ON THE RESONANT FREQUENCIES OF THE OJA

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ABSTRACT
A method for calculating the unblown resonant frequencies of an 'Oja' (a traditional Nigerian musical instrument) is developed. Support for the theory is provided by data derived from experimentally measured spectra of typical oja tones. It is also shown that for resonant frequencies below about 2000Hz, the differences between the theoretical and experimental data are in qualitative agreement with the findings of earlier investigators with regard to the effect of a mouth-cavity resonance on the resonant frequencies of a wood-wind instrument.

LIST OF MAIN SYMBOLS

a	-	radius of a tone pipe
a_m	-	radius of main pipe
B_j	-	junction susceptance due to tone pipes for any fingering configuration
B_m	-	subsceptance at mouth of main pipe due to mouth inertance
C	-	velocity of sound in air
F	-	frequency
K	-	wave number
I	-	length of a tone pipe
I_m	-	length of main pipe
Y_{in}	-	input admittance at mouth of main pipe
Y_m	-	admittance at mouth of main pipe due to radiation and inertance
Y_s	-	source admittance at mouth of main pipe due to lips and buccal cavity of flautist
λ	-	wavelength
ρ	-	density of air
Z_{jc}	-	impedance of a closed tone pipe referred to the plane of the junction
Z_{jo}	-	impedance of an open tone pipe referred to the plane of the junction.

1. INTRODUCTION

1.1 General

Nigeria has a rich assortment of indigenous musical instruments. Many of these are still used today especially on cultural occasions and on such cultural revivalist radio and television programmes as *Music of our Land* and *Nigeria Dances.*

Pioneer studies[1 2], of Nigerian traditional musical instruments, have been conducted essentially from the viewpoint of ethno-musicology. Nothing seems to have been done to describe and explain the performance of these instruments using the methods of musical acoustics. As a result, the instruments have, by and large, remained in their crude pristine form. Without a doubt, a scientific approach will be invaluable in any attempt to develop and improve the traditional instruments beyond their received state.

The work reported in this paper is a preliminary theoretical and experimental investigation of the 'oja', a woodwind instrument of the flute class. The primary objective was to perform theoretical calculations of the resonant frequencies of an 'oja' and obtain experimental data based on the spectral analysis of the waveforms of typical 'oja' tones to verify the theory.
Echezona[1] describes the 'oja' and a related instrument called the 'ogbo'. He however restricts the term 'oja to a bamboo instrument found in the Nsukka area of Igboland. We use

the term more broadly to correspond with the 'ogbo' in Echezona's terminology.

1.2 Description and method of note production

As shown in fig. 1, the oja consists of four cylindrical air pipes of varying lengths and diameters in a cruciform arrangement.

There are three tone holes: one at the bottom of the instrument terminating the lower tone pipe, and the other two about the middle of the body on diametrically opposite sides. The side holes terminate the side tone-pipes. The tone pipes terminated by the tone holes are narrow in diameter (of the order of a few mm) while the top air column (the main column) is of a much wider diameter compared with the tone pipes. The main pipe ends in a mouth piece or embouchure which is slightly flared and cut to form a double vee-notch. The material is of soft wood such as ‘egu’, ‘udara’ which does not crack in the harmattan[1]

The instrument is played while being held vertically with both hands, the flautist's lips pursed and pressed tightly against one of the vee-notches. The thumb and the first finger of the right hand cover the two side tone holes while the second finger of the left hand covers the lower tone hole. A note is sounded by the flautist blowing a stream of air across the embouchure and fingering the tone holes as desired. The lips, the embouchure, the tone pipes and the main pipe form a tightly coupled resonant system which radiates sound waves into the surrounding air though the second vee-notch. As with all woodwind, the player's skill in choosing the correct blowing pressure as well as the shape of the mouth and the lip pressure against the embouchure exercises considerable effect on the quality of the note produced.

2. THEORETICAL A ALYSIS

2.1. Condition for resonance

Acoustically, the oja consists of four air pipes in a cruciform arrangement (fig. 1). The four pipes meet in a transition region which we shall term the junction. We shall neglect the finite axial extent of the transition region. Richardson[3] has discussed the determination of resonant frequencies of compound air pipes using impedance concepts drawn from the theory of the radio frequency transmission lines. We adopt the method in the paper.

We assume that the system resonances of interest, i.e., those that couple substantial acoustic power to the surrounding air, are the resonances of the main pipe. The function of the tone pipes is to provide reactive loading at the junction which serves to 'tune' the resonating main pipe. The tone pipes are therefore 'stub- tuners' of fixed length and fixed position, each of which may be open or closed at the tone hole end. The system is assumed to be symmetrical about the central axis.

The resultant transmission line model is shown in fig. 2, where B_j (w) represents the total susceptive loading at the plane of the junction, U is the volume-velocity source function generated by the player, Y_m is the admittance at the mouthpiece due to radiation and the effect of mouthpiece/surrounding-air interface, and Y_s, is the source admittance Y_s, accounts for the effect of the player's lips and mouth cavity. We shall ignore Y_s, to obtain the so called unblown resonance of the system.

From fig. 2. using the well-known transmission- line admittance transformation equation, the input admittance Y_{in} at the mouthpiece is given by:

$$Y_{in} = j^{Y_0}\frac{B_j + Y_0 \tan K1_m}{Y_0 - B_j \tan k1_m} + Y_m \qquad (1)$$

For resonance, the imaginary part of Y_{in} must vanish. This gives the resonance condition:

$$B_j = -\frac{B_m/Y_0 + \tan K1_m}{1-(B_m/Y_0)\ \tan K1_m} \qquad (2)$$

Where B_m is the imaginary part of Y_m

2.2 Solution of the Resonance Equation

Expressions for B_j, and Y_m; occurring in equation (2) are derived in the appendix. The equation itself can only be solved numerically for any given set of instrument dimensions. The dimensions of the oja selected for the study are shown on fig. I. The resulting theoretical estimates of the unblown resonances for the six distinct fingering configurations are shown in column (2) of table I.

3. EXPERIMENTAL STUDY

The objective of the experimental study was to obtain a practical verification of the resonant frequencies calculated in the previous section. The method adopted was to obtain recordings of waveforms of individual notes produced by the oja when played by a skilled performer, and by carrying out a spectral analysis of the waveforms to deduce the corresponding resonant frequencies.

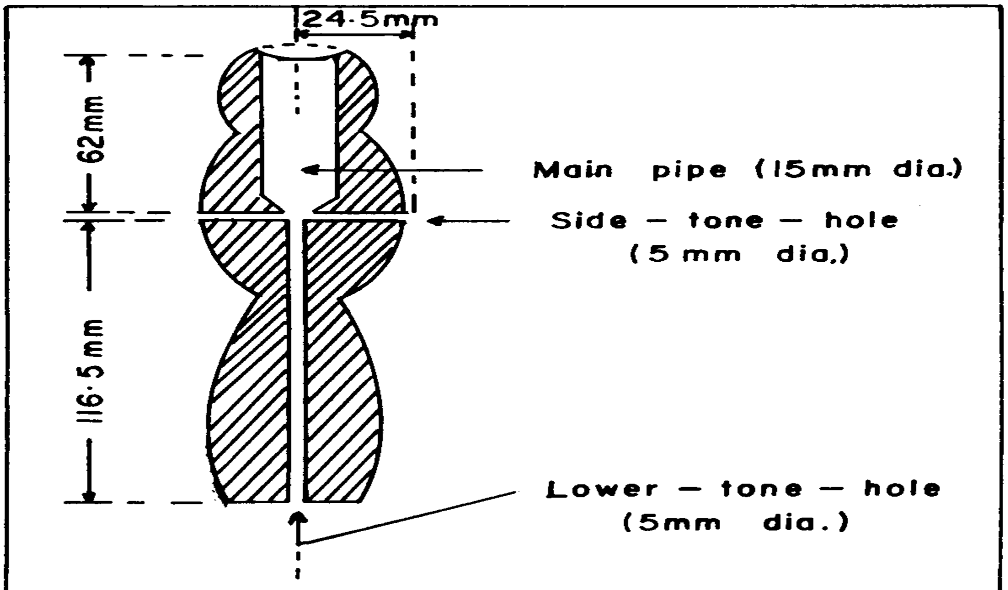

Fig. 1: Vertical section of an oja.

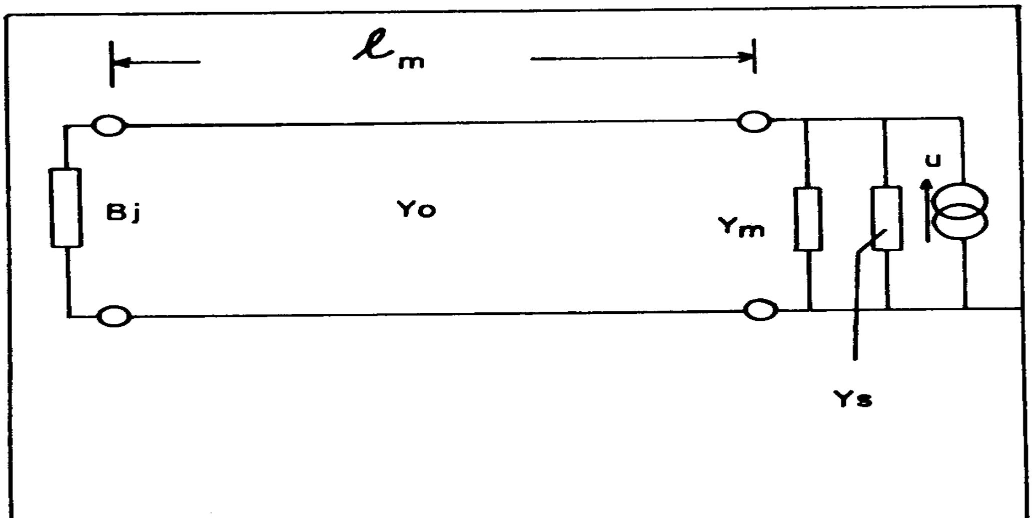

Fig. 2 : Transmission line model.

TABLE 1: RESONANT FREQUENCIES OF OJA

Fingering (1)	Theoretical frequency F_T (H_Z)	Experimental				Aural pitch Match	% Deviation $((F_T-F_X)/F_X)$ 100%
	(2)	Label (3)	db above noise (4)	Frequency, F_X, (H_Z) (5)	Ration to dominant frequency (6)	(7)	(8)
All tone holes closed.	687	a	9	748	0.64	E_5 (660 Hz)	-8.2
	1274	b	30	1175	1.00		-8.4
	2244	c	15	2344	1.99		-4.3
	3002	d	11	3548	3.02		-15.4
One side tone hole open. other tone holes closed.	720	a	10	780	0.62	F_5(740 Hz)	-7.7
	1412	b	35	1259	1.00		-12.2
	2200	c	16	2582	2.05		-14.8
	3068	d	15	3981	3.16		-22.9
	3565						
2 side tone holes open, lower tone hole	704					G_5 (786)	
	1524						
2 side tone holes closed, lower tone hole open.	1108	a	25	1023	1.00	C_5 (520 Hz)	-8.3
	1551	b	13	2118	2.07		-26.8
		c	11	6668	6.52		
One side tone hole closed, other tone holes open.	1232					D_5/A_5b/A_5	
	1607					620/830/880	
All tone holes open	- /1302	a	9	760	0.53	A_5 /880 Hz	
	1302/1662	b	25	1432	1.00		-9.1/16.1
	2881/3810	c	13	3162	2.21		

3.1 Signal Recording and Waveforms

To record and display the waveforms the instrument system shown in fig. 3 was assembled. The 1/2 inch Bruel and Kjaer (B & K) condenser-type microphone was supported horizontally on a level with the performer's mouth and at a distance of about 1 metre from him. The signal from the microphone, after amplification, was fed to a B & K type 7003 4- channel F. M. tape recorder. The tape recorder was equipped with a 2.5m long endless tape-loop cassette (B & K type UD 0035) and operated at a tape speed of 15 ins/see for which the cut off frequency was 10KHz. At the tape speed used, the loop allowed about $6^1/2$ seconds of recording time per channel.

The tape was allowed to run up to speed before the performer was signalled to start. Recording took place in a 'quiet' room except that there was a low frequency air-conditioner hum in the background. The performer sounded and sustained a note for about 5 seconds. One note was recorded on one channel of the tape.

The waveforms were captured on play-back by means of a Phillips PM 3234 storage oscilloscope with long persistence and one-shot facility. The waveforms displayed on the oscilloscope were photographed using an f2.0 Asahi Pentax Camera at a distance of about 38 cm and an exposure time of 1/2 second.

Samples of the observed waveforms are shown in fig. 4(a) - (h).

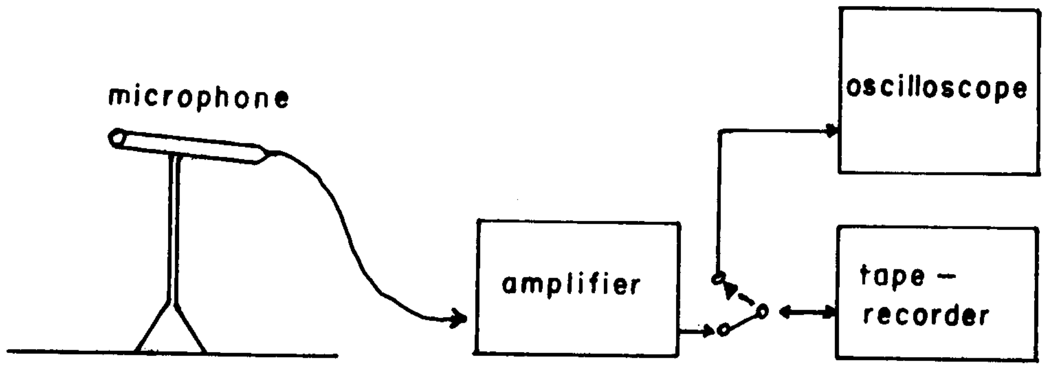

Fig. 3. Signal – recording system.

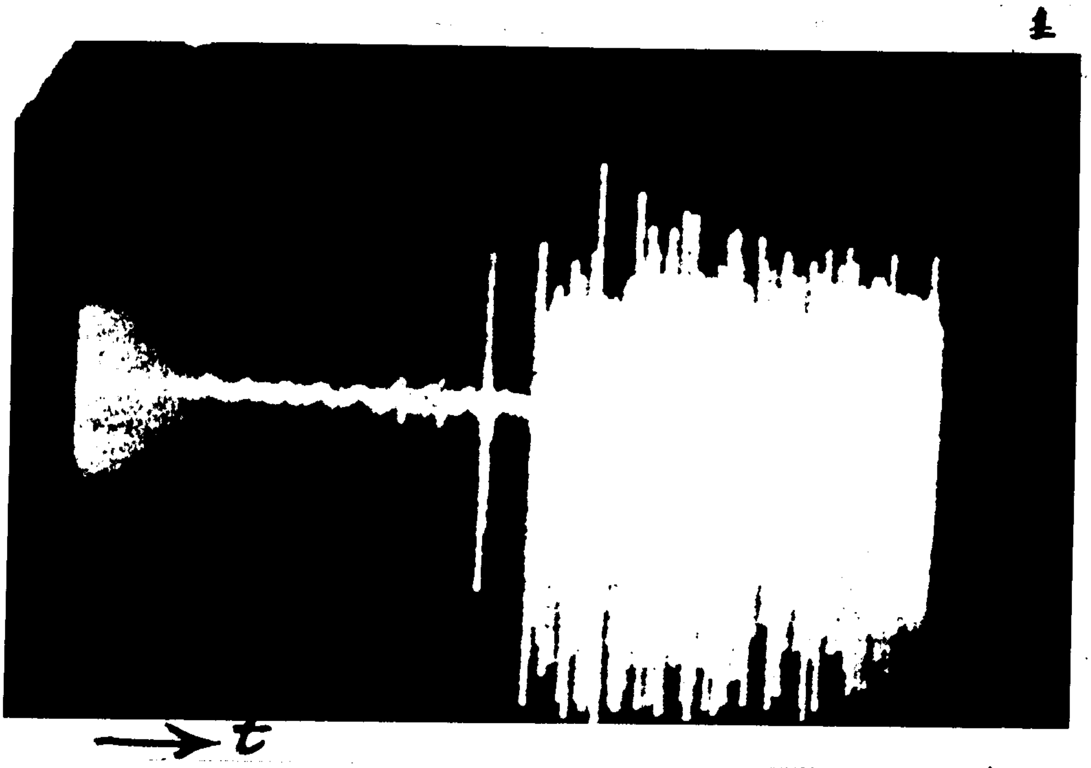

Fig 4(a): all holes closed, 10ms/div scale

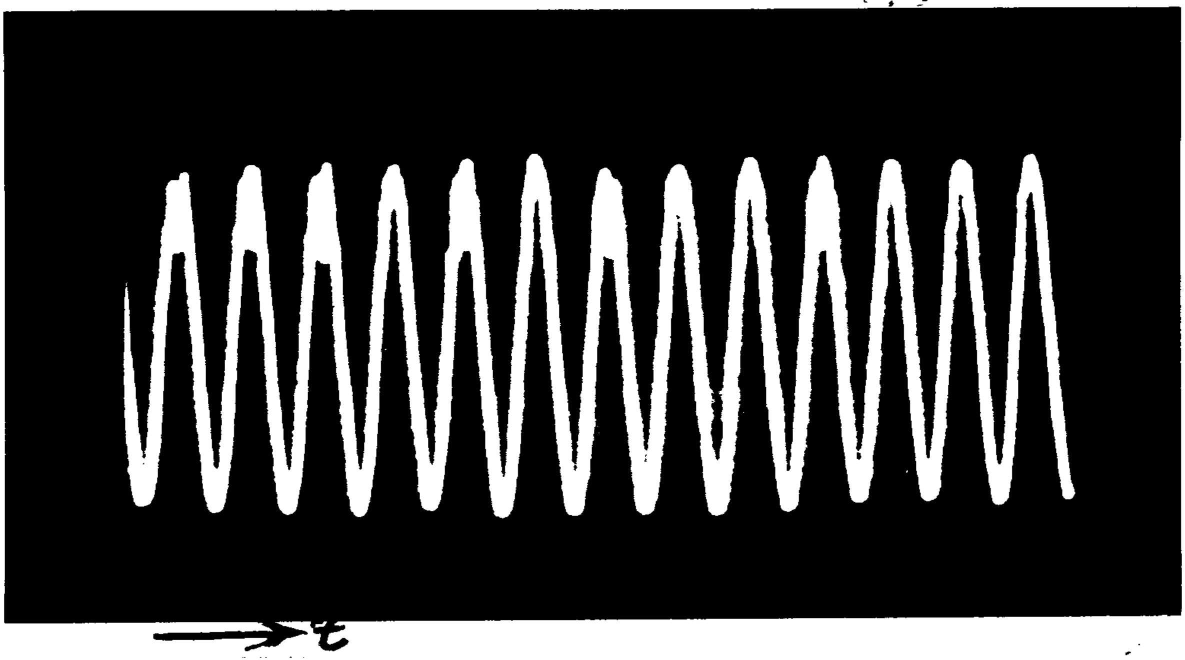

Fig 4(e): all holes closed, 0.5ms/div scale

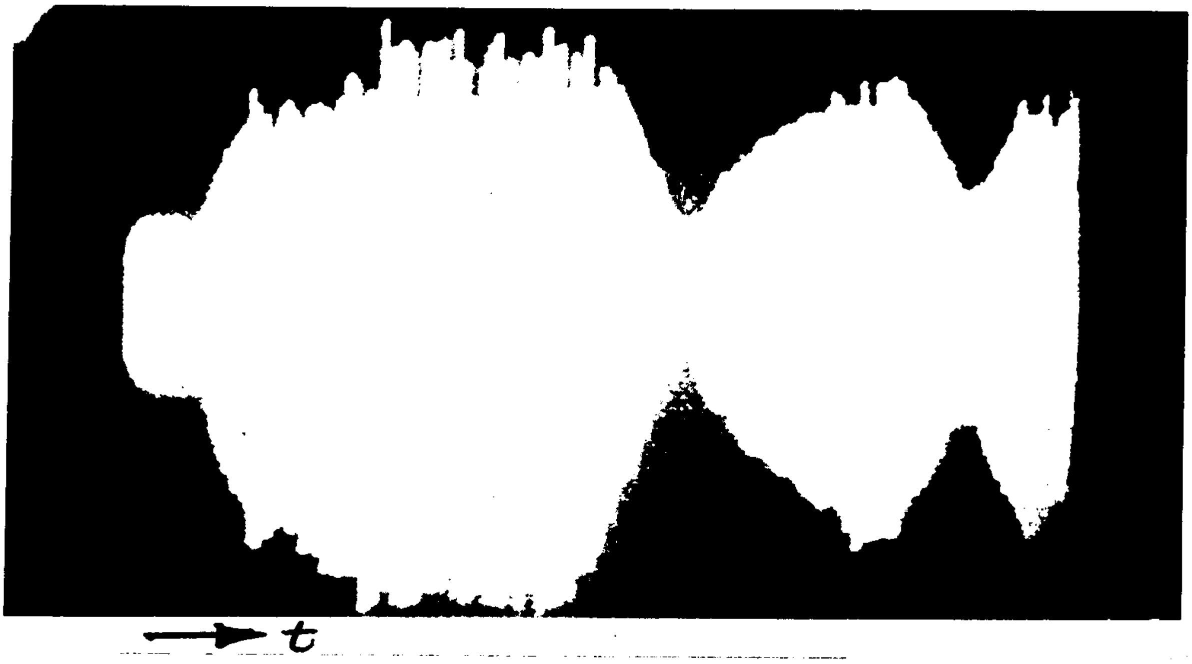

Fig 4(b): one side hole open, 0.5ms/div scale

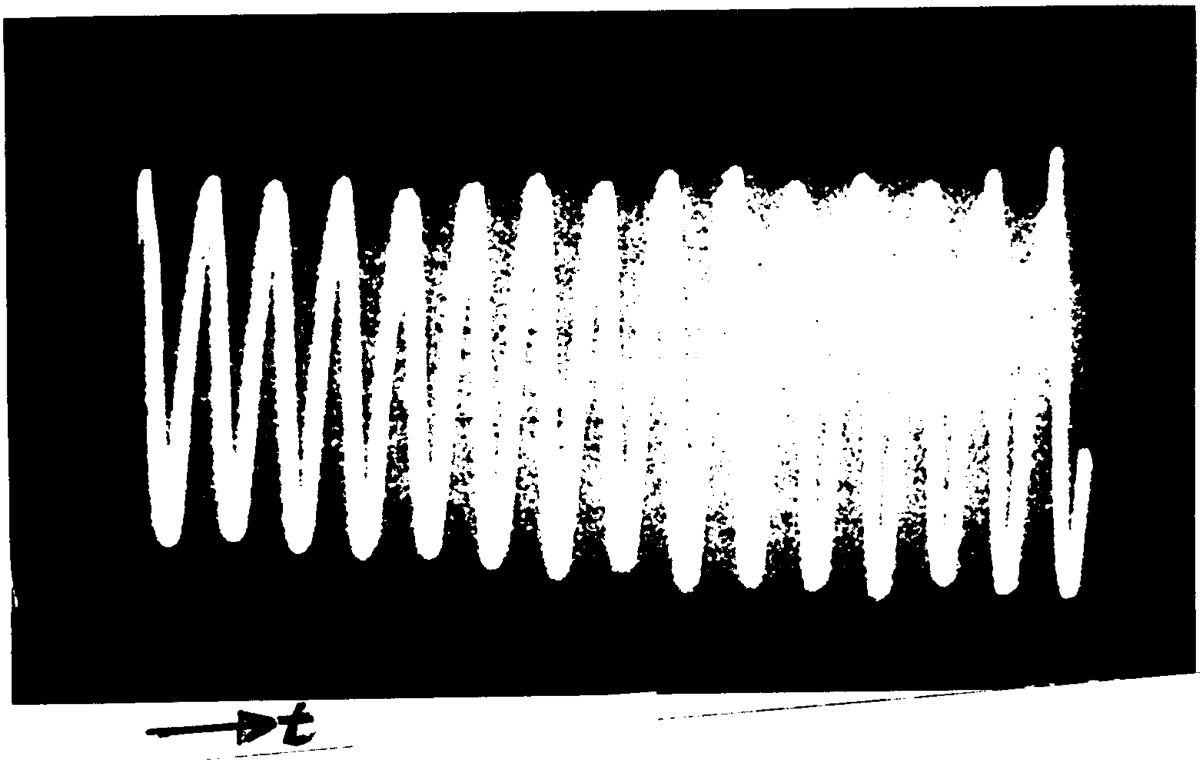

Fig 4(f): one side hole open, 0.5ms/div scale

Fig 4(d): all holes open, 10ms/div scale

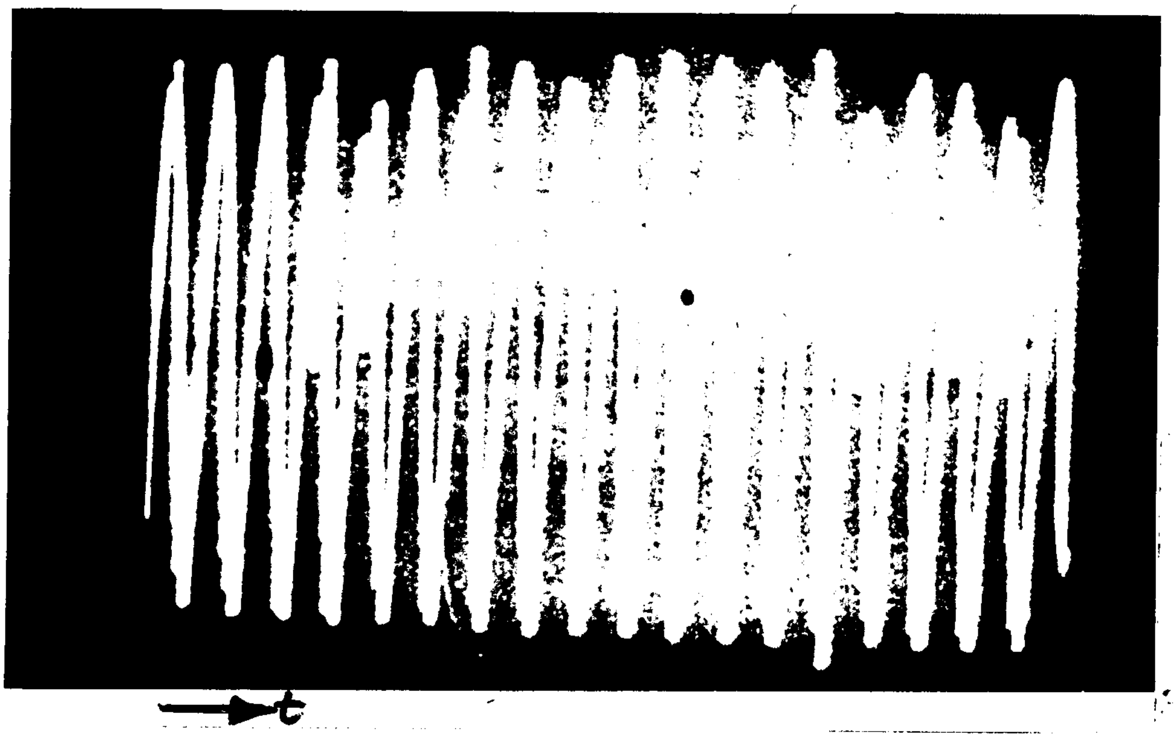

Fig 4(h): all holes open, 0.5ms/div scale

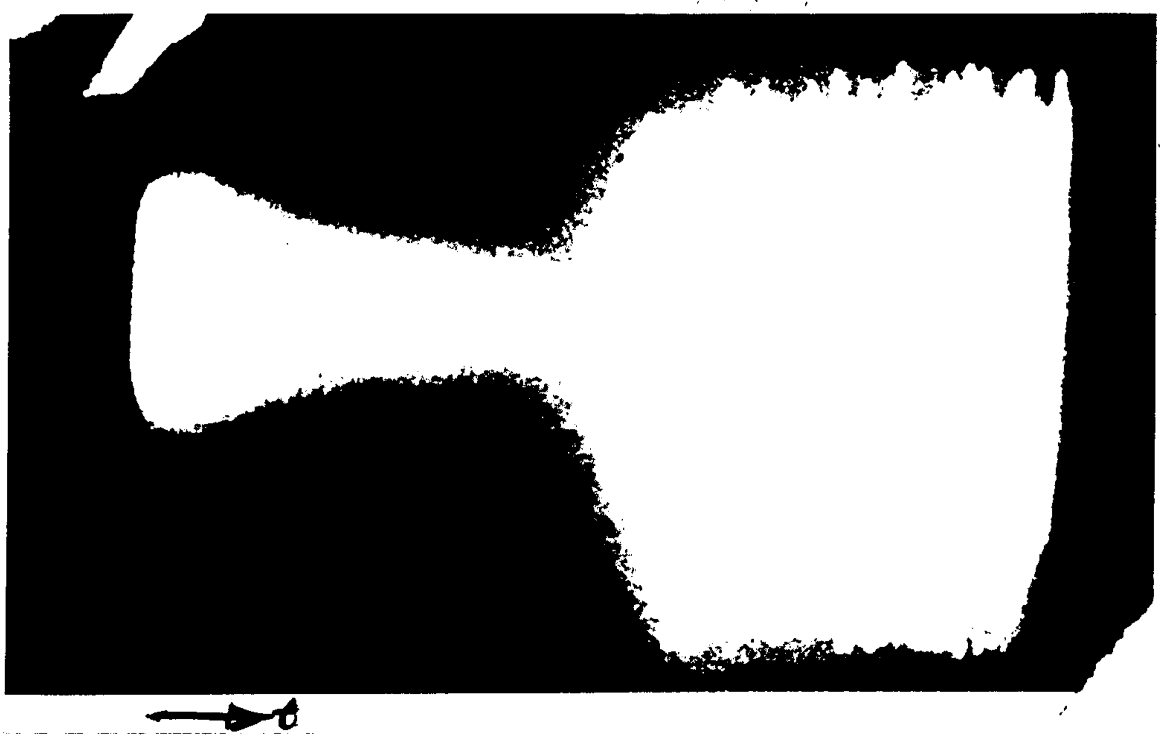

Fig 4(c): two side holes closed, 10ms/div scale

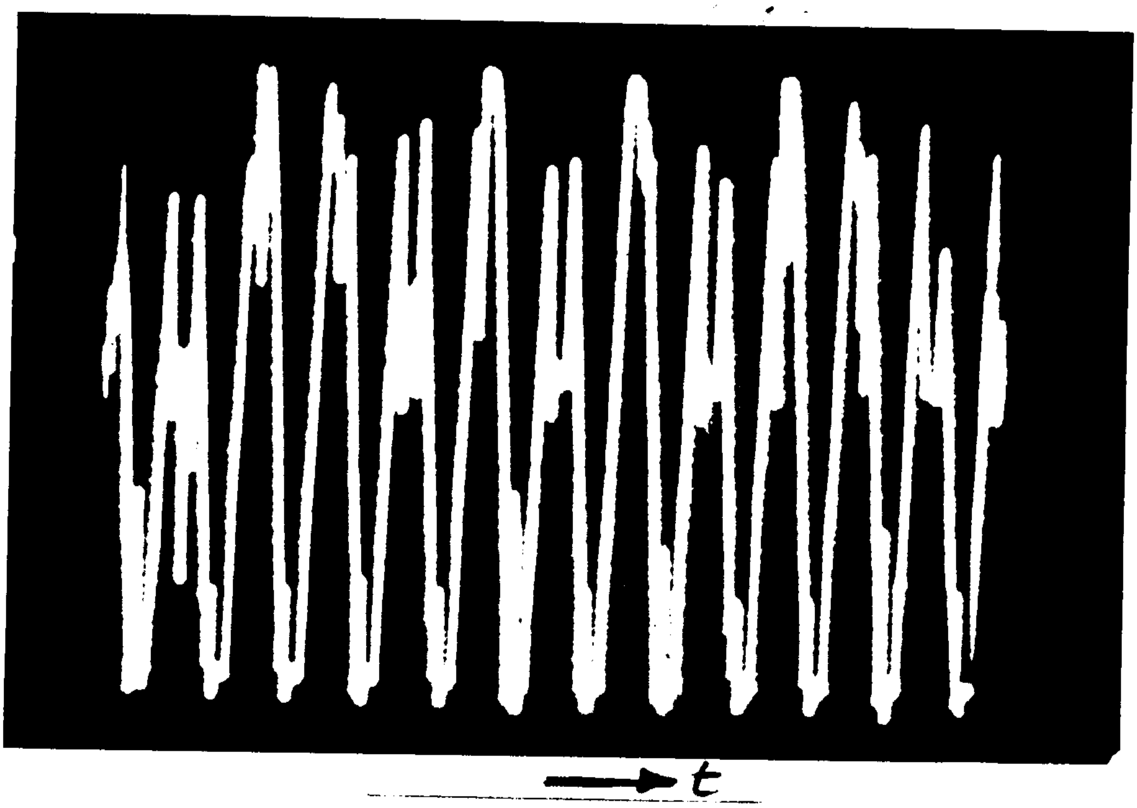

Fig 4(g): two scale holes closed, 0.5ms/div scale

3.2 Spectral Analysis

The instrumentation for the spectral analysis is shown in fig. 5.

Playback was at the same speed as that used in the record mode, namely 15 ins/second. The output from the tape recorder was connected to the input of the measuring amplifier and thence to the input of a continuously tunable B & K heterodyne slave filer operating in the frequency range 0 to 20 KHz. The slave filter was tuned electrically by signals supplied by the B & K sine-wave generator. The analysis bandwidth selected was 10Hz.

At a given setting of the filter centre frequency, the signal component with that frequency is extracted and passed through the measuring amplifier to the B & K level recorder. The recording paper had a logarithmic frequency scale and was driven at a speed of 0.1 mm/sec. The movement of the paper was synchronised with the sweep of the sine-wave generator (and therefore the filter pass frequency) by means of a flexible drive cable coupling the generator mechanically with the level recorder.

Spectral analysis of each channel lasted some 25 minutes. The observed spectra are shown in figs. 6 (a) - (d).

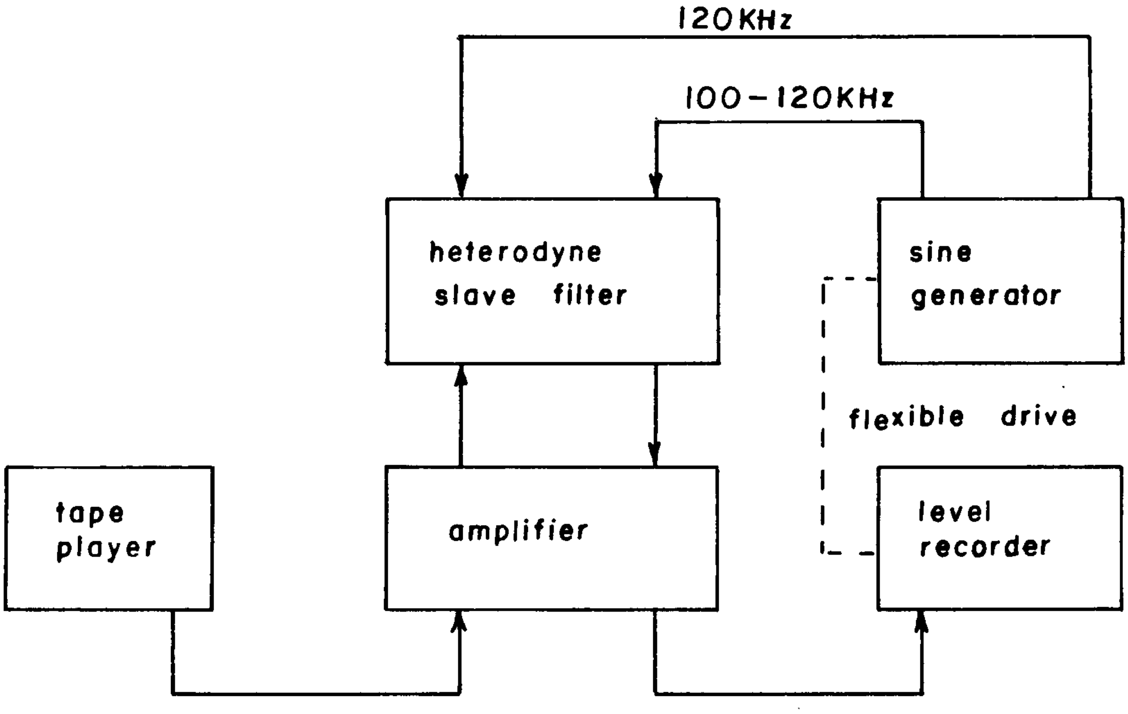

Fig. 5: Signal frequency analysis system.

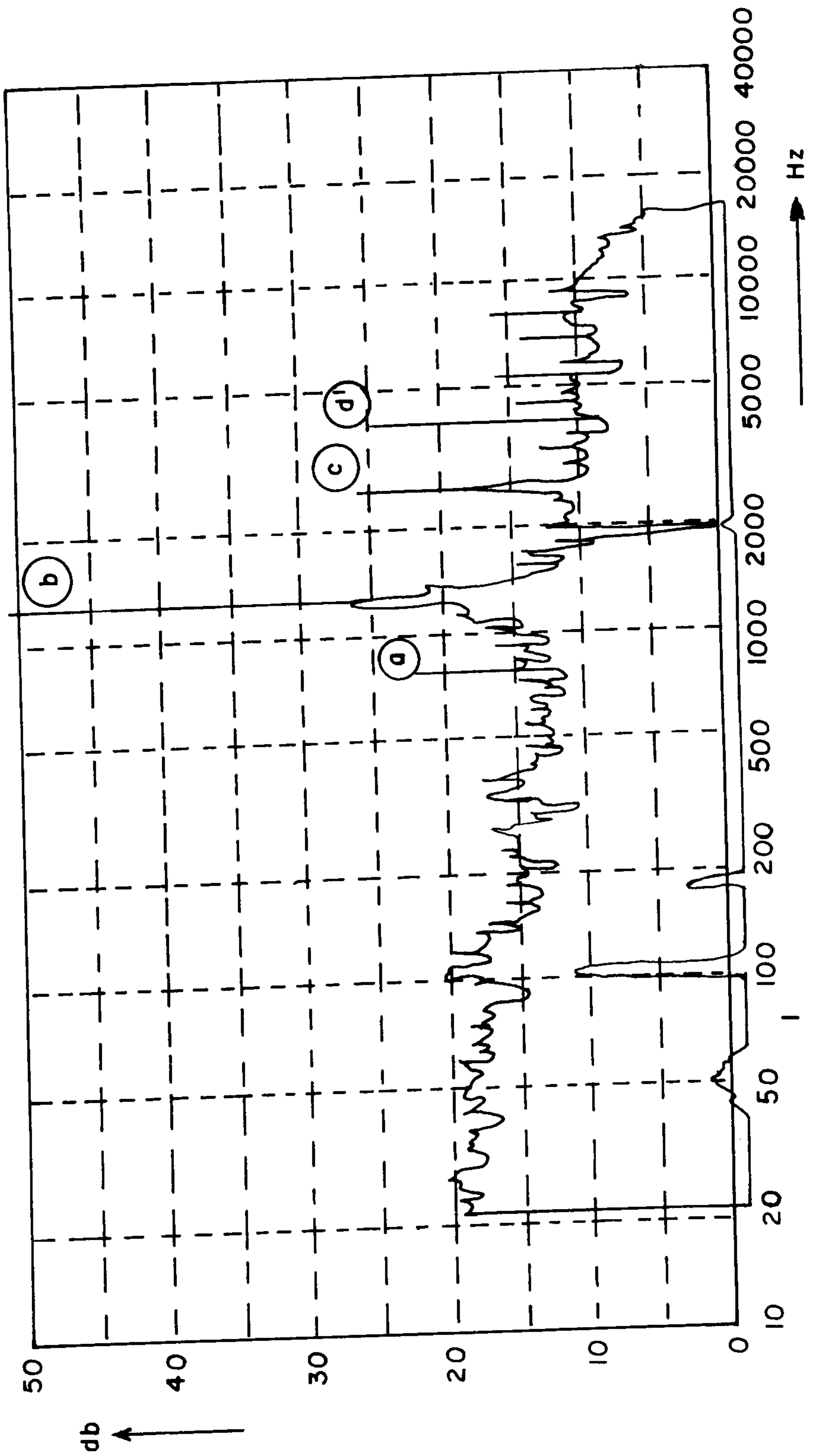

Fig. 6b : Spectogram of oja sound (one side tone hole open, other tone hole closed).

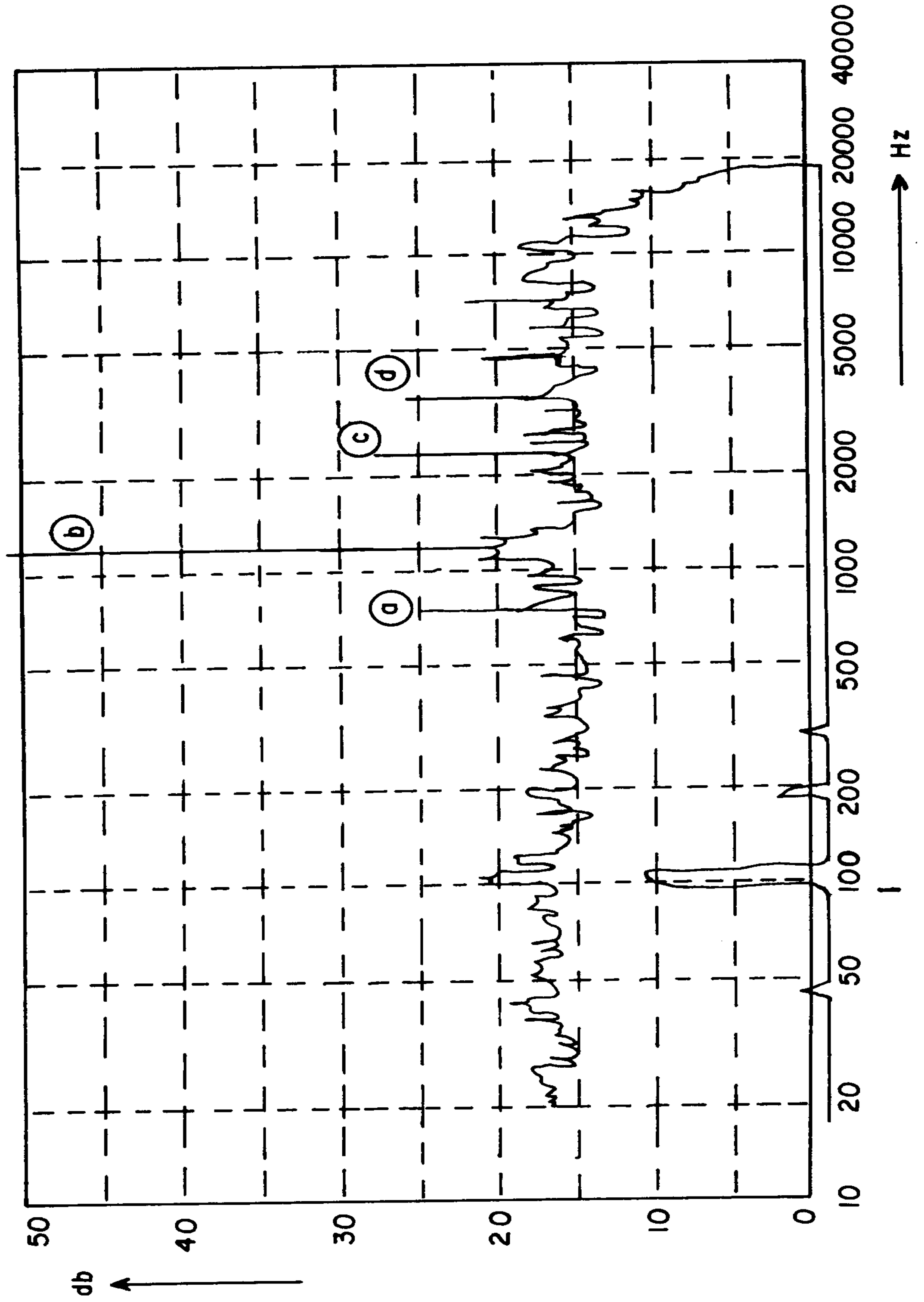

Fig. 6a: Spectrogram of oja sound (all tone holes closed).

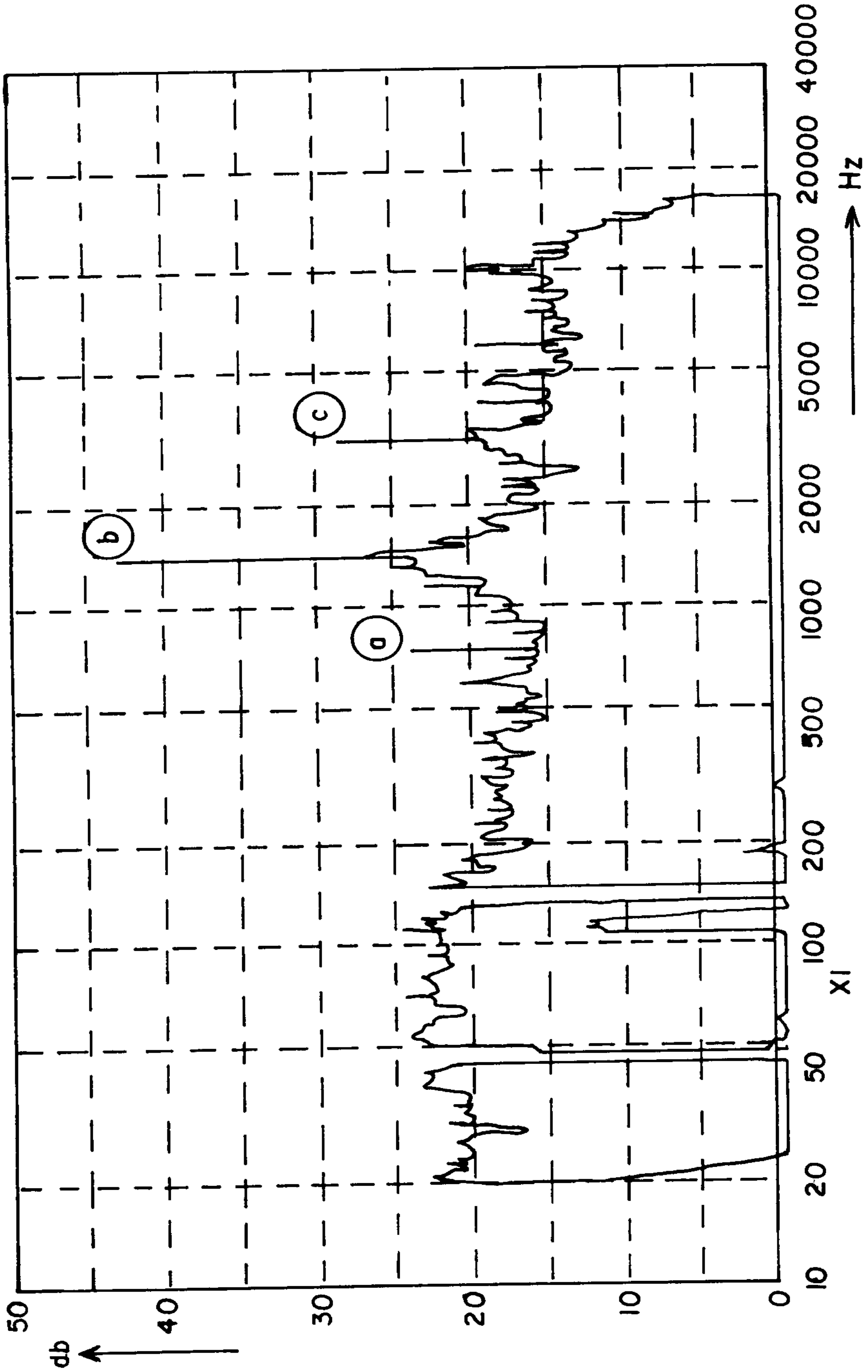

Fig. 6d: Spectrogram of oja sound (all tone holes open).

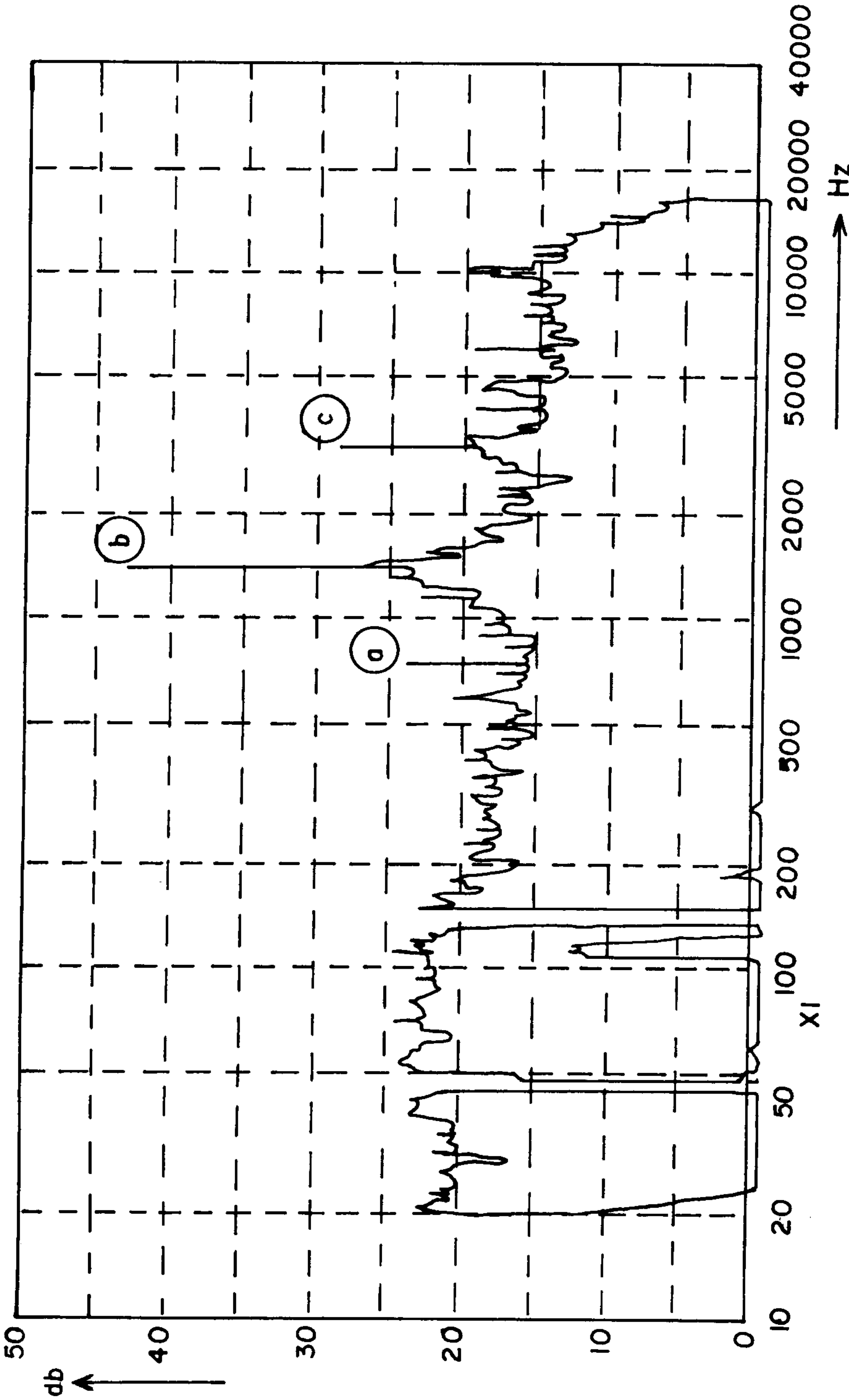

Fig. 6d: Spectrogram of oja sound (all tone holes open).

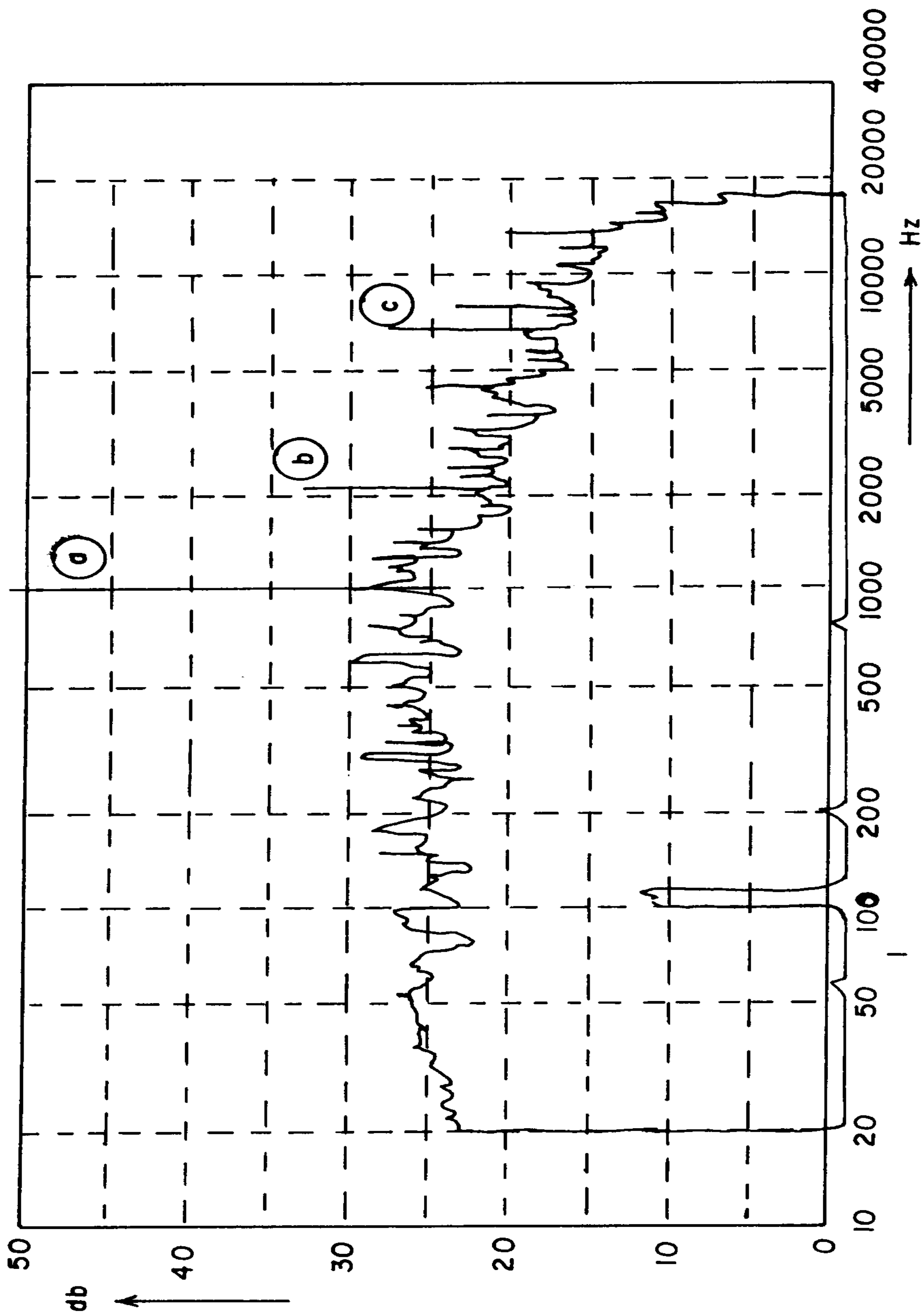

Fig. 6c : Spectrogram of oja sound (both side tones closed, lower hole closed).

4. DISCUSSION

4.1 Interpretation of Experimental Data

In general, the observed waveforms show considerable amplitude fluctuation. The pictures fig. 4 (a) - (d) taken with a time base factor of 10 ms/division show clearly an 'attack' portion characterised by a rapid build up of oscillations, and a more or less steady-state portion. In some of the waveforms amplitude modulation effects are apparent, but these are related to the modulation of the blowing effort on the part of the fluatist. Not evident in the pictures but also observed in the oscillography is a decay portion. Fig. 4 (e) - (h) are segments of the steady-state portion of the waveforms taken with a time base factor of 0.5 ms/division. One of these (fig. 4 (f) is almost a pure sinusoid while the others show a dominant sinusoidal component.

Coming now to the spectra, it is recalled that continuous signal joined to form an endless loop for playback, is an artificially periodic function, and as such will have a line spectrum with frequency separation of $1/T_r$, Hz where T_r, is the repetition time of the signal in second[4].

Further, since the signal is in effect time-multiplied with a rectangular window, the spectrum will show the well-known effect of frequency domain convolution with the (sin x)/x function, i.e. sidelobes will appear on either side of the lines of the true signal spectrum. Finally, swept-frequency analysis with finite band-width filter introduces an averaging process which converts the line spectrum into a continuous spectrum.

Under the conditions used in the spectral analysis, l/T_r, is only a small fraction of 1 Hz and is thus considerably less than the constant filter bandwidth of 10 Hz. Again, with a logarithmic frequency scale, the dwell time[5] varies from 8 seconds at 250 Hz to 0.5 second at 4000 Hz, halving for every octave change in frequency. This means that different segments of the fluctuating amplitude waveform are used to determine the spectral magnitude at different frequencies.

The combined effect of these various factors causes the observed spectra to exhibit considerable fluctuation in the side-lobe structure, with the possibility of some of the side-lobes being mistaken for signal spectral peaks.

However, the spectra do show distinct spectral prominence such as those marked a,b,c etc., in the respective charts, and where amplitudes are 10 db or more greater than other spectral amplitudes in their immediate neighbourhood. These prominence are definitely associated with the true resonant frequencies of the oja-mouth cavity resonant system. These frequencies are listed on table I column (4) for each fingering configuration. Column (6) of table 1 gives the ratio between the frequency of each of the lower amplitude components and the dominant resonant frequency. In general, there is no harmonic relationship between these frequencies.

It may be pointed out that the perceived pitch of the note may not be determined by the dominant spectral term at all. On table 1 column (7), we have shown for each note the frequency assigned to the note by a skilled musician using aural matching[6], With some notes the musician is unable to assign a definite pitch a fact no doubt related to the inharmonicity of the partials (see for example ref. 7).

4.2 Comparison of Theory and Experiment

Comparison is made on table 1, and in graphical form in fig. 7.

The theoretical analysis seems to be most successful in the case when all the tone holes are closed. In this case, the resonant frequencies of the first three unblown resonances are within ± 10% of the experimentally determined values. In general, however, the theory holds better at lower frequencies (below about 1000 Hz) than at higher frequencies. In this regard, it appears that the theory is less successful for cases when the lower tone pipe is open. In fact, the lowest resonant frequency observed when all three tone

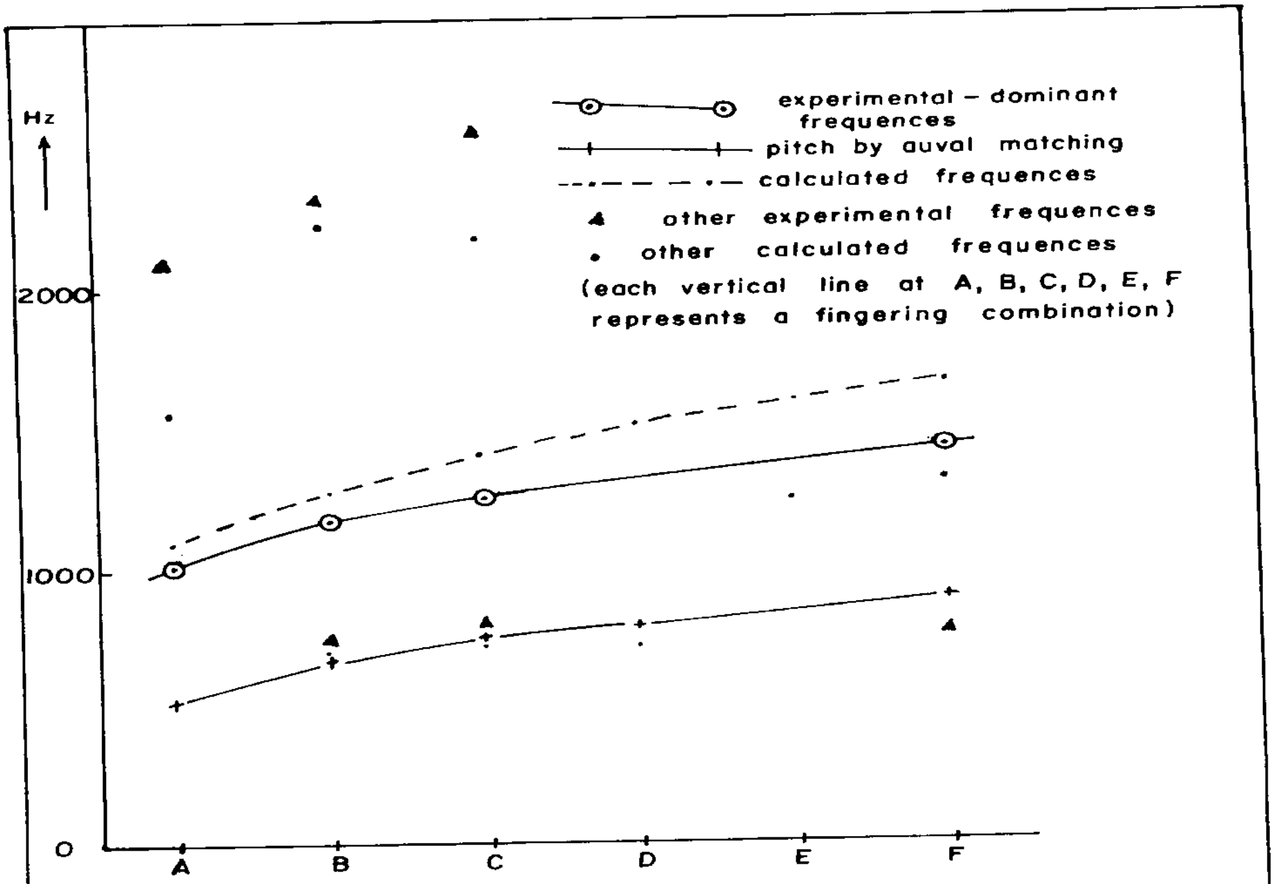

Fig. 7. Resonant frequencies of an oja.

holes are open, namely 760 Hz is not predicted by the theory.

Various factors contribute to the difference between the calculated resonances and the experimentally determined ones. Among these are:

i) error in the actual dimensions used in the theoretical calculations:

ii) effect of the neglected series transition reactance;

iii). error in estimating the tone pipe admittance functions;

iv) effect of the player's lips and mouth cavity.

The first two factors become increasingly more important at higher frequencies. The third comes into play with open tone pipes and is all the more serious when the lower tone pipe is open because of its greater length; it is also frequency - dependent.

With regard to the fourth factor, the findings of Coltman who has investigated the effect of the mouth cavity on the frequency of the flute[8], are relevant. Coltman's study showed that a mouth resonance occurring in the vicinity of 1000 Hz causes a shift in the resonant frequency when the flute is played. There is an upward shift in frequency as the mouth cavity resonance is approached from below, followed by a downward shift as the mouth cavity resonance frequency is passed.

In fig. 8, the deviation of the experimental oja frequencies from the corresponding theoretically determined unblown values have been plotted with the unblown frequency as the independent variable. Resonances below 2000 Hz have been considered. The behaviour seems to accord qualitatively with Coltman's findings with a cavity resonance in the neighbourhood of 900 Hz.

One further point has to be discussed, namely, the differences in amplitudes among the spectral prominence. In all the observed spectra, the frequency component in the vicinity of 1000 Hz exhibits the largest spectral amplitude, and there is a falling-off on either side. This preference for the resonances in the neighbour of 1000 Hz is probably associated with the greater excitation efficiency of these resonances which are close to the mouth cavity resonance[7]. The higher frequency are probably over-estimated as a result of the greater radiation efficiency at higher frequencies.

5. CONCLUSION

The oja is a simple wood wind instrument with a limited tonal range. Waveform and spectral studies have shown that the typical oja note has only a few inharmonically related overtones. This accounts for the 'transparent' quality of the notes. Perhaps more important than these preliminary findings is the fact that unblown resonances can be estimated to a fair degree of approximation using impedance methods of electrical transmission line theory. It should thus be possible to develop a practical design procedure for making an instrument with specified musical characteristics. Extended studies aimed at developing the instrument to its ultimate capability can therefore be carried out. Such a development can only enrich the musical culture which this instrument has traditionally been used to give expression.

6. ACKNOWLEDGEMENTS

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Mr. S. U. Obiejesi, formerly of the Department of Music, University of Nigeria, Nsukka, elicited from the tones which were recorded for this study. The author thanks Dr. W.W.C. Echezona for discussions on Igbo Musical Instruments.

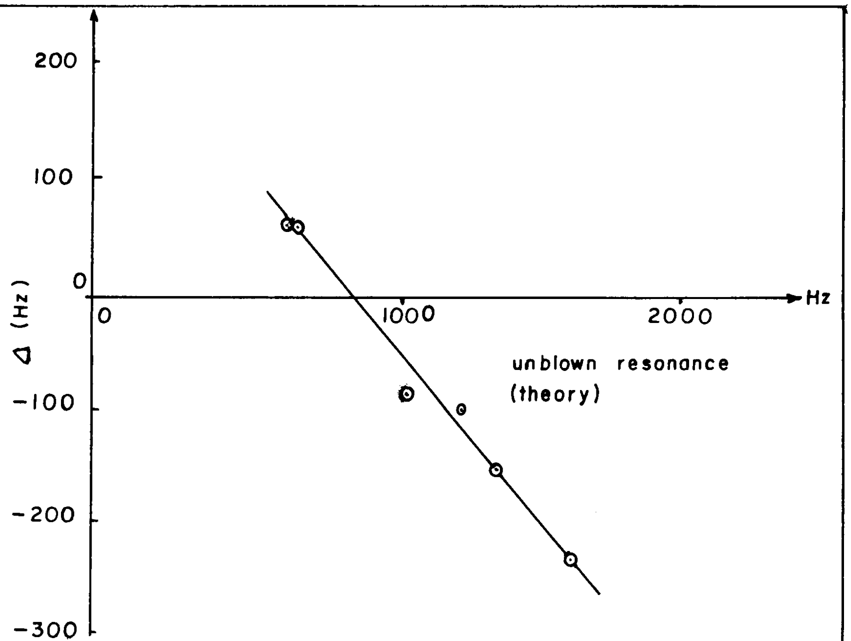

Fig. 8. Deviation (Δ) of experimental frequencies from theoretical.

$$z_{j0} = j\frac{\rho c}{\pi a^2}k1' + \frac{\rho c}{\pi a^2}\left[\frac{Z_1 + \rho c/(\pi a^2 \tan k1)}{\rho c/(\pi a^2) + jZ_1 \tan k1}\right] \quad (3)$$

Using equation (2) in equation (3) gives

$$Z_{j0} = \frac{\rho c}{\pi a^2}\left[jk1' + \frac{\alpha + j(\tan k1 + \beta)}{(1 - \beta \tan k1) + j\alpha \tan k1}\right] \quad (4)$$

Where

$$\alpha = \frac{K^2 a^2}{4} \quad 5(a)$$

and

$$\beta = 0.6ka. \quad 5(b)$$

Equation (4) is rearranged to give

$$Z_{j0} = \frac{\rho c}{\pi a^2}\left[\frac{\alpha(1 - k1' \tan k1) + j(\beta + k1' + (1 - \beta k1')\tan k1)}{(1 - \beta \tan k1) + j\alpha \tan k1}\right] \quad (6)$$

In equation (6), we shall neglect the terms containing α, the radiation factor, to get the final form:

$$Z_{j0} = 1/Y_{j0}, \frac{\rho c}{\pi}Y_{j0} = -ja^2\left[\frac{1 - \beta \tan k1}{\beta + k1' + (1 - \beta k1')\tan k1}\right] \quad (7)$$

Equations (1) and (7) are used to calculate the admittance functions of the tone pipes with reference to the plane of the junction for a given tone-hole fingering combination. The total admittance at the junction is found by summing the contribution of each of the three tone pipes. This quantity is denoted by B_j.

8.2 Admittance at Mouth of Main Pipe

The main pipe is open at the mouth and the impedance due to radiation and inertance is given by equation (2) with the appropriate radius a_m, substituted. We write

$$Z_m = 1/Y_m = \frac{\rho c}{\pi a_m^2}\left[\frac{K^2 a_m^2}{4} + j\,0.6Ka_m\right] \quad (8)$$

Again we neglect the radiation term to get the admittance B_m at the mouth of the main pipe:

$$Y_m = jB_m = -j\frac{\pi a_m^2}{\rho c}\left(\frac{1}{0.6ka_m}\right) \quad (9)$$

Or

$$\frac{\rho c}{\pi}B_m = \frac{a_m^2}{0.6ka_m} \quad (10)$$

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8. APPENDIX

8.1 Junction Admittance Due to Tone Pipes

A branch pipe of length l and radius a closed at one end has terminating impedance $Z_1 = \infty$.The impedance referred to the junction is given by[9]

$$Z_{jc} = j\frac{\rho c}{\pi a^2}(K1' - \cot K1) \qquad (1)$$

Where

k = 2π/λ (rd/m)

ρ = density of the air (Kg.m^3)

c = velocity of sound in (m/sec)

1' = 0.85 a (for ka <<1)

(represents the effect of the inertance of the air in the vicinity of the branch point).

If the branch pipe is open, the terminating acoustic impedance is given by[9]

$$Z_1 = \frac{\rho c}{\pi a^2}\left(\frac{K^2 a^2}{4} + 0.6jka\right) \qquad (2)$$

In equation (2), the first term in the brackets is due to the radiation coupling through the open orifice to the external medium while the second term is due to the inertance of the air at the orifice.

Upon transforming the impedance equation (2) to the plane of the junction and allowing for the effect of the inertance of the air at the entrance to the branch, we have