EFFECT OF CHANNEL BENDS ON TRANSVERSE MIXING

BY

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ABSTRACT

Velocity and tracer concentration measurements made in a meandering channel are used to discuss the effect of bends on the transverse mixing of a conservative tracer introduced into the flow. It is shown that bend induced spiral motion greatly enhance the mixing potential of meandering channel flows; The magnitude of the normalised transverse mixing coefficient is observed to be greater than the normlised transverse diffusion coefficient for straight channel flows.

1. INTRODUCTION

Transverse (across channel) mixing is extremely important in river flows because it, exerts a large influence on the rate of longitudinal dispersion (1). Also, knowledge of the transverse mixing coefficient is of considerable practical importance in studies dealing with mixing down-stream of sources of pollution.

The first study of transverse mixing in bends of turbulent open channel flows was reported by Fischer (2). He derived an expression for predicting the transverse dispersion coefficient based on the following assumptions;

(i) transverse dispersion is analogous to longitudinal dispersion.

(ii) the transverse velocity distribution in the vertical can be described by Rozovskii's transverse velocity distribution for fully developed turbulent flow in an open channel bend.

Fischer demonstrated that the dominant effect of curvature is generally to increase the transverse mixing coefficient, E' w compared to that usually observed in straight open channels.

Yotsukura et al (3) employed a simulation procedure to predict the transverse mixing coefficient in a six-mile reach of the Missouri River near the city of Blair, Nebrasta. They obtained an average value for the transverse mixing coefficient, E_{zw} of 0.6u.T (u. is shear velocity and D is average flow depth) compared with O.7u.D from the method of moments technique.

Chang (4) also proposed simulation and integral methods to find the transverse mixing coefficient. Chang's results indicated a strong periodic variation of E_{zw} in the longitudinal direction. This phenomenon was also observed by Engmann (5) who used the moments technique of Aris in his analysis.

The objective of this study is to demonstrate and explain the pronounced effect on the mixing potential of meandering channel flows caused by the bend induced spiral motion.

2. THEORETICAL CONSIDERATIONS

The starting point of analysis of transverse mixing is the convective diffusion equation. The time averaged convective diffusion equation in a curvilinear coordinate system is given by Sayre and Fukuolca (8) as

$$\frac{\partial c}{\partial t}+\frac{1}{h_1}\frac{\partial (cu)}{\partial x}+\frac{\partial (cv)}{dy}+\frac{\partial (cw)}{dz}=\frac{1}{h_i^2}\frac{\partial}{\partial x}\left(E_x\frac{\partial c}{\partial x}\right)$$
$$+\frac{\partial}{\partial y}\left(E_y\frac{\partial c}{\partial y}\right)+\frac{1}{h_i}\frac{\partial}{\partial z_i}\left(h_i\frac{\partial c}{\partial z}\right) \quad \ldots\ldots\ldots..(1)$$

Where x coincides with the center line of channel; Y is the vertical coordinate measured upwards from channel bed; z is lateral distance from centerline of channel; u, v, w are the time-averaged velocity components in the x, y and z directions respectively; c is the time averaged concentration at a point; E , E , and E are local turbulent diffusion coefficient; $h_1 = 1 \pm \frac{z}{r_c}$ where the plus sign is for a circular bend curving to the left and the negative sign when bend curves to the right; r is the radius of the channel centerline.

For steady flow and negligible effect of longitudinal diffusion, (9) averaging Eq (1) with respect to the f'low depth gives

$$\frac{\partial}{\partial x} D\{(\overline{uc} + \overline{u^{\iota}c^{\iota}})\} + \frac{\partial}{\partial z}\{Dh_i\,(\overline{wc} + \overline{w^{\iota}c^{\iota}})\} = \frac{\partial}{\partial z}\left\{Dh_i\,\bar{E}_z\,\frac{\partial \bar{c}}{\partial z}\right\} \ldots\ldots(2)$$

in which u^i, W^i, and c^i represent a local deviation from the depth averaged values $\bar{u}\,\bar{w}$ and $\bar{c}$ respectively.

Since the main interest is in the overall mixing due to the combined effects of turbulent diffusion and transverse velocities, the transverse mixing coefficient E_{zw} is defined as follows:

$$E_{zw}\frac{\partial \bar{c}}{\partial z} = E_z\frac{\partial \bar{c}}{\partial z} - \overline{wc} - \overline{w^{\iota}c^{\iota}} \quad \ldots\ldots\ldots.. \quad (3)$$

Similar approximations nave previously been made by Chang (4) and Engmann (5). E_{zw} is assumed to represent the average transverse mixing coefficient over a small reach of the channel.

Eq. (2) therefore reduces to

$$\frac{\partial}{\partial x} D\{(\overline{uc} + \overline{u^{\iota}c^{\iota}})\} = E_{zw}\frac{\partial}{\partial z}\left(Dh_i\,\frac{\partial \bar{c}}{\partial z}\right) \quad \ldots\ldots\ldots.. \quad (4)$$

using the moments technique of Aris (10), Eq. (4) is multiplied by η and integrated from the left ban, η, = W_1, to the right bank, η = W_2 (where η = z – z_o and z is the z coordinate of the point of tracer release). The result is divided by the tracer flux $\int_{w_1}^{w_2}\overline{Ducd}\eta$ which is a constant to obtain

$$E_{zw} = \frac{1}{2}\,\frac{dM_2(x)}{dx}\,/A(x) \quad \ldots\ldots\ldots.. \quad (5)$$

Where

$$A(x) = \frac{\int_{w_1}^{w_2} D\bar{c}\left(1+\frac{2\eta+z_0}{r_c}\right)d\eta + \int_{w_1}^{w_2}\frac{\partial}{\partial \eta}\left\{D\bar{c}\eta\,\left(1+\frac{2\eta+z_0}{r_c}\right)d\eta\right\}}{\int_{w_1}^{w_2}\overline{Ducd}} \quad - - \quad (6)$$

and

$$M_2(x) = \frac{\int_{w_1}^{w_2} D\,(\overline{uc}\ \overline{u^{\iota}c^{\iota}})\,\eta^2 d\eta}{\int_{w_1}^{w_2}\overline{Ducd}\eta} \quad \text{--------------} \quad (7)$$

Equations (5) to, (7) formed the basis for evaluating the transverse mixing coefficient, E_{zw}

3. EXPERIMENTAL EQUIPMENT AND PROCEDURE

The experiments were performed in a meandering flume 19.05. long, 0.76m wide and 0.35m deep as shown in Figure 1A. The flume had two identical but reversed 180^0 circular bends with a centerline radius of 2.74m. A 1.52m long straight reach connected the first bend to the head tank. This flume rested on ten wooden supports, each fitted with bolts for slope adjustment.

Water velocities in the flume were determined by a three-tube yaw probe capable of giving the magnitude and the direction of the velocity vector component in a horizontal plane. The principle and calibration of the probe are fully described by Rajaratnam and Muralidhar (6). The yaw probe could be positioned at any desired location within a cross-section electronically.

Longitudinal and transverse velocity measurements were made at selected cross-sections (see Figure 1B) for 5 verticals located at Z/W - 0, + 0.20 and ±0.40 and 10 points in the vertical, where W is the channel width. Table 1 summarises the hydraulic data for the experiments.

TABLE 1: HYDRAULIC DATA FOR EXPERIMENTS

Test Run	Mean Flow Depth, D cm	Channel Slope so	Average Shear u. cm/s	Average Yelocity U_o m/s	Froude Number $U_o/\sqrt{g^D}$
302	7.05	0.000636	1.93	0.243	0.292

304/6	4.24	0.002121	2.82	0.342	0.530
307/8	7.17	0.000424	1.59	0.239	0.285

The tracer concentration measurements consisted of injecting a steady stream of Rhodamine WT20 fluorescent dye at a velocity which approximated that of the water. The tracer source was located at mid-depth for both centerline and bank injections. The tracer injection in all the tests was made at the beginning of the first bend. The spread of the tracer downstream of the source was measured with a sampling rake having 24 sampling probes (tubes) spaced at 3.05cm intervals. This rake enabled simultaneous sampling at 24 points across the flume. The samples collected were analysed for concentration with a fluorometer (Turner Model III). Tracer concentration measurements were made at three levels: Y/D = 0.25, 0.50 and 0.75, for all verticals at each cross-section, where D is the mean flow depth and Y is vertical distance from channel bed to point of measurement. The average tracer concentration in a vertical was given by the average of the three readings.

4. RESULTS AND DISCUSSION OF RESULTS

FLOW CHARACTERISTICS

Flow in meandering channels differs from that in straight channels because the presence of centrifugal forces results in the formation of transverse pressure gradients and the associated transverse circulation (spiral motion) in the plane of a cross-section. The growth, decay and reversal of the spiral motion cause the flow structure to change in the downstream direction.

A typical transverse distribution of the depth-averaged longitudinal velocity, $\bar{u}$, at different cross-sections is given in Figure 2. It is evident from this figure that higher velocities initially occur near the inside at the beginning of each bend. This observation is in agreement with potential flow theory for flow around a bend which predicts a free vortex type of motion with $\bar{u} \propto 1/r$. The high velocity zone however start to shift away from the inner towards the outer bank from about θ^1or θ^2 = $\pi/3$(θ^1or θ^2 are radial angles from beginning of 1st and 2nd bends resp.) due to the development of the spiral motion.

Figure 3 shows a typical set of measurements of the transverse velocity component, w. The plot clearly reveals the growth, decay and reversal of the spiral motion. The strength of the spiral motion defined by Yen (7) as: $\int_o^d /w/$ dy, where d is the flow depth, was found to vary laterally in all cross-sections. As the spiral motion due to the second bend develops it displaces the residual spiral motion from the first bend towards the outer bend causing it to decay completely around the middle of this bend.

Transverse Tracer Distribution

Transverse tracer distribution at the middle of the first bend at various level and for different source location are presented in figure 4.The plots indicate that the transverse spreading of the, tracer is consistent; with the pattern of transverse velocities. It is obvious from Figure 5 that the tracer initially spreads much more rapidly for centerline release of tracer than for side release. Hence complete mixing of tracer is achieved within a shorter distance of channel when tracer source is located near the centerline than at other points within the cross-section.

Longitudinal Variation of E_z/U.R

Table 2 gives values of M (x) and A(x) for some of the Runs. The transverse mixing coefficient, E_{zw} is computed from Eq. 5 and then normalized with the product of the shear velocity, $U^{\cdot}$, and the hydraulic mean depth, R. The results are summarised in Table 3 and plotted in Figure 6. Runs 302, 304 and 307 achieved virtually full mixing of the tracer near the beginning of the second bend hence no furthle readings were necessary.

From the longitudinal variation of the normalised mixing coefficient, E_{zw} /U· R it can be inferred that the mixing coefficient is influenced by source position and this probably reflects the role of transverse velocities in the mixing process. The value of E_{zw} /U· R was observed to be greater within the first half of the first bend for centerline release of tracer compared with bank release for the same flow conditions. This is clearly seen from Figure 4 which shows the tracer to be better mixed with the flow in Run 304 than in Run 306. Maximum values of E_{zw} /U· R generally occured within the middle third of both bends. Low and occasional negative values E_{zw} /U· R were observed at the beginning of the second bend.

TABLE 2: Values M_2 (x) and A (x)for Runs 302, 306 and 308

RUN	302		306		308	
SECTION	$M_2(X)$ m^2	A(x) $(m/s)^{-1}$	$M_2(x)$ m^2	A(x) $(m/s)^{-1}$	$M_2(X)$ m^2	A(x) $(m/s)^{-1}$
0_1 0	0	3.496	0	3.691	0	3.800
/6	0.0042	3.496	0.0030	3.691	0.0020	3.800,
/3	0.0303	3.484	0.0091	3.468	0.0157	3.582
/2	0.0749	3.820	0.0198	2.900	0.0526	3.723
2/3	0.1363	3.620	0.0332	2.749	0.1140	3.892
5/6,	0.1616	2.286	0.0449	2.569	0.1517	3.061
0_2 0	0.1982	1.070	0.0527	2.605	0.1868	1.895
/6	0.2009	0.792	0.0647	2.369	0.1907	1.562
/3	0.1869	0.236	0.0829	2.545	0.1884	0.645
/2			0.1013	2.377	0.1862	0.182
2/3			0.1145	2.088		
5/6			0.1341	1.565		

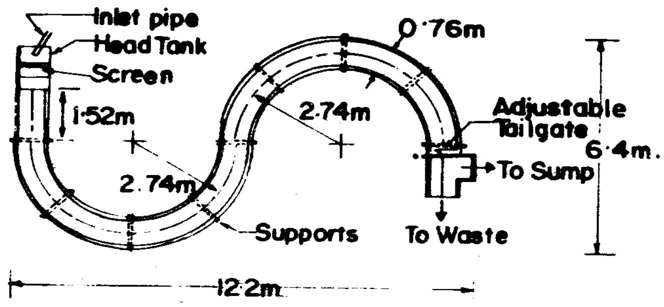

FIGURE 1A PLAN VIEW OF MEANDERING FLUME

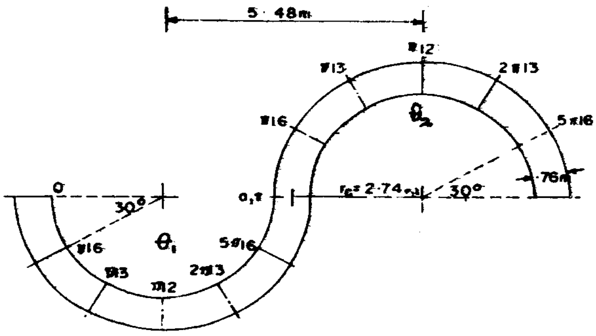

FIGURE 1B SKETCH OF MEANDERING FLUME SHOWING SECTIONS USED FOR VELOCITY AND CONCENTRATION MEASUREMENTS

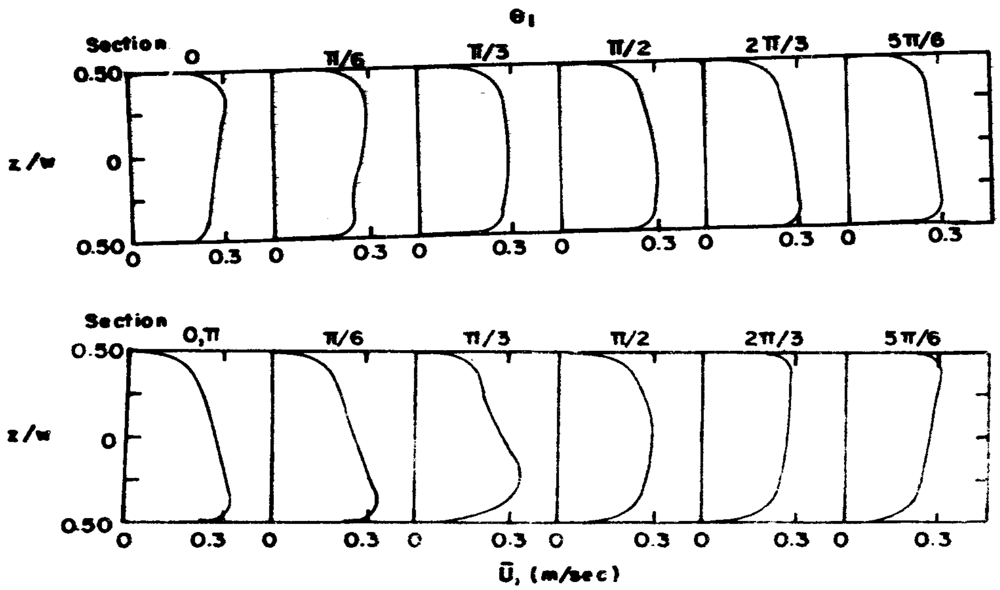

FIGURE 2. LATERAL DISTRIBUTION DEPTH-AVERAGED LONGITUDINAL VELOCITY RUN 301/2/3

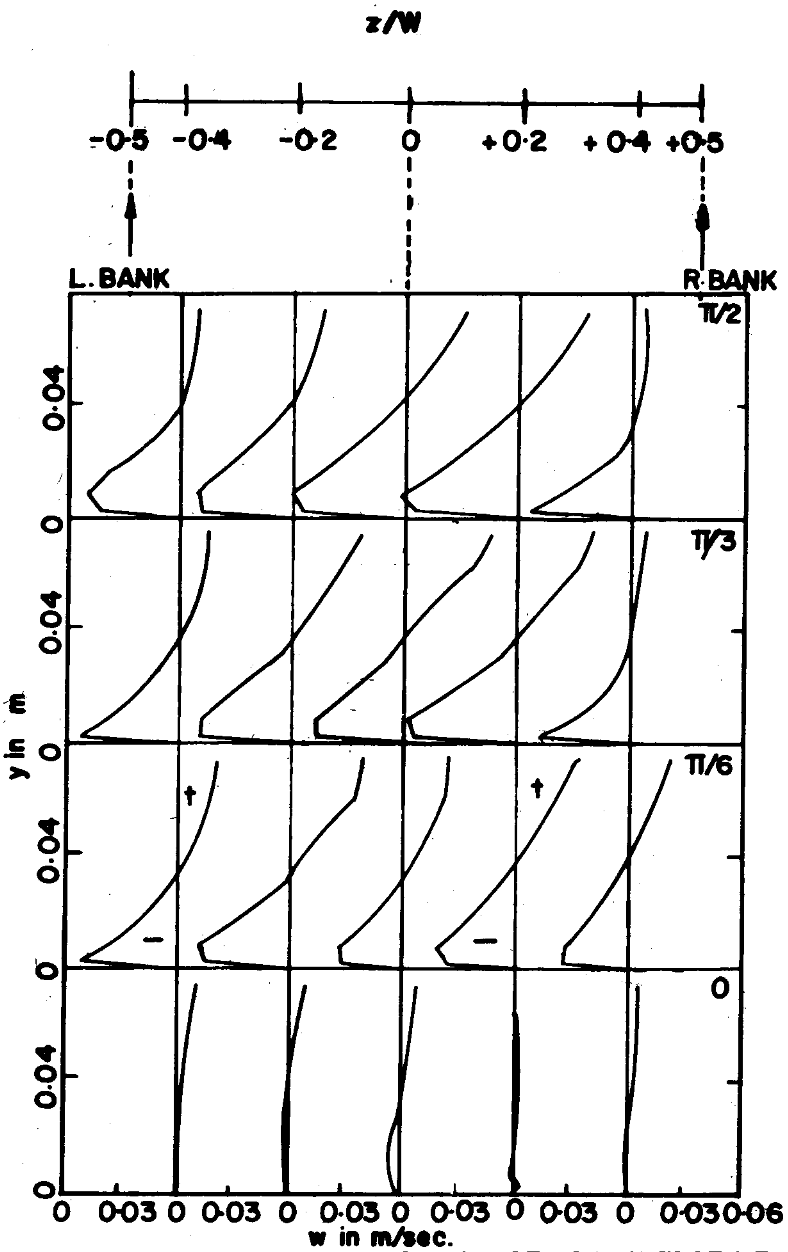

FIGURE 3a VERTICAL DISTRIBUTION OF TRANSVERSE VELOCITY RUNS301/2/3 SECTIONS $\theta_1 = 0$ TO $\pi/2$.

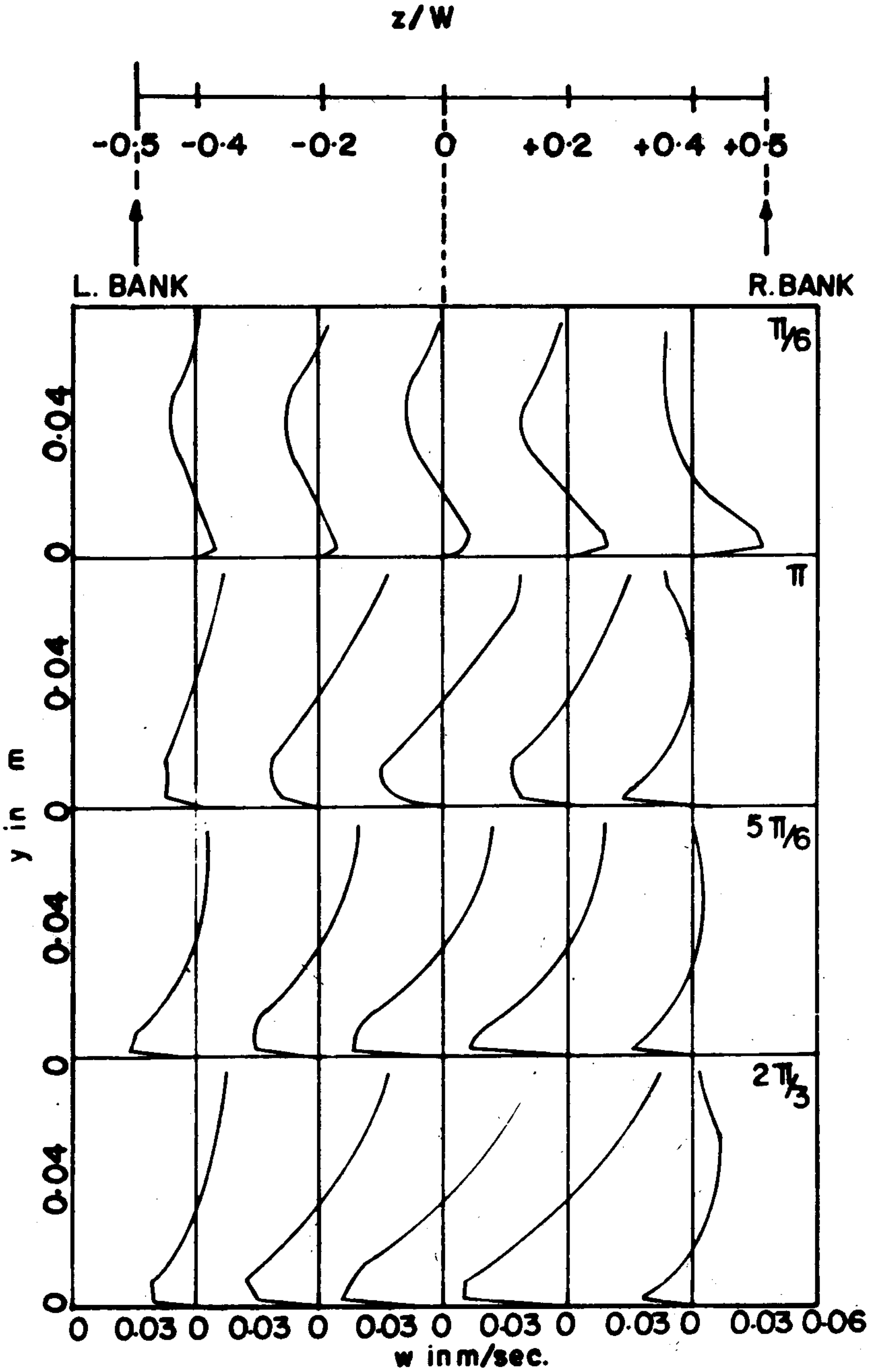

FIGURE 3b VERTICAL DISTRIBUTION OF TRANSVERSE VELOCITY. RUNS 301/2/3 SECTIONS $\theta_1 = 2\pi/3$ TO $\theta_2 = \pi/6$

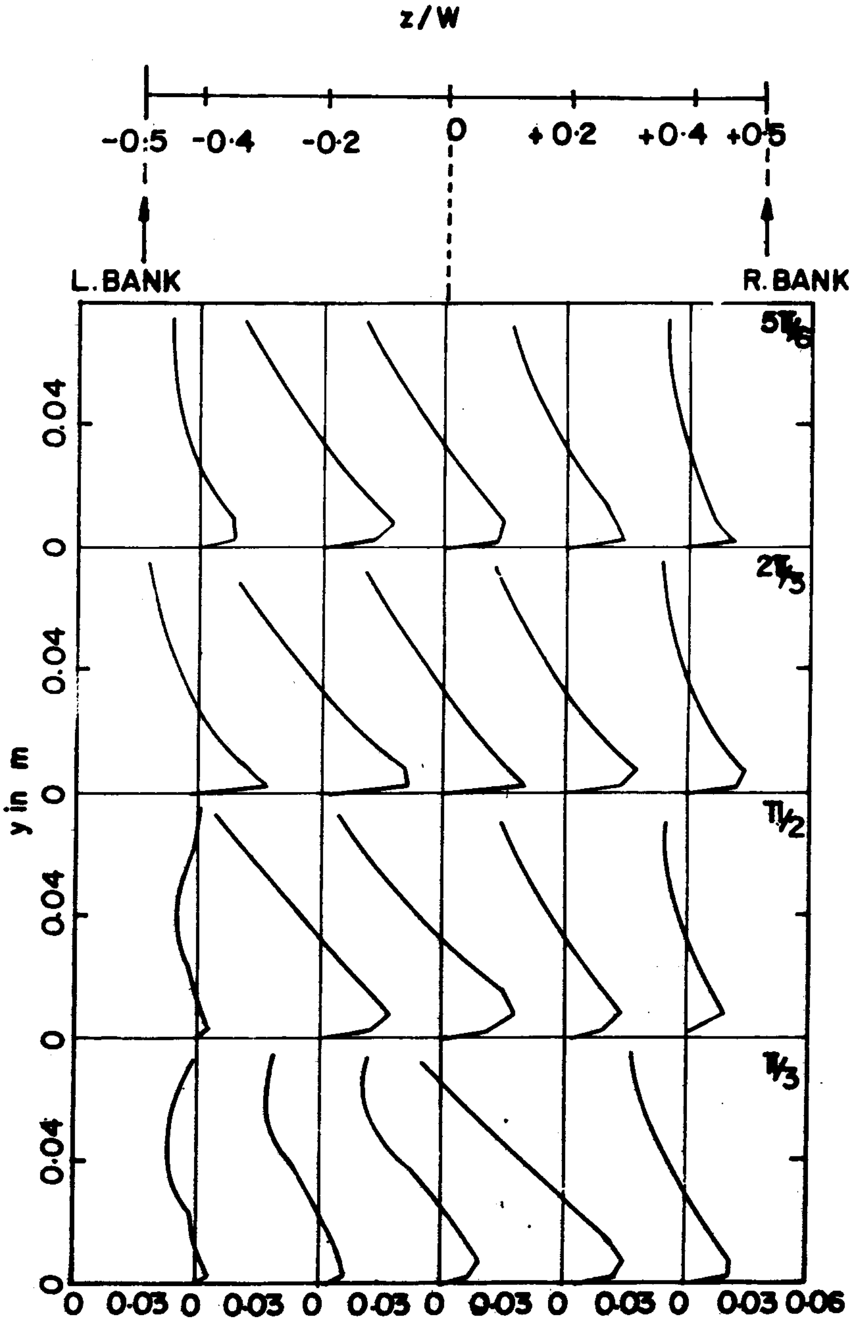

FIGURE 3c VERTICAL DISTRIBUTION OF TRANSVERSE VELOCITY RUNS 301/2/3 SECTIONS $\theta_2 = \pi/2$ TO $5\pi/6$

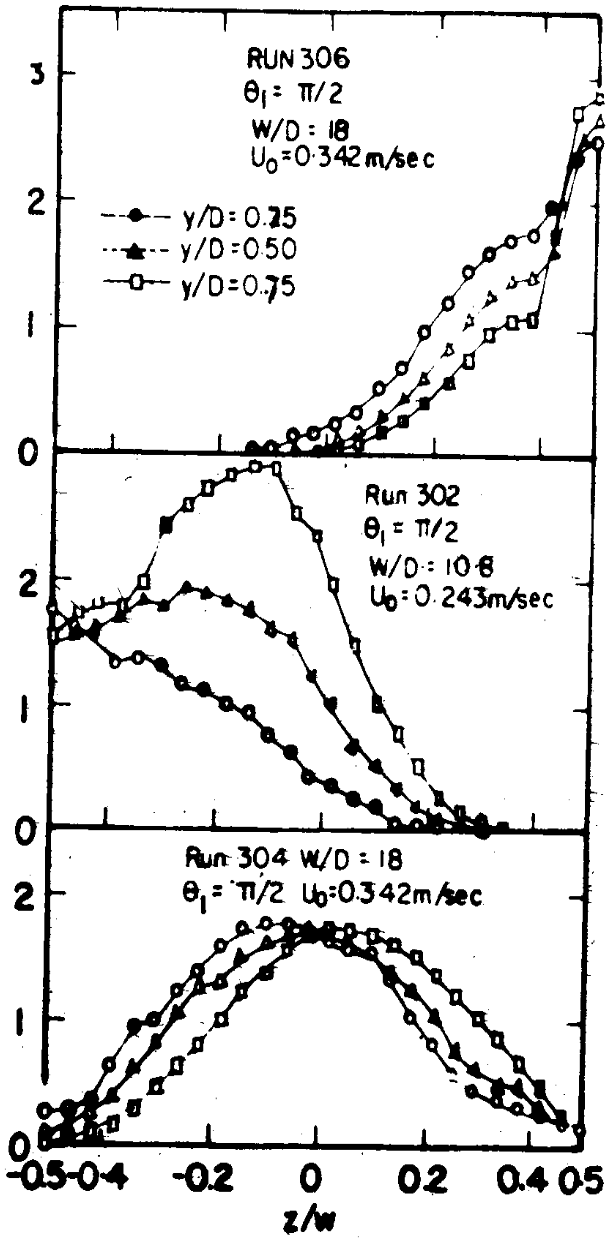

FIGURE 4. TRANSVERCE CONCENTRATION PROFILE AT VARIOUS FLOW LEVELS AND AT SECTION $\theta_1 = \pi/2$

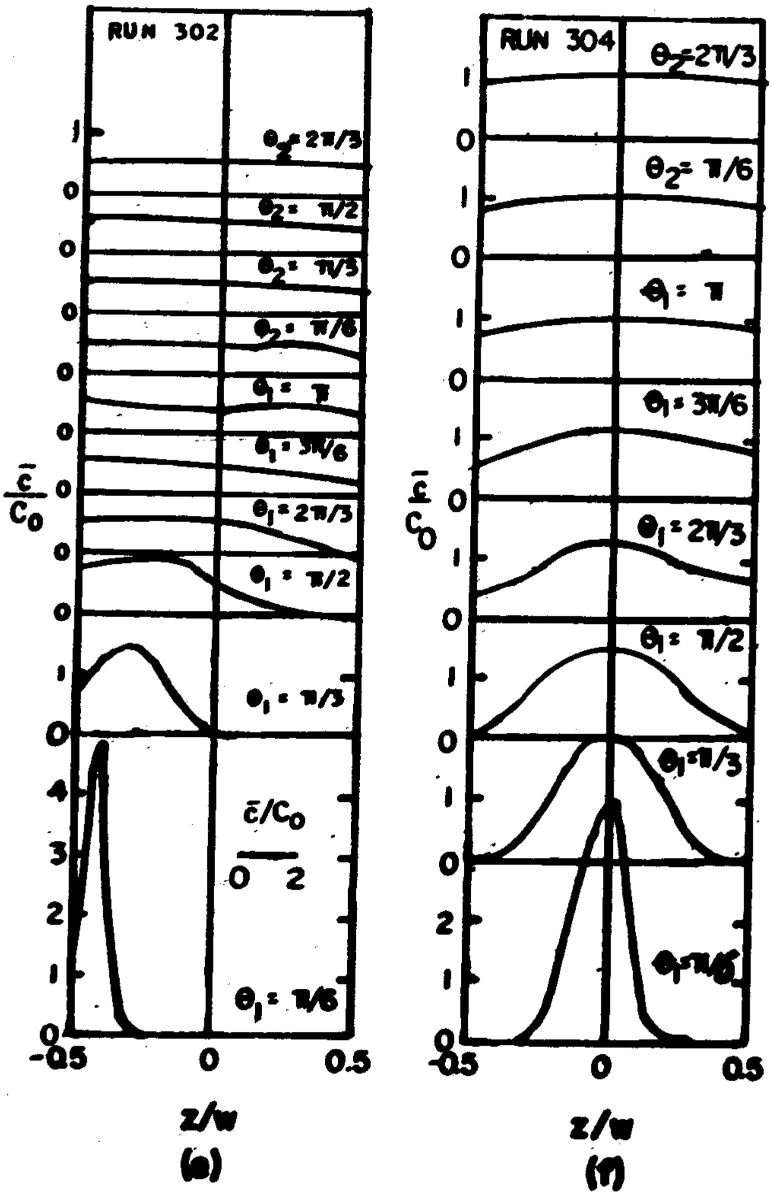

FIGURE 5 TRANSVERSE DISTRIBUTIONS OF DEPTH-AVERAGED TRACER CONCENTRATION AT VARIOUS SECTIONS DOWNSTREAM OF SOURCE.

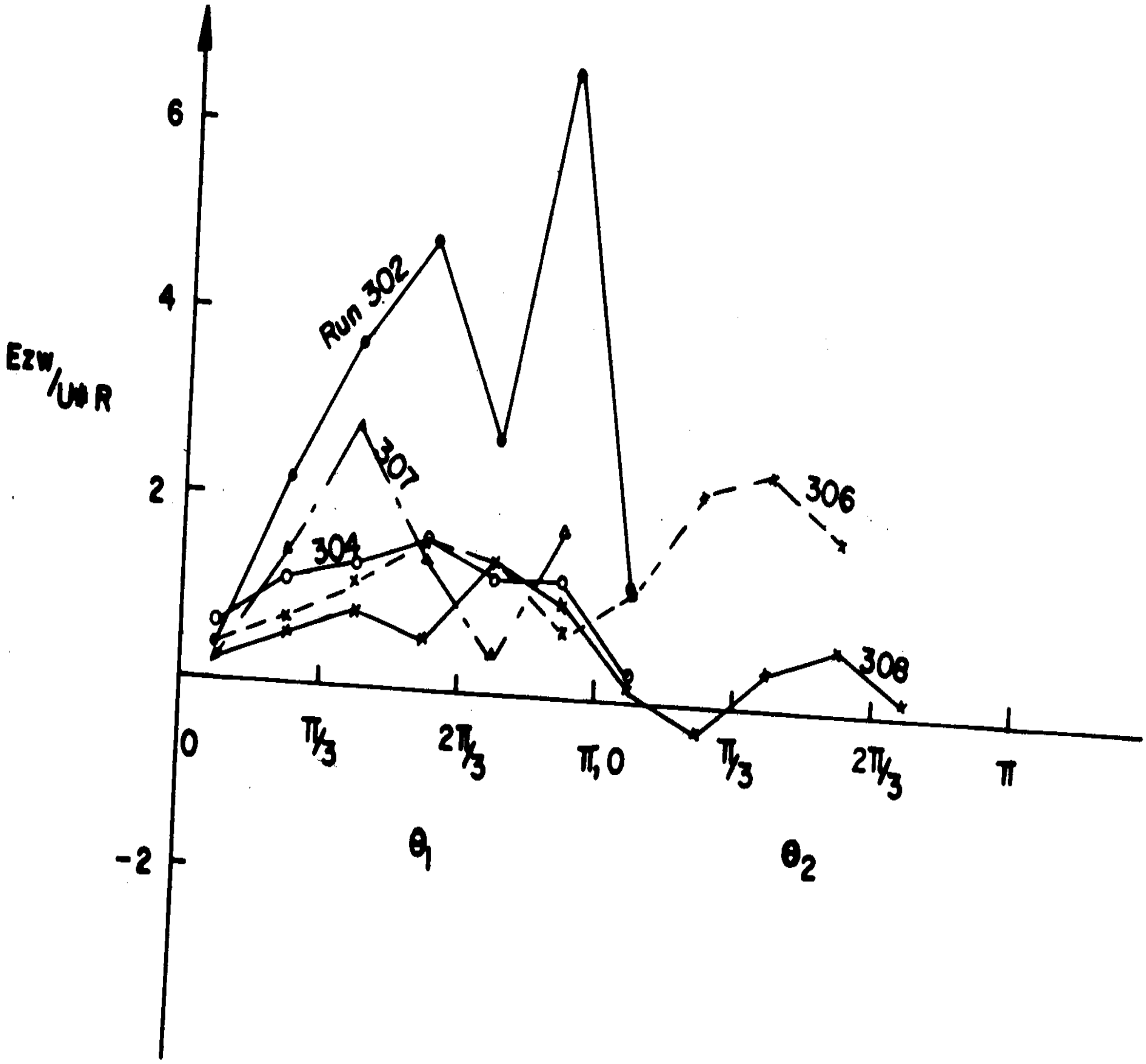

FIGURE 6. LONGITUDINAL VARIATION OF THE NORMALISED TRANSVERSE MIXING COEFFICIENT.

Although a negative E_{zw} /U· R violates the concept of gradient-type mixing, it is realised that the gradient-type mixing concept is an approximation and may not hold true for all flow situations. It is possible to obtain negative E_{zw} /U· R values in regions of bend curvature reversal as this is initially accompanied by a net lateral discharge towards the outer bank. This net flow can inhibit spreading of tracer released from the outer bank and produce a negative E_{zw} /U· R value.

The longitudinal variation of E_{zw} /U· R was found to be strongly related to the growth-decay-reversal cycle of the spiral motion. With only two bends, it was not possible to relate the probable cyclic variation of E_{zw} /U*R to that of the spiral motion strength.

The average E_{zw} /U*R values obtained for the various Runs ranged from about 0.5 to 3.0 with an overall average of about 1.50 compared to the normally observed values of about 0.15 - 0.25 in straight open channels. Hence meandering channel flows have a higher mixing potential than straight open channel flows.

TABLE 3: LONGITUDINAL VARIATION OF E_{ZW}/U.R

Test Run / Source Location	E_{ZW}/U.R				
	302 Left Bank (LB)	304 Center-line (CL)	306 Right Bank(RB)	307 Center-line(CL)	308 Left Bank(LB)
θ_1 0	0.38	0.59	0.41	0.24	0.21
/6	2.20	1.17	0.70	1.43	0.55
/3	3.69	1.35	1.16	2.80	0.81
/2	4.87	1.63	1.65	1.44	0.52
/3	2.76	1.29	1.46	0.45	1.41
5 /6					
	6.76	1.30	0.68	1.84	1.03
θ_2 0	1.27	0.29	1.20	-	0.17
/6	-	-	2.29	-	-0.26
/3	-	-	2.53	-	0.42
/2	-	-	1.86	-	0.68
2 /3	-	-	3.11	-	0.17
5 /6					

Transverse Spreading and Mixing of Tracer

The root mean square concentration deviation, C defined as

$$C_v = \frac{1}{C_o}\frac{1}{N}\sum_{I=1}^{N}(\bar{c}-c)^2 \;\; ^{\frac{1}{2}} \ldots\ldots\ldots(8)$$

is a good indicator of the degree of transverse mixing. Low values of C_v imply a high degree of mixing and vice versa. In Eq (8) C represents the tracer concentration at fully mixed conditions with no tracer losses; c_i is the depth-average concentration at vertical i.

According to the procedure of Ward (II) the distance, X ,in straight channels required to achieve a given percentage degree of mixing, defined as. (1 - c_v) x 100% is given by

$$\frac{X_m}{W^2} = \frac{T_1 U_o}{E_z}$$

where

T_1 is a variable which depends on source position and the specified degree of mixing.

W is the channel width

U_o is the average flow velocity

E_z is the transverse diffusion coefficient which can be approximated by 0.15 u.D

Run 304 for example, achieved approximately 95% mixing at $\theta_2/3$. (see Fig. 7).

According to Ward's method an equivalent straight channel with the same W, D, u. and U_o as in Run 304 would require about 8 times the distance from $\Theta = 0$ to $\theta_2 = /3$ to achieve the same degree of mixing. The bend induced spiral motion therefore enhances transverse mixing by rapidly spreading the tracer while turbulence evens out the vertical differences in tracer concentration. There is evidence from Figure 7 that the tracer undergoes different rates of mixing within the same bend for the same flow conditions depending on the source position. This is consistent with the observation that the strength of the spiral motion varies both laterally and longitudinally.

In the experiments the magnitude of $\bar{w}$ was observed to be of the same order as the error in the measurement of W hence $\overline{wc}$ could not be accurately determined. The major effect of the bends on the mixing was assumed to be caused by the dispersion term $\overline{w^{\iota}c^{\iota}}$. A general comparison of the relative importance of transverse velocities and transverse diffusion due to turbulence only was made. Using the data for Run 304 in which the average velocity was 0.342m/s, the average maximum relative transverse velocity between tracer particles located near the water surface and those near the channel bed was about 7.9cm/s. The transverse displacement rate of tracer due to this relative transverse velocity would be 0.079/0.342 = 0.23m/m of channel length. An estimate of the diffusion rate can be obtained by assuming that the concentration distribution due to turblent diffusion is Gaussian and that E_z = 0.15u.R. If it is noted that in Run 304, U_o/u = 12.14, the above assumptions can be used to show that the standard deviation of the tracer (which is a measure of rate of transverse spreading by diffusion),

$$z = \frac{2\,E_z.X^{1/2}}{U_o}$$

would grow at a rate of about 0.058m/m of channel length. The displacement rate of tracer due to transverse velocities is, therefore, much greater, about 4 times for Run 304, than that caused by turblent diffusion.

CONCLUSIONS

The normlalised transverse mixing coefficient, E_{zw}/u.R, varied longitudinally regardless of tracer source location. Also an apparent dependence of E_{zw}/u.R on source position was observed. These effects seem to be related to the growth decay and reversal of the spiral action. E_{zw}/u.R is approximately an order of magnitude greater than the normalised diffusion coefficient. The longitudinal distance required to achieve a give degree of mixing in a meandering channel flow is considerably shorter than the corresponding straight channel flow, showing that the mixing potential of meandering channel flows is much greater than that of straight channel flows.

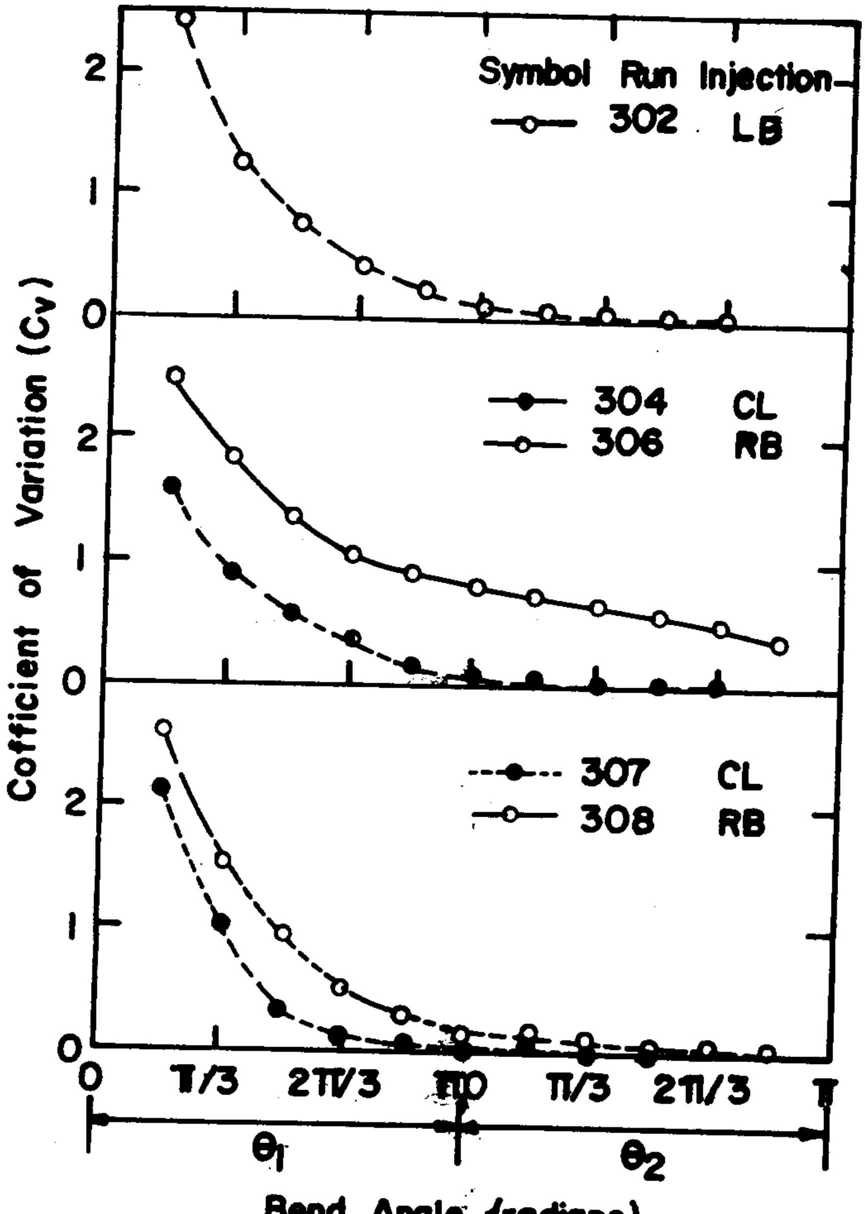

FIGURE 7 COEFFICIENT OF VARIATION CURVES FOR VARIOUS TEST RUNS.

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NOTATION

List of Selected Symbols Used

C - depth averaged concentration of tracer
C_O - tracer concentration at fully mixed conditions

C_V - root mean square concentration deviation

D - average flow depth
$\bar{E}_z$ - average transverse diffusion coefficient over a channel reach
E_{zw} - average transverse mixing coefficient over a channel reach
M_2 - second moment of tracer flux distribution
r_c - channel centre line radius
R - hydraulic radius
S - strength of spiral motion
$u.$ - shear velocity
U_O - mean flow velocity
W - width of channel
W_1, W_2 -transverse of Z-coordinate of channel sides
Z_O - lateral distance from channel axis to point of tracer release
E_x , E_y , E_z - turbulent diffusion coefficients in the x,y,z directions respectively
- lateral distance from point of tracer release
θ_1, θ_2 - radial angle from beginning of first and second bends, respectively.