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STUDY OF THE INCEPTION LENGTH OF FLOW OVER STEPPED SPILLWAY MODELS

S. Munta [1,*], J. A. Otun [2]
[1,2] DEPT. OF WATER RES. & ENVIRONMENTAL ENGINEERING, AHMADU BELLO UNIVERSITY, ZARIA. NIGERIA

Email addresses: [1] *muntjen@yahoo.com*, [2] *johnsonotun@yahoo.com*

ABSTRACT
This study is the modification of Bauer's development length in order to determine the excess in his length which has not been investigated by earlier researchers. The observations were performed in the non-aerated flow region of wooden stepped spillway models fabricated and installed at the Hydraulics Laboratory of the Department of Water Resources and Environmental Engineering, Ahmadu Bello University, Zaria-Nigeria. A total of 40 experiments were conducted over three different chute geometry to determine Bauer's development length which were plotted on a drawing paper to a scale of 1:1 to obtained the development length in this study. The results showed that the inception (development) length increases as the unit discharge increases and it decreases with an increase in both stepped roughness height and chute angle. The ratio of the development length, in this study, to that of Bauer's was found to be 4:5. Finally, SMM-5 produced the least velocity of flow and the highest depth of inception; indicating that it could be employed optimally, considering this study, in curbing or preventing cavitation as a flood controls structure.

Key words: *Stepped spillway models, skimming flow, development length, practical method, chute angle*

1. INTRODUCTION
A spillway is a hydraulic structure that regulates and releases excess flow from a dam. Stability of a dam depends on the competency of its spillway. Faulty design of such structure could undermine the dam, which may cause loss of lives and property downstream. The excess flows over the chute of the spillway move with high velocity and may cause cavitation which may eventually affects thestructure. [1] stated that the non-aerated region is the region that is prone to this cavitation.
The report in [2] has classified the flow over the stepped spillway into three: nappe associated with low discharges, transition which is between the two regimes and the skimming flow associated with large discharges. Paper [3] has considered skimming flow as a pseudo bottom defined by a straight line connecting the edges of each step. To attain skimming flow (eq. 1) and aeration (eq. 2), [4] proposed the equations for a range of slopes from ($25^\circ < \theta < 55^\circ$) as:

$$\frac{d_c}{S_h} \geq 0.91 - 0.14 \tan\theta \quad (1)$$

$$\frac{H_{dam}}{d_c} \geq 15 - 20 \quad (2)$$

Where H_{dam}=height of stepped spillway model, d_c=critical depth and S_h=step height
Skimming flow is usually found for typical design discharges of stepped spillways [5]; hence it has captured the attention in this study. On a stepped spillway with skimming flow, [6] clearly described the flow conditions down a stepped chute as the boundary layer grows until free surface aeration begins. The boundary layer grows from the stepped cascade floor in the non-aerated flow region close to the cascade crest.[7, 8] have reported that there are two distinct regions of skimming flow over the length of the spillway chute viz: non-aerated and aerated regions which are separated by a critical point or point of inception. They further stressed that clear water was observed in the non-aerated region where the water enters the chute and white water in the aerated region which begins when the turbulent boundary layer from the floor intersects the water surface view figure 1.

*** Corresponding author, Tel: +2348150770847**

The location of the onset of air entrainment (surface aeration) is significant for the designer of a stepped chute; it delimits the reach of the spillway which can be prone to cavitation damage. Also, [3] has stated that the location of the inception point on stepped spillway was significantly closer to the spillway crest than on smooth chutes because the growth of the boundary layer depends on the bottom roughness. Stepped spillway was used in this study because of its contribution to the enhancement of the boundary layer development. [9] have stated that the step geometry could be horizontal, inclined or pooled; the different chute configuration have the tendency to enhance the development of the boundary layer and are of remarkable interest in this study.

Many equations have been proposed to determine the development (inception) length of flow over the stepped chute. Typical examples are those by

[10], who gave $L_c = 14.7q^{0.53}$ (3)

[11], who gave $\frac{d_e}{X} = \frac{0.024}{(X/K_s)^{0.13}}$ (4)

[12], who gave $\frac{d_e}{L_i} = 0.01$ (5)

[l3] who recommended values of $\frac{d_e}{L_i}$ between 0.016 and 0.01 (6)

Where L_i L_e and L_c are distance from start of boundary growth, X = [11] development length, d_e =boundary layer thickness (inception depth), q =unit discharge and K_s =step roughness height

Bauer's investigation [11] on the length of inception of flow over spillway is the only revealed practical method of estimating the inception length. [14] acknowledged that [11] computation for the length of inception was an approximate one; even then the length of inception defined by [12,15,16,17,18,19] amongst others, as the distance measured from the chute entrance to the onset of aeration (fig.1) made the description of [11] length to be longer. In [11] length may not give engineers designing stepped spillway a precise information and there has not been a study relating Bauer's length with the one described by the earlier researchers. This research focuses on determining both [11] length and the modified length of flow over plain, end-sill and inclined stepped chutes, particularly, the effect of flow rate, stepped roughness height and the chute angle for the purpose of optimization and also correlates the Bauer's length with that of the present study.

2. MATERIALS AND METHOD

2.1 Materials

The following are the materials used during the experiments: Flume and its appurtenances, supply line, pump, workable basin, stop watch, pointer gauge.

2.2 Physical Models

Ten physical models of stepped-channel chute were built and fixed at the Hydraulics Laboratory of the Department of Water Resources and Environmental Engineering, Ahmadu Bello University, Zaria. The physical models were fabricated of wooden materials. The crest of the models was broad-crested, [20]. The physical models dimension was selected based on the condition presented by [4] for skimming flow (eq.1) and aeration (eq. 2)

The stepped spillway model (SSM) was fabricated from wood and painted to reduce its roughness coefficient.

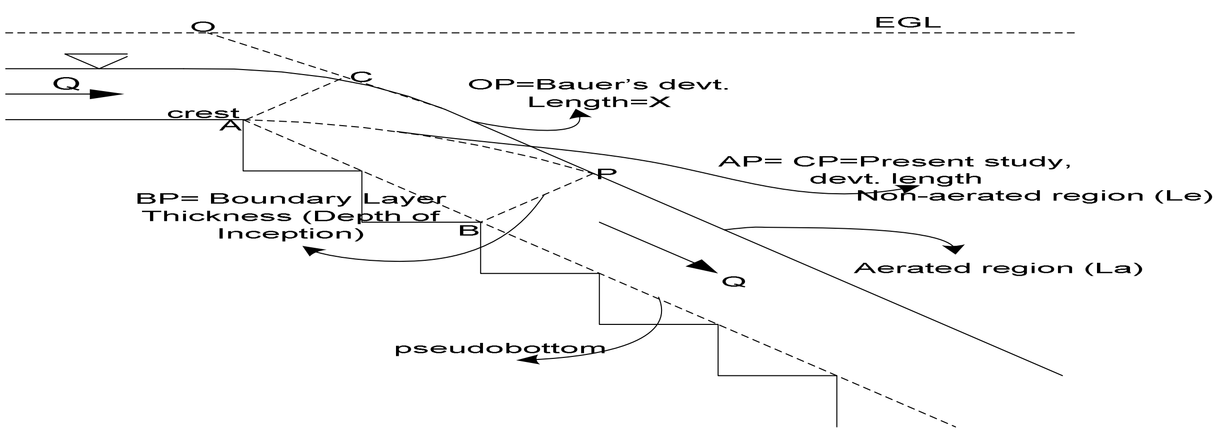

Figure 1: Air entrainment Flow Regimes, after [7, 8]

The SSM used for this experiment were designated as SSM-1 to SSM-10. SSM-1 and SSM-6 were plain steps, SSM-2, 3, 7 and 8 had end sill configuration, and SSM-4, 5, 9 and 10 were made up of inclined steps. SSM-1, 2, 3, 4 & 5 had chute angle of 38.7° and 26 numbers of step height respectively; while SSM-6, 7, 8, 9 & 10 had chute angle of 45° and also, 26 numbers of step height. Wooden pieces (26 pieces set) measuring 1cm in thickness by 2cm in height and 15cm in width were placed at the edge of plainsteps to convert them to the end-sill (SSM-7). Also, wooden pieces (26 pieces set) measuring 1cm by 4cm by 15cm were placed at the edge of plain steps to convert them to the end-sill (SSM-8). Also, wooden pieces (26pieces set) measuring 1.25 cm by 2cm by 15 cm were placed at the edge of plain steps to convert them to another end-sill (SSM-2). In the same vein, wooden pieces (26 pieces set) measuring 1.25 cm by 4 cm by 15cm were placed at the edge of plain steps to convert them to the end-sill (SSM-3). Similarly, triangular wooden pieces (26 pieces set) of 4cm base, 2cm in height and 15 cm in width were added at the bottom of plain steps in order to produce inclined steps (SSM-9) with 26.6^o angle of inclination. Also, triangular wooden pieces (26 pieces set)of 4cm by 4cm by 15cm were added at the bottom of plain steps to produce inclined steps (SSM-10) with 45° angle of inclination. Likewise, triangular wooden pieces (26 pieces set) of 5cm by 2cm by 15cm were added at the bottom of plain steps to produce inclined steps (SSM-4) with 21.8^o angle of inclination. Finally, triangular wooden pieces (26 pieces set) of 5cm by 4 cm by 15cm were added at the bottom of plain steps to produce inclined steps (SSM-5) with 38.7° angle of inclination. Refer to Fig.2 and table 1 for details.

2.3 Method

2.3.1 Experimental Arrangement and Procedures.

The experiments were conducted in the Hydraulics Laboratory of the Department of Water Resources and Environmental Engineering of Ahmadu Bello University Zaria-Nigeria. A schematic representation of the experimental set-up is shown in Figures 3 and 4, which consisted of a workable basin of capacity 433 litres placed inside one of the laboratory channels which was used to supply water for the models; three pumps of sizes 3hp (175L/s), 1.5hp (3L/s) and 1hp (1L/s) were used to supply energy for transfer of the water from the basin through 2 inches, 11/4 inches and 1inch pipes, in that order, into a stilling storage

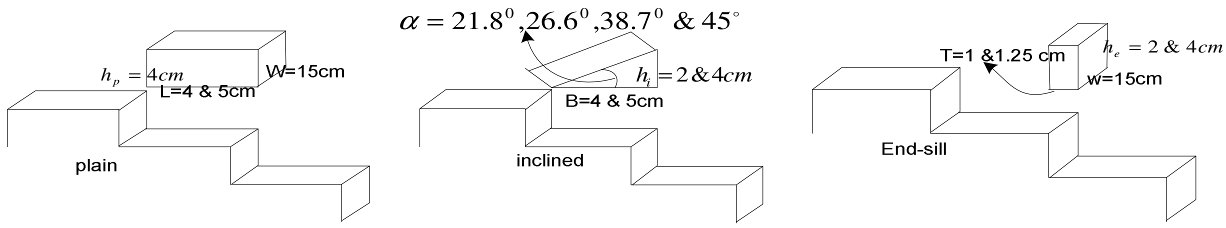

Figure 2: Details of Stepped Spillway Models

Table 1: Geometry of the Developed Stepped Spillway Model

Model Type	Model Height(cm)	Number Of steps	Step height, S_h(cm)	Steproughness Height, K_s (cm)	Chute Length (L) (cm)	Chute Angle θ (deg)
SSM-1	104	26	4	3.12	166.3	38.7
SSM-2	104	26	6	4.10	166.3	38.7
SSM-3	104	26	8	5.47	166.3	38.7
SSM-4	104	26	6	4.68	166.3	38.7
SSM-5	104	26	8	6.24	166.3	38.7
SSM-6	104	26	4	2.83	147	45
SSM-7	104	26	6	3.60	147	45
SSM-8	104	26	8	4.80	147	45
SSM-9	104	26	6	4.24	147	45
SSM-10	104	26	8	5.66	147	45

$K_s = S_h \cos\theta$, where, S_h = Step height given as: for plain chute, $S_h = h_p$ (plain step height), end-sill chute, $S_h = h_p + h_e$ (end-sill height) and inclined chute, $S_h = h_p + h_i$ (inclined height); $L = 104/\sin\theta$; because of the thickness of the end-sill, the angle used to calculate their K_s was determined as: for SSM-2 & 3, 46.9^0 and SSM-7&8, 53.1^0 Taken after [21, 22, 23].

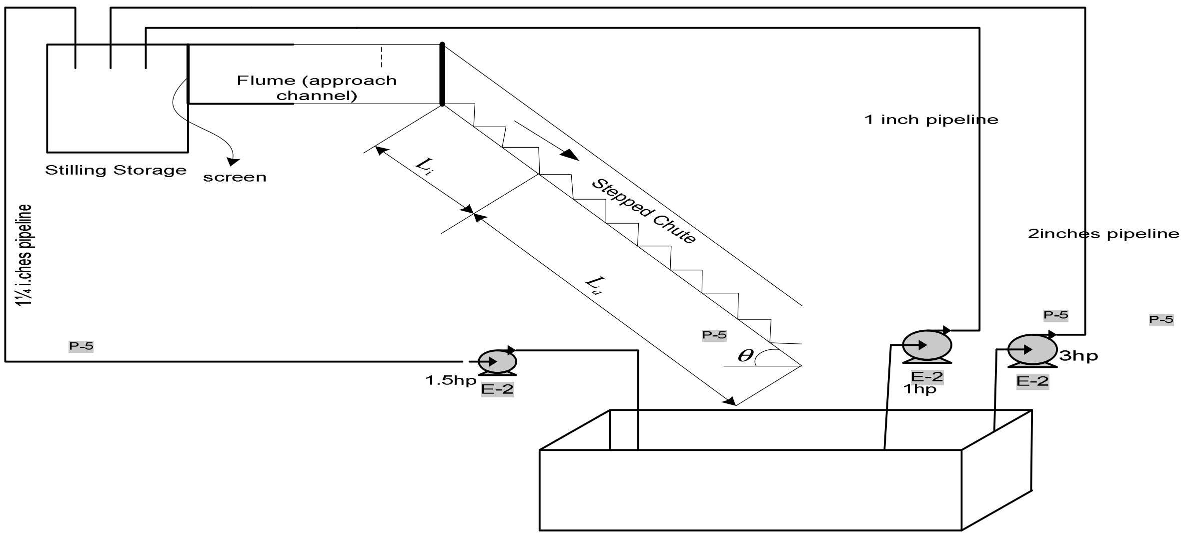

Figure 3: Experimental arrangement for Stepped Spillway Model

Figure 4: Set-up and Experimental performance of the Development length

To reduce turbulence of the flow, a honeycomb was installed vertically at the outlet of the stilling storage. Then an approach channel of size 1.5 by 0.3 by 0.50m received the water with its bed 1.04m above the laboratory floor. A prismatic rectangular stepped chute channel, 0.15m wide and 0.9m deep, was fixed to the head of the flume. This is the channel for the experiments. One of the wall sides, where measurements were taken, was made of transparent Perspex material to visualised flow regime.

2.3.2 Discharge Measurement

A rectangular weir was incorporated to the outlet of the approach channel which was used to measure the flow rate in this experiment. To calibrate the flow, head of water above the crest of the weir for every discharge was measured by a point gauge and was collected inside a container of a known volume after a particular time period. The procedures were repeated for several different values of discharges. The discharge over a rectangular weir is generally given as

$$Q = \frac{2}{3} L \sqrt{2g} C_d H^{3/2} \quad (7)$$

2.3.3 Experimental procedures for the Location of the onset of Aeration Concept

To determine Bauer's development length, an arbitrary length, X in cm was assigned and continued to vary, for every flow rate, until the boundary layer thickness (d_e) is equivalent to the computed water surface at the point of inception (fig. 5); this was done by using a Microsoft Office Excel 2007. When [11] is applied the relationship on a concrete overflow spillway as given in equation (4). The method of computation is shown on Table 2.

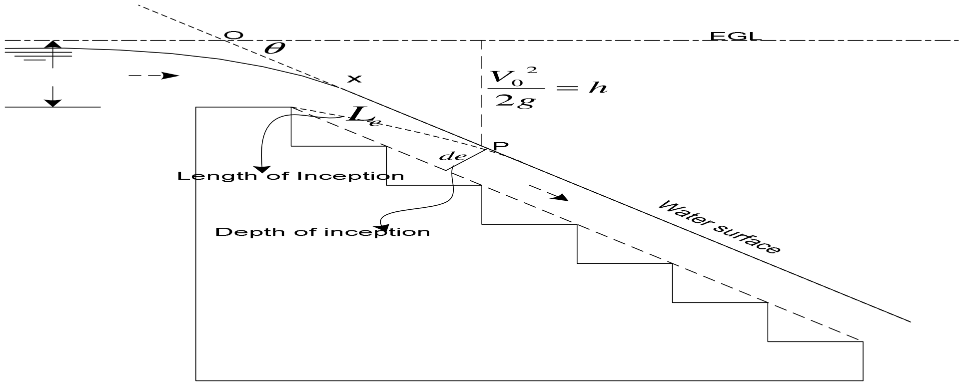

Figure 5: Growth of Boundary Layer over the surface of stepped spillway

Table 2: Method of computation of the development length

x (1)	x/k_s (2)	d_e/x (3)	d_e (4)	$v_0^2/2g$ (5)	v_0 (6)	q/v_0 (7)	y(8)

Col.1-arbitrary assigned length of x in cm, measured from O, Col.2-values of X/K_s, Col.3-values of d_e/X computed from eq (4), Col.4-values of d_e(cm), Col.5-velocity head $v_0^2/2g$(cm), which is equal to $x\sin\theta$, Col.6-velocity in cm/s corresponding to the velocity head in column 5, Col.7-potential thickness of flow in cm, equal to a given discharge divided by the velocity v_0, Col.8-actual thickness of flow in cm equal to potential thickness plus displacement thickness, which is assumed to be 10% of the boundary layer thickness (depth of inception).

New Concept: The [11] lengths and their respective boundary layer thickness were plotted on a drawing paper to a scale of 1:1 to determine the present study development lengths (fig. 5).

2.3.4 Development of relevant Mathematical Relationships

Eq.(7) is of the form:

$$Q = \phi H^j \quad (8)$$

where, Q is the actual discharge L/s; H is the measured head cm; Where, ϕ and j are determined experimentally. Comparing the expressions in eq. (7) & (8) gives Coefficient of discharge as,

$$C_d = \frac{3\phi}{2L\sqrt{2g}} \quad (9)$$

[11] inception length was related to the length determined in this study and is given as

$$L_e = r_T X \quad (10)$$

Where, r_T = the ratio of L_e to X

3. RESULTS AND DISCUSSION

3.1 The Flow Model

Seven experiments were conducted to determine the flow rates. Refer to Table 3 for the discharge and head data. These data were analyzed using a Microsoft Excel 2003, as illustrated in figure 6, to obtain the values of $\phi = 0.33$ and $j = 1.5$; the head-discharge relationship in eq. (8) and $C_d = 0.75$ in eq. (9). The theoretical discharge equation over a rectangular weir has been given in eq. (7) and the results in figure 6 gives the exponent of H to be 1.497. Comparing it with the standard exponent of H (1.5), gives a percentage error of

$$\%error = \left|\frac{Theretical - Actual}{Theoretical}\right| \times 100 \quad (11)$$

$$= \frac{1.500 - 1.497}{1.500} \times 100 = 0.2$$

$R^2 = 0.999$=indicator of the reliability of the Excel 2003 model, as given in Figure.6. Eq. (8), the developed flow model, is adequate to be employed in running this experiment.

Table 3: Measured Head and Flow Rate

S/N (-)	1	2	3	4	5	6	7
H (cm)	2.40	3.50	4.30	5.10	5.80	6.30	7.20
Q (L/s)	1.28	2.16	2.94	3.91	4.73	5.34	6.51

3.2 Length of Inception

The results of the development length in this study and the [11] development length were determined and can be viewed on table 4 for the ten models under consideration.

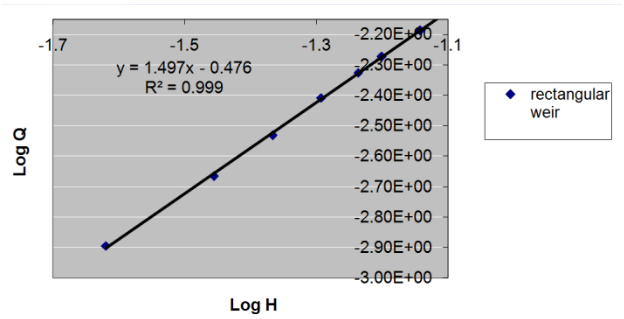

Figure 6: Head-Discharge Relationship

The results revealed that for each model the length of inception increases with an increase in unit discharge. This is because the flow depth increases with increasing discharge and the boundary layer reaches the free surface further downstream. For a given discharge, inception length decreases with an increase in stepped roughness height. This agrees with the statement of [24] that the section of inception length was closer to the crest for larger step heights. For every unit discharge and the same stepped height, inception length of flow decreases on models with higher chute angle. This was in line with the statement of [25] that inception length decreases with an increase in chute angle. For models of the different configurations, lower inception length of flow was observed among the inclined types than that of end sill of the same stepped height. This is attributed to the higher stepped roughness height of inclined than the end-sill. The results also showed that the ratio of depth to length of inception in this study is 0.02. The ratio of depth to length of inception of [12] eq.5 and [13] (eq.6) are respectively, 0.01 and 0.016. There is a better depth of inception in this study, which shows a reduced free stream velocity. Hence for a given flow, the stepped chutes in this study could curtail the potential occurrence of cavitation than the chutes provided by [12, 13]. This agreed with the statement of [26] that cavitation could be prevented or minimized by reducing the velocity of flow to increase the depth of flow. Finally, the ratio of the length of inception, in this study, to that of [11] is about 0.8.

Table 4: Summary of Inception (development) length of Flow

		SSM-1					SSM-2		
q(L/s.m)	26.1	31.5	35.6	43.4	q(L/s.m)	26.1	31.5	35.6	43.4
V_0(cm/s)	279	301	313	336	V_0(cm/s)	276	296	309	332
d_e(cm)	1.03	1.16	1.26	1.43	d_e(cm)	1.05	1.18	1.28	1.45
X(cm)	63.8	73.1	79.7	92.3	X(cm)	62.0	71.2	77.9	90.0
L_e(cm)	51.1	58.5	63.8	73.8	L_e(cm)	49.6	57.0	62.3	72.0
		SSM-3					SSM-4		
q(L/s.m)	26.1	31.5	35.6	43.4	q(L/s.m)	26.1	31.5	35.6	43.4
V_0(cm/s)	272	292	305	328	V_0(cm/s)	275	294	307	330
d_e(cm)	1.06	1.20	1.30	1.47	d_e(cm)	1.06	1.19	1.29	1.46
X(cm)	60.3	69.3	76.0	87.6	X(cm)	61.4	70.3	77.1	88.9
L_e(cm)	48.2	55.4	60.8	70.1	L_e(cm)	49.2	56.2	61.7	71.1
		SSM-5					SSM-6		
q(L/s.m)	26.1	31.5	35.6	43.4	q(L/s.m)	26.1	31.5	35.6	43.4
V_0(cm/s)	270	290	303	326	V_0(cm/s)	292	314	328	353
d_e(cm)	1.07	1.21	1.31	1.48	d_e(cm)	0.99	1.11	1.20	1.36
X(cm)	59.5	68.6	75.2	86.6	X(cm)	61.1	70.5	77.1	89.0
L_e(cm)	47.6	54.9	60.1	69.3	L_e(cm)	49.3	56.4	61.7	71.2
		SSM-7					SSM-8		
q(L/s.m)	26.1	31.5	35.6	43.4	q(L/s.m)	26.1	31.5	35.6	43.4
V_0(cm/s)	289	309	323	348	V_0(cm/s)	285	306	319	343
d_e(cm)	1.00	1.13	1.22	1.39	d_e(cm)	1.02	1.15	1.24	1.40
X(cm)	60.1	68.8	75.2	87.4	X(cm)	58.6	67.3	73.4	84.9
L_e(cm)	48.1	55.0	60.2	69.9	L_e(cm)	46.9	53.8	58.7	67.9
		SSM-9					SSM-10		
q(L/s.m)	26.1	31.5	35.6	43.4	q(L/s.m)	26.1	31.5	35.6	43.4
V_0(cm/s)	286	307	321	345	V_0(cm/s)	283	303	317	341
d_e(cm)	1.01	1.14	1.23	1.40	d_e(cm)	1.03	1.16	1.25	1.42
X(cm)	59.0	67.9	74.1	86.0	X(cm)	57.8	66.3	72.3	83.7
L_e(cm)	47.2	54.3	59.3	68.8	L_e(cm)	46.2	53.0	57.8	67.0

4. CONCLUSION

Laboratory experiments were carried out on SSM with different step shapes (plain, end sill and inclined) to determine the excess in [11] inception length which has not been investigated by previous researchers and to study the influence of step roughness height, chute angle and unit discharge on the inception length of flow over the stepped spillways under consideration. Based on these findings, the following conclusions can be drawn:

1. The ratio of the length of inception in this study to [11] length was found to be 4:5
2. The inception length increases with an increase in unit discharge but decreases with increase in both the stepped roughness height and the chute angle.
3. For a given unit discharge, reduce inception length was noticed among the inclined chute geometry than the end-sill and plain chute. This is attributed to the higher value of stepped roughness height over the inclined.
4. SMM-5 produced the least velocity of flow and the highest depth of inception; indicating that it could be employed optimally in curbing or preventing cavitation as a flood control structure.

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