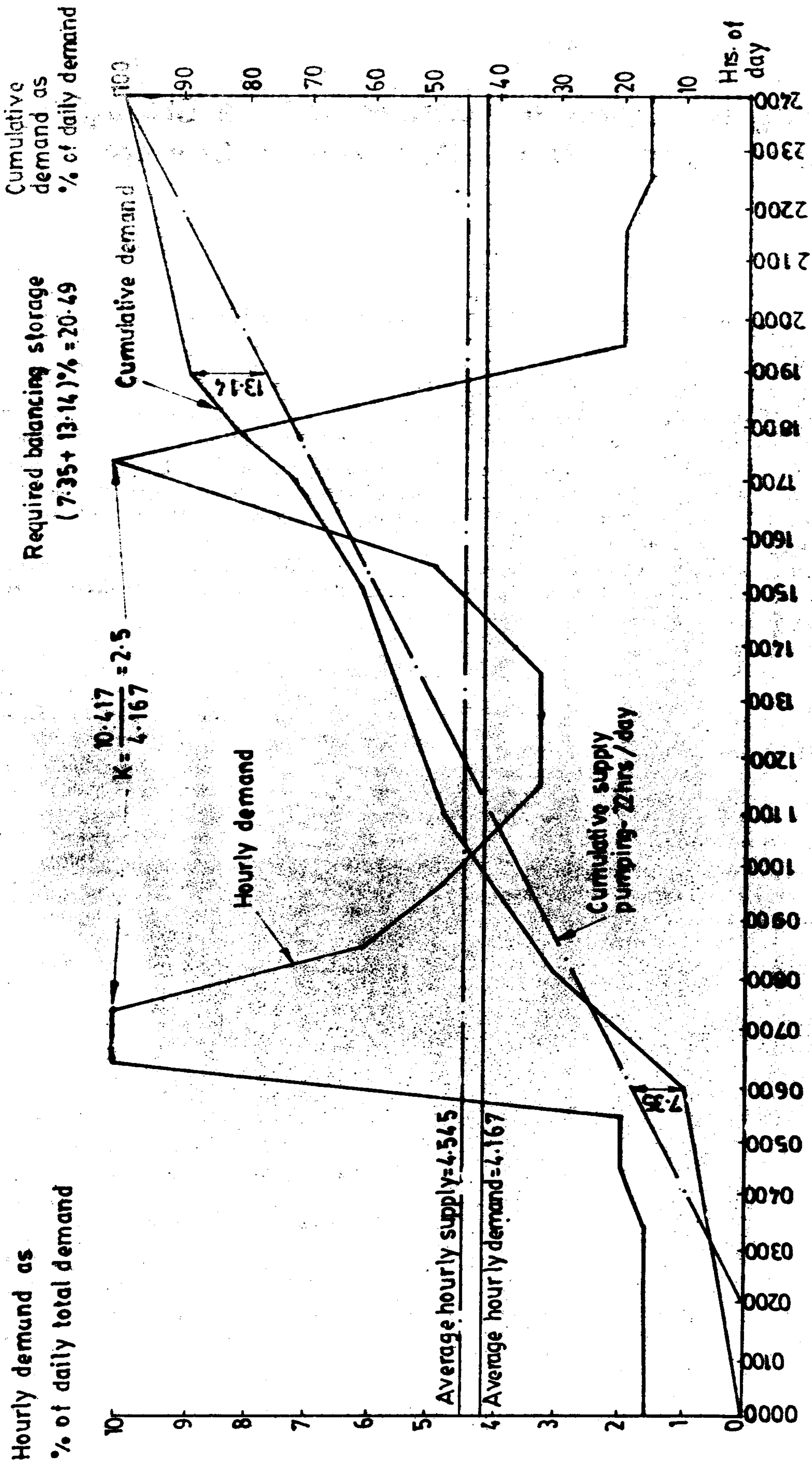

FIG. 1 IJEBU - ODE
TYPICAL DAILY WATER DEMAND CURVE
(Source: Balasha Jalon [2])

While the usual practice in water supply and distribution (WSD) systems is to locate one line reservoir at the end of the distribution system opposite the source [7]. Deb [8] found that the position of one large line reservoir was an important factor in the network cost optimisation. He found that the elevated storage tank located at the centre yielded minimum cost solution and that the optimum cost solution his system was about 1.33 times more when the service reservoir is located at the one corner than when it is located at the centre. Consequently, in the location of a single reservoir used is assumed to be at the centre of the network

1.3 Pipe Systems.

In order to identify the system components involved in a multiple reservoir system cost trade-off, some terms are defined. The Source (pipe) network, consists of the pipelines which may run directly from each demand centre to the closest line storage reservoir.

In the source network the flows are steady and assumed constant throughout a 24-hour day. The flow in each pipe in this network is the average daily demand at the demand centres receiving part of their demand from the line storage reservoir under consideration.

In the demand network, the flows depend on the nature of the demand and the total supply time to each demand centre as well as the degree of building in the network. The flows in the demand network may be continuous or intermittent as in some developing countries.

2. Cost Relationships

2.1 Reservoir Cost

Normally, only a fraction of the flow Q that is intended for a given node is stored in a given reservoir. Let this fraction be denoted by α.

As N, the number of storage reservoirs is increased, the flow αQ, is divided by more line storage reservoirs, the flow into each reservoir is (αQ/N) and the size of the reservoir is based on this flow. Hence the average cost for each reservoir declines with N.

The economics of scale in the storage reservoir is given by the equation [9]

$$C_{SR} = K_{cen}\left(\frac{\alpha Q}{N}\right)^{*}$$

For N storage reservoirs, the cost of $C_{SR}(N)$ is given by

$$C_{SR}(N) = K_{cen}N\left(\frac{\alpha Q}{N}\right)^{*}$$

$$= K_{cen}(\alpha Q)^{X}\, N^{1-K}$$

In figure 2 is plotted the cost function from the above equation for the storage reservoirs as the number of reservoirs increases from 1 for the same amount of water stored.

2.2 Tradeoffs

If the total amount of water stored remains the same, it has been shown [10] that as the number of storage reservoirs increases, the costs of storage reservoirs and the source of pipe network both increase, while the costs of the demand pipe network decreases.

Consequently, a trade-off exits between the storage reservoir and source network cost and the demand network costs. The optimal number of storage reservoirs is that number which gives a system of least total cost while, satisfying all the constraints. This is depicted in figure 3.

3. Model Construction

A model to determine the most appropriate number of storage reservoirs should contain constraints which indicate the minimum consumption of consumption, and the minimum head required at each demand centre. The objective of the model is to minimise the system costs while respecting the .restrictions imposed on the problem.

3.1 Assumptions of the Model

A number of assumptions are made to make the problem tractable and include the following:

- a demand centre is linked to the nearest line reservoir;
- the cost of each, link depends on the flow rate, the link length, and the pressure in the link;
- flow between two demand centres from a line reservoir may be a direct link or may be forwarded through a series of links;
- all demand centres have identical consumption and consumption characteristics (i.e., the consumption curve is the same at all demand centres),
- the system with several reservoirs offers the same service as that with one reservoir(i. e, with one only at the centre),
- the pressure ,head at each storage reservoir is adequate to supply all demand centres assigned to the reservoir,
- the cost of each line storage reservoir is independent of the location. That is, a fixed size of reservoir costs the same irrespective of where it is located on the plane.

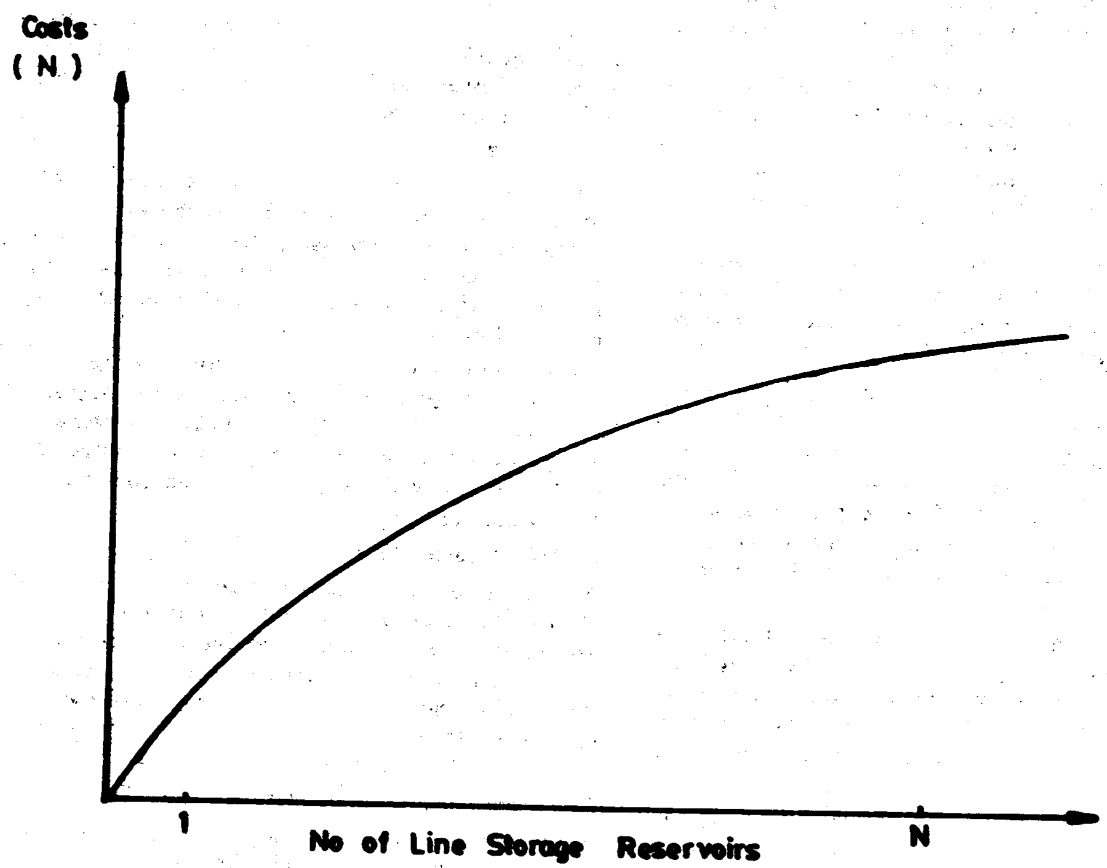

Figure 2: COST FUNCTION FOR LINE STORAGE RESERVOIRS VS NUMBER OF LINE RESERVOIRS (FOR SAME AMOUNT OF WATER STORED) (Adopted from Buhr & Woodside [9])

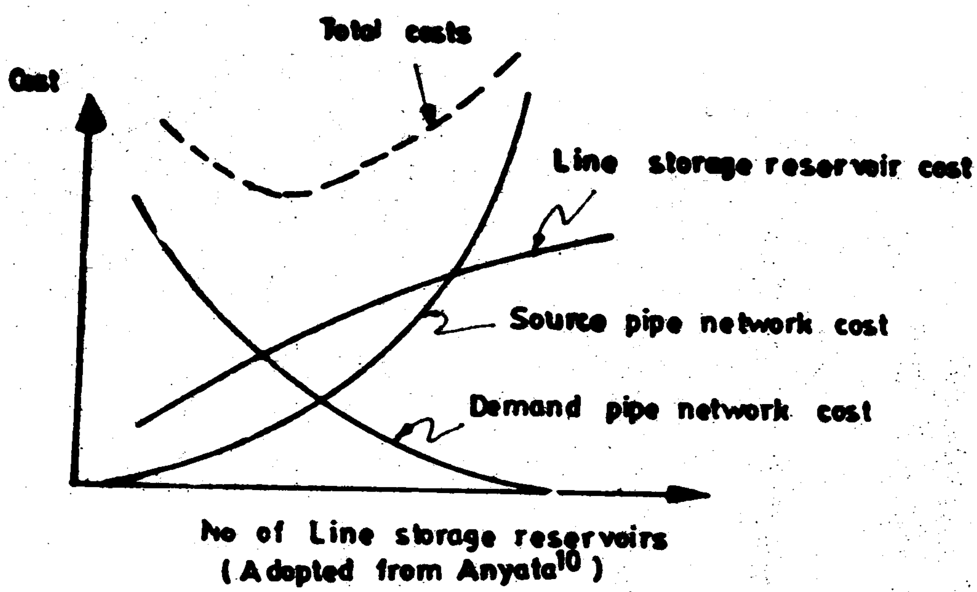

Figure 3: THE EFFECTS OF NUMBER OF LINE RESERVOIR ON COMPONENT COST OF WATER DISTRIBUTION SYSTEME FOR THE SAME AMOUNT OF WATER STORED.

3.2 Problem Formulation-State for 1 Period Model

Demand centres m and potential sites, for line storage reservoirs are assumed in the network. The potential sites may occasionally coincide with some of the m demand centre locations. The problem can be modelled as a simple plant location problem [l1] in the following manner.

3.3 Objective Function

The objective of this mathematical model is to find the values of Q_1, the capacity of reservoir located at site i, (i = 1...s) and q_{ij} the capacity of pipelines between source and reservoirs, and reservoirs and demand centres (i = 1...m) to minimise the total cost of storage reservoirs and pipelines used in transshipment of water. The objective function is:

$$F = C_{SR.i}(Q_i) + \sum_{i=1}^{s} \sum_{o=o}^{s} PC(q_{oi})L_{io} + \sum_{j=i}^{m} \sum_{i=1}^{s} PC(q_{ij})L_{ij} \quad (1)$$

where, $C_{SR,j}(Q_i)$ = Cost of Storage reservoir of capacity Q_i at i $PC(q_{io})$ = unit cost of pipe link between source at o and storage reservoir at i (i.e., in source network) for flow of q_{oi} in link oi.

$PC(q_{ij})$ = unit cost of pipe link between node i and demand centre j (i.e. in demand network) for flow of q_{ij} in link.

q_{io} = Pipeline capacity from source, o to storage reservoir at i(i.e., in. source network).

q_{ij} = Capacity of pipelink from node i to demand centre j(i.e. in demand network).

L_{oi} = Length of pipelink between source o and storage reservoir i

L_{ij} = Length of pipe link between nodes i and j.

3.4 Constraint

Continuity at node j (pot a reservoir)

$$\sum_{i=o}^{s} q_{ij} - dem_j = \sum_{k} q_{jk,} \forall_{j;}\, j = 1,..m \qquad (2)$$

Power form all - demand at = flow from
nodes i to j node j to all
node j node k

At nodes j (a reservoir):

$$\sum_{i=o}^{s} q_{ij} - dem_j = \sum_{k}^{s} q_{jk,} \forall_{j;}\, j = 1,..m \qquad (3)$$

Flow to reservoir		Sum ,0f all		Flow required
site j from source	=	demand from	+	at other reservoir
or another reservoir		demand center		sites, k through
		fed by reservoir		reservoir j
		at j = $Q_i + q_{ij}$		

Capacity Constraint on the Reservoirs

For node i

Let a_{ij} be the fraction of total demand at node j that is stored in reservoir i and let Q_i be the reservoir capacity, Then

$$Q_{ij} \geq \sum_{j=1}^{M_i} a_{ij}\, q_{ij};\ \forall_{i,j;}\, i, \dots, s \qquad (4)$$

$$j, \dots M_i (where\ M_i\ is\ number)$$

Capacity of		Fraction of total	of demand centre
storage	≥	quantity shipped	fed by reservoir at i)
reservoir		from reservoir	
at i		site to demand j	

The Head Constraints at the Junction Points

Reservoirs Nodes and Demand Centres

The head loss in pipe link i,j is given by

$\Delta H_{ij} = G_{ij} L_{ij}$ (5)

Where G_{ij} the hydraulic gradient between i and j depends OD the pipe; ifF0perties diameter and roughness and discharge L_{ij} is the pipe length between i and j Using the Hazen – William equation gives

G = const. (Q/C_{Ew}) 1.952_d-4.87

where Q is the discharge (C_{mv}) the Hazen-William coefficient, and d the pipe diameter, (if d is the cm, Q in m^3/sec, const. = 8.51 x 10^5).

Starting from any node in, the system o, at which the head is known in advance (for. example, at the source or at a reservoir), the head constraint for the node e is

$H_{min}\theta < H_o \pm \sum_{ij} G_{ij} L_{ij} < H_{max}\theta$ (6)

where the summation is overall links connecting node o to node θ. The sign of the term depends on the flow direction. The equation represents two nonlinear constraints in q_{ij} The $H_{min}\theta$ constraint usually result from service performance requirements. The $H_{max}\theta$ constraint may result from service performance constraints or from technological limitation on the pressure-bearing capacity for the pipes [1]. However, in the analysis, we ignore this constraint and use only the H_{min} constraint, so that we have

$\pm \sum H_{ji} > H_o - H_{min}\theta$ (7)

Sum of all head > Known - Min allowable
Losses between head at head at node θ
The known head source
and node at θ say

$V_{\theta;}$ θ = All nodes including reservoir and demand centres

The non-negativity constraints

The non-negativity requirements are that

$q_{ij} \geq O; \forall_{ij}, i = 1, 2 \ldots s$
$j = 1, 2, \ldots m$ (8)

The optimisation mode then is Minimise

$$\sum_{i=1}^{s} C_{SR}, i(Qi) + \sum^{s} \sum_{\substack{i=0\\j=1}}^{s} PC(q_{ij})\, L_{ij} + \sum_{\substack{j=1\\i=1\\i\neq j}}^{m} \sum PC(q_{ij})L_{ij} = 1 \quad (1a)$$

Subject to

$$\sum_{i=1}^{s} q_{ij} - Dem_j = \sum_{qk}, \forall_j\ ; \ j = 1, \ldots, m \quad (2a)$$

At node j (a reservoir)

$$\sum_{i=o}^{s} q_{ij} = \sum_{k=1}^{Mi} Dem^s_{jk} + \sum q_{jk}, \forall_j;\ j = 1, \ldots, m \quad (3a)$$

Mi = number of demand centre fed

$$Q_i \gtreqless \sum_{j=1}^{Mi} a_{ij} q_{ij} \ \forall_i,\ i =, \ldots, s \quad (4a)$$

$$\mp \sum\sum \Delta\ H_{ij} > H_o - H_{min\theta};\ \theta = \text{all node} \quad (5a)$$

$q_{ij} \geq 0\ \forall_{ij}$; i – 1, 2, …,s (6a)
j = 1 ……..,2,…, m

4. Model Solution

The analysis involves a search for a globally optimal solution. Several algorithms could be used to solve this problem. However, most of them almost are impossible to apply on large scale networks due to the large amount of computation time involved or because the solution might diverge [12, 13].

Therefore, a computer programme for designing cost effective water distribution net works called Economic Number of Reservoir Technique (ECONORTEC) [10] was developed. The solution is based on two different approaches to reducing system distribution costs:

(i) Flow concentrations (or combining flows). This method combines flows to two or more nodes into one flow link to minimize the pipe costs.

(ii) The other method is to locate fine system reservoirs in each, geographically divided area (or sectors), and to supply the demand nodes from these reservoirs.

The pipe lengths in the net work are minimized and thus distribution system cost reduced.

5. Evaluation Method

The criteria used for evaluating alternative distribution configuration costs are restricted to the pumping costs pipe material and labour cost storage reservior costs and; where applicable, booster pump cost Watanatada [13] and Deb[8] showed that these represent major components operation and capital costs and are assumed to be a good surrogate for total -distribution system costs.

The general procedure followed was to initially prescribe 4 reservoirs within the network. By successive1y dropping one reservoirs at a time (a recursive procedure) and rearranging the lowest cost network for the specified number reservoirs was found, while maintaining a minimum pressure head of 14.1 m (138 KN/m^2) at each demand centre[14]

The reservoirs were initial1y located to nodes 6, 7, and 11.(figure 4a). Pressure head at inlet reservoir (node 6) is 105m (691.2kN/m^2). The reservoirs have a life of 25 years and pipes a life of 50 years cost of line storage reservoirs = N171.4 x 10^3 $Q^{0.73}$ where Q is the capacity of the reservoir. Discount rate is 10% For the purposes of this work, number of reservoirs which gives the least system cost is the optimal number of line storage reservoirs in the system. The starting and final (least costly) configurations for a system serving 16 demand centres are given in figures 4(a) and (b). The costs for 1 to 4 reservoirs are shown in Table 1

Table 1:System Costs (N/Annum) for 1 to 4 Line Reservoirs when the total amount of water stored remains constant

No. of Line eservoirs	4	3	2	1
Pipe + Energy Cost (₦)	191, 772	195, 137	198, 545	208, 177
Line Reservoir Costs (₦)	47, 210	42,427	37, 645	208, 064
System Costs (₦)	238, 982	237, 566	236, 190	240, 181

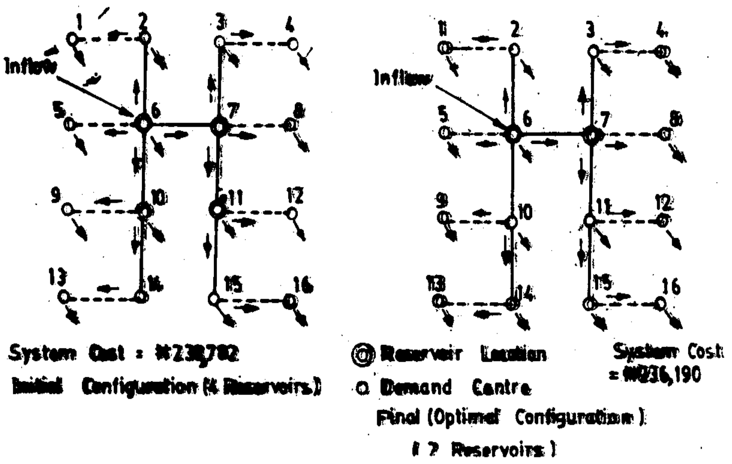

Figure 4: INITIAL AND FINAL CONFIGURATIONS FOR RESERVOIR LOCATION IN THE LEAST COST NETWORK.

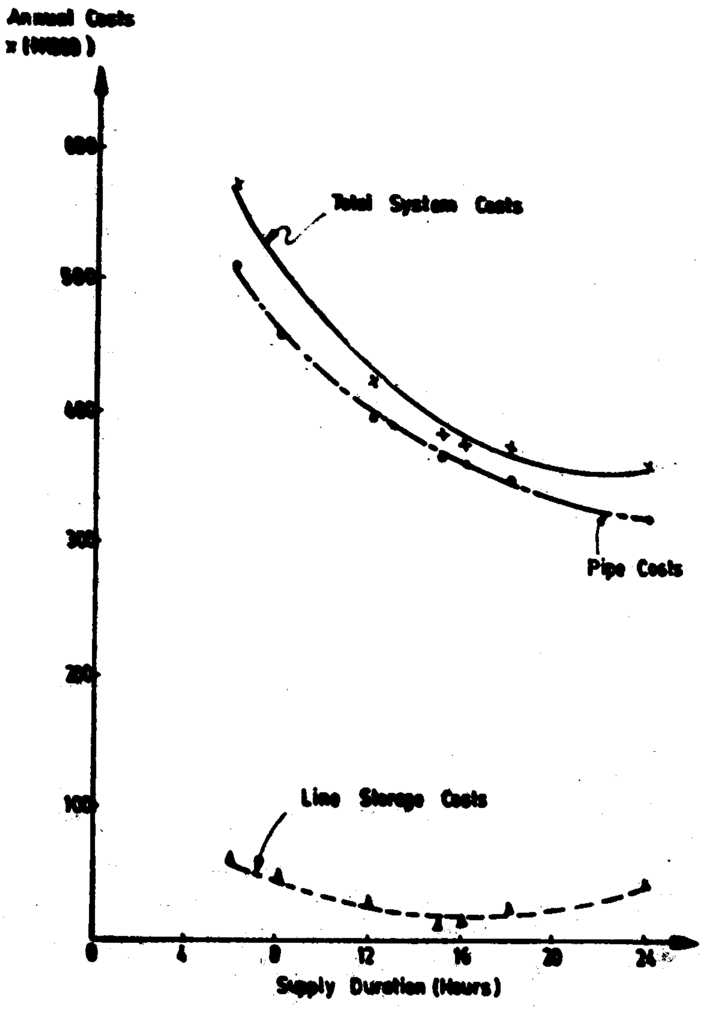

Figure 5: THE EFFECTS OF SUPPLY DURATION ON SYSTEM AND COMPONENT COSTS FOR LINE STORAGE.

5.1 The Effect of Supply Duration on the System Costs

The effect of supply duration at each demand centre on the system costs was also examined. Starting with 2 line reservoirs which from, Table provided the least costly system, and assuming a trapezoidal consumption curve for each demand centre the system-costs were computed for supply durations of 6,8,12,14,16,18 and 24hours

The results are shown in figure 5.

6. Discussions and Conclusion

When the supply duration to demand centre was Varied from 6 to M hours and using a trapezoidal consumption curve, the results indicate that the total System cost initially decreases as the supply duration to each demand centre increases (figure 5). The total system cost attains a minimum when the duration of supply is 15 hours and increase beyond the minimum value. It is observed that the system cost are primarily determined by the pipe costs. The roost contribution of line storage is small to comparison to pipe costs rnai1y due to economic of scale.

While the volume of storage required for short supply durations is appreciable, scale economies are captured by using large reservoir volumes. As the supply duration increases, the sizes (and costs) of the storage reservoirs decrease, reach a minimum at 15 hours of supply, and then increase.

For a short duration the flow rate in the distribution system pipe network is high, resulting in large pipe sizes and high pipe costs. As the supply duration increases, the flow rate in the pipes (and the pipe costs) decreases attaining the minimum value when the supply is continuous. The total system and component costs for varying duration of supply (figure 5.) clearly show that:

(i) a continuous supply system is not the least costly, and

(ii) when intermittent supply is being considered, the supply duration is an important factor in determining overall least-cost system.

Consequently, cost comparisons made at arbitrary values of supply durations can be biased in favour of one of the systems.

The limitations to the model are mainly that only ground storage tanks were evaluated. Also combinations of both types of reservoirs were not evaluated. This was done, because there appears to be no cost advantage between ground and elevated storage.

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