

DETERMINATION OF UPPER MANTLE CONDUCTIVITY USING QUIET DAY IONOSPHERIC CURRENT

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Abstract

The electrical conductivity of the upper mantle has been studied using data from L'Aquila (Latitude of 42° 23'N; longitude of 13° 19'E). The method of analysis employed was the Gauss spherical Harmonic analysis (SHA) technique. The results of the study indicate that the average electrical conductivity of the upper mantle in L'Aquila rose from very low sub-crustal value of 0.07 S/m to 0.17 S/m at about 290 km depth and dropped to a value of about 0.16 S/m at a mean depth of approximately 380 km and again rises sharply to a value of about 0.28 S/m at 705 km with no indication of leveling off. Results further show good correspondence between high conductivity and low seismic velocity zone. The cause of enhanced conductivity obtained from this study in the area under study could be attributed to the complex structure of the lower crust and upper mantle which consequently produces large lateral variation in electrical conductivity.

Keywords: *Sq*, spherical harmonic analysis, conductivity, ionospheric current
And upper mantle.

1. Introduction

The systematic flow of varying electric current in the part of the earth's upper atmosphere, the ionosphere, give rise to a magnetic field which in turn induces electric current in the earth's crust and mantle. The composite of both the external field and the internal field is measurable at the earth's surface in magnetic observatories. Such measurements, when made on quiet electrical conductivity of the upper mantle in the region concerned. This work is concerned with the use of fields due to ionospheric current variation at quiet

conditions in L'Aquila, Italy to obtain the conductivity-depth profile of the area.

Schumucker (1970) initiated the method of obtaining the electrical conductivity depth profile from the SHA external and internal coefficients of fields. Campbell *et al.* (1992, 1993) used this method to investigate the external source currents and deep - earth electrical conductivity for the India - Siberia region. Arora *et al.* (1995) also investigated the upper mantle electrical conductivity in the Himalayan region. They determined the conductivity profile of the earth at depths of about 50 to 500 km using the quiet

ionospheric current variations observed at a line of stations near 75°E longitude. They found conductivity values of about 0.06 S/m from 50 to approximately 350 km depth with slight relative maxima near 125 and 275 km, the day can be used to determine the interspersed by relative minima near 210 and 330 km. According to them the conductivity increased sharply after this, a value of about 0.18S/m at 500km with no indication of leveling off.

Campbell *et al* (1998) carried out a research on quiet-day ionospheric currents and their application to upper mantle conductivity in Australia, employing SHA, which allowed the separation of the external and internal field contribution of the Sq variation. They used special transfer function to compute the equivalent conductivity-depth profile of central Australia from paired external and internal coefficients of the SHA. A regression line through their conductivity estimates gave a profile that starts at 0.025 S/m for a depth of 130 km, rising gradually to about 0.045 S/m at 210 km, then steepens to 0.11S/m at 360 km and rises moderately to 0.13 S/m at 470 km close to the base of the upper mantle. Okeke (2000) suggested that a new model of Sq current system could be used to obtain a mantle electrical conductivity profile for hemispheres.

2. Source of Data

The data set obtained from the geomagnetic observatory in L'Aquila consists of the hourly mean values of the five quietest days for each month and from

the years 1989 to 1999. Both horizontal (H) and vertical (z) components of the field from the solar quiet (Sq) day variations in the region were utilized.

3. Method of Analysis

Spherical harmonic analysis (SHA) technique devised by Gauss (1838) was used to separate the external and internal fields and we henceforth used the SHA coefficients generated to determine the deep-Earth conductivity.

The theory of upper mantle conductivity determination begins with the Maxwell's formulation of electromagnetic field equations. Maxwell (1873) had generated a concise list of differential equations which summarized the laws governing electric and magnetic processes. The magnetic fields in these equations are due to real currents flowing in the ionosphere or equivalent (substitute) currents.

For the case in which the fields are measured about the surface of a sphere across which current does not flow, Gauss (1838) provided a separable series solution of Maxwell's differential equations for his spherical harmonic analysis (SHA) of the Earth's main field. Schuster (1889, 1908) used the Gauss SHA technique for Sq and showed that ionospheric current flow in the upper atmosphere of the earth was the cause of the quiet daily field variations. With the separated external and internal fields, Chapman (1919) succeeded in achieving the first recognizable earth conductivity determinations and also drew greater attention to the method of considering the

magnetic observatories as moving below an external Sq current system fixed with respect to the sun in order that the 24 hour Sq variation at a station represent 360° of longitude.

Schmucker (1970) provided equations expressing a conductivity and depth where an induced current could flow for the fields. Campbell (1998) amended and henceforth summarized Schmucker's work with the following equations:

$$d_n^m = z - p \tag{1}$$

where d_n^m is depth; z and p are real and imaginary parts of a certain complex number transfer function and m and n are order and degree, respectively.

$$\sigma_n^m = \frac{5.4 \times 10^4}{m(\pi p)^2} \text{ S/m} \tag{2}$$

where σ_n^m is conductivity.

$$z = R \frac{\{A_n^m/n(a_n^m) - (n+1)(a_n^m) + B_n^m[n(b_n^m) - (n+1)(b_n^m)]\}}{n(n+1)[(A_n^m)^2 + (B_n^m)^2]} \tag{3}$$

$$P = \frac{R\{A_n^m[n(b_n^m) - (n+1)(b_n^m)] - B_n^m[n(a_n^m) - (n+1)(a_n^m)]\}}{n(n+1)[(A_n^m)^2 + (B_n^m)^2]} \tag{4}$$

where $(a_{ei})_n^m$ and $(b_{ei})_n^m$ are the external cosine and sine SHA coefficients $(a_{ii})_n^m$ and $(b_{ii})_n^m$ and are the internal cosine and sine SHA coefficients. A_n^m and B_n^m are coefficient sums for the external and internal SHA coefficients, respectively.

This work however involves the use of only a single observatory data quiet unlike those of Campbell and Stiffmacher (1987,1988b), Arora *et al* (1995) and Campbell *et al* (1998) and instead of the one or two- year data used by most of the early researchers, a six - year data was employed in this present study in the L'Aquila region. The data processing routine employed was adapted from that of Campbell *et al.* (1998) and it is shown in Fig. 1.

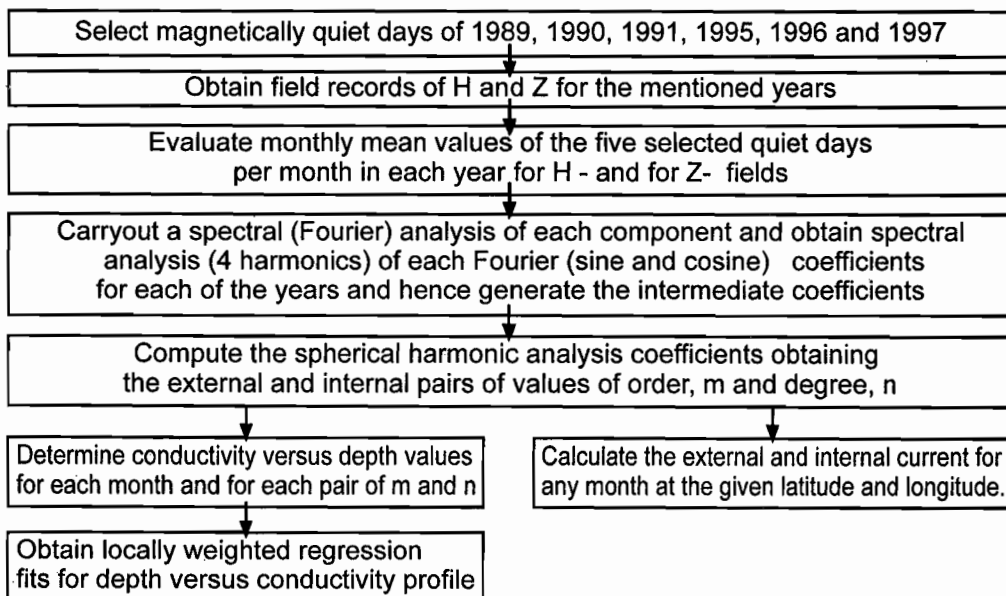


Fig. 1: Data processing routine

4. Results and Discussion

From the flow diagram, Fig. 1, the monthly mean values of the five selected quiet days for each month of the six years utilized were evaluated for both the H - and the Z- components and a Fourier analysis was carried out to obtain the Fourier coefficients and the Fourier- analyzed values of H and Z. Associated Legendre polynomials (Schmidt functions) for the study area (colatitude $47^{\circ}37'$) were computed. The intermediate coefficients were calculated using the Fourier-analysed values of H- and Z-components. Spherical harmonic analysis coefficients were determined from the intermediate coefficients and the coefficient sums A_n^m and B_n^m were evaluated. The real and imaginary parts, z and p respectively of the special transfer function related to the depth, d (km) and conductivity σ (S/m) of the earth were determined from the sparated external and internal SHA coefficients and the values^m of d_n^m and for various pairs of m and n were evaluated with $n = 1$ to 12, $m = 1$ to 4 and $m \leq n$. The conductivity-depth values were analyzed graphcially and the statistical method employed was the locally weighted regreson fitting described by Cleveland (1979). This gave the most likely profile for each year's data. By using the lowess regression fitting with two fitting iterations and appropriate smoothing factors, the profiles obtained are as

shown in Figs. 2 to 7.

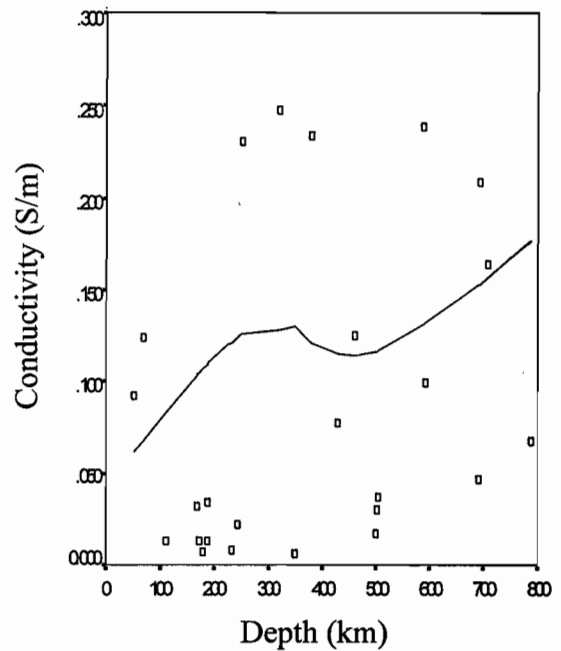


Fig. 2: Conductivity depth-profile of L'Aquila for 1989 Sq data

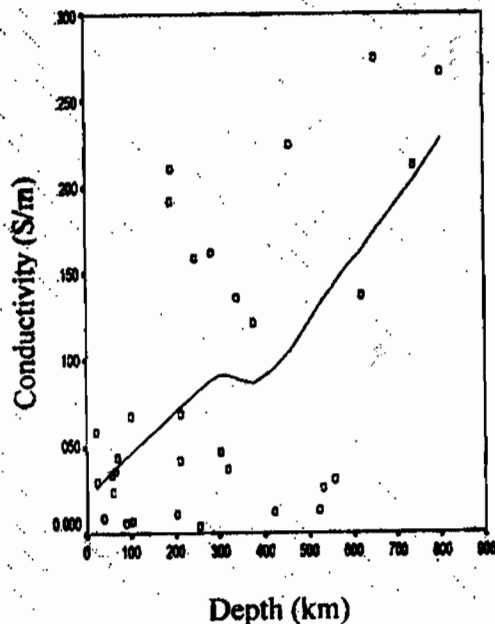


Fig. 3: Conductivity - depth profile of L'Aquila for 1990 Sq data

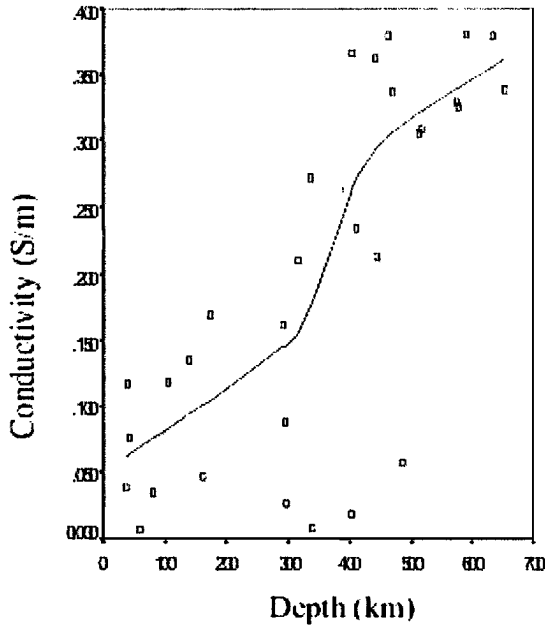


Fig. 4: Conductivity-depth profile of L'Aquila for 1995 Sq data

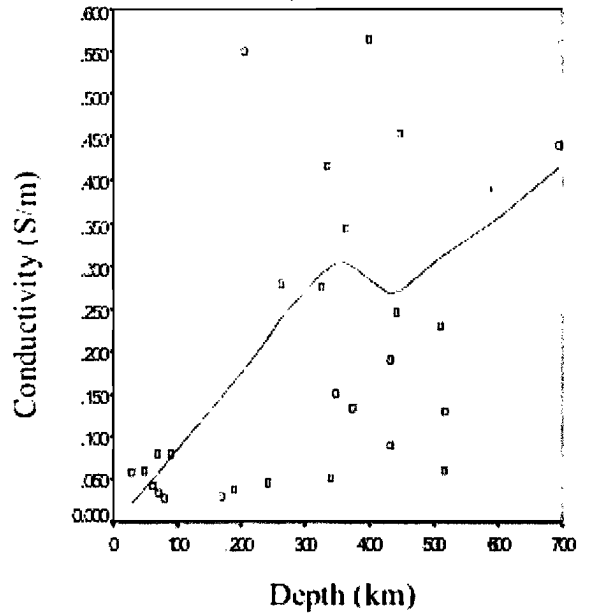


Fig. 6: Conductivity-depth profile of L'Aquila for 1997 Sq data

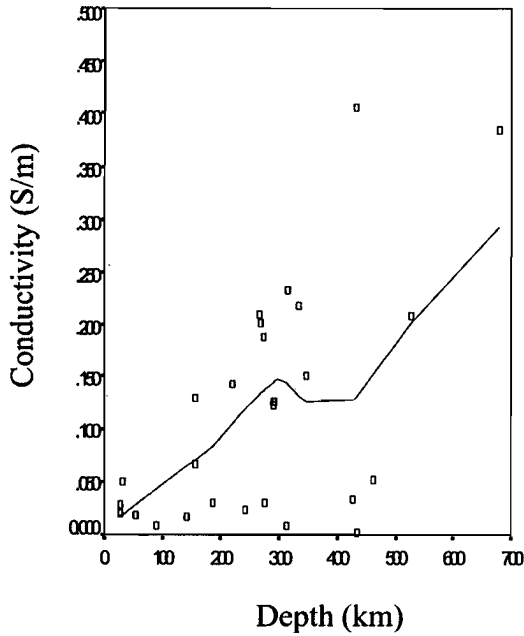


Fig. 5: Conductivity-depth profile of L'Aquila for 1996 Sq data

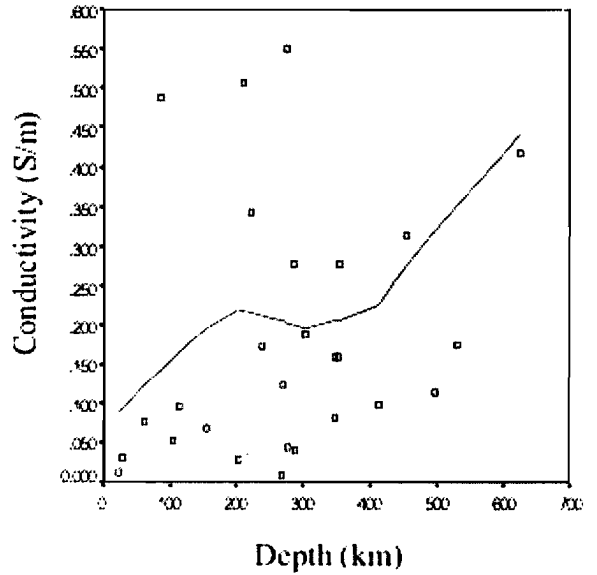


Fig. 7: Conductivity-depth profile of L'Aquila for 1999 Sq data

The open squares in Figures 2 to 7 illustrate the conductivity-depth computation results. The solid lines are the regression fitted values. From Fig. 2, it is clearly seen that, the conductivity rises from a low sub-crustal value of about 0.06 S/m to an almost relative maximum of about 0.13 S/m near 295 km depth. A relative minimum conductivity of about 0.12 S/m occur near 450 km depth. Thereafter, the conductivity rises sharply toward a value of 0.17 S/m at 780 km depth.

In Fig. 3, the conductivity is observed to rise moderately from a very low sub-crustal value to a value of about 0.09 S/m near 300 km depth. Thereafter it decreases gently to about 0.08 S/m at 380 km depth and rises rapidly from there to about 0.25 S/m at 800 km depth with no sign of leveling off. In fact, it is clearly evident from these profiles, including those of Figs. 4, 5, 6, and 7, that:

- (i) The profiles exhibit about the same trend of variation of conductivity with depth. This is suggestive in the similarity of the shape of the regression fittings.
- (ii) The conductivity in all the profiles increases generally with increase in depth except at few depth ranges. This is in excellent agreement with global models.
- (iii) Almost all the profiles show two turning points -relative maxima and

minima with the maxima always before the minima.

Correlating the conductivity-depth profiles with seismic wave velocity models, the shapes of Figs. 2, 3, 4 and 6 suggest some mantle zonation. On this ground, it becomes important to match our models with seismological and petrological boundaries. According to Arora *et al* (1995), current seismic wave velocity models for the mantle such as PREM (Dziewonski and Anderson, 1981) and IASP 91 (Kenneth and Engdahl, (1991) show abrupt rise in velocity at 410 km and 670 km depth and provide strong evidence for the existence of a zone of low velocity gradient between about 100 and 220 km depth. The conductivity profiles in this present study do not resolve a discontinuity at the base of the lithosphere (before the low velocity layer) of 80 km depth, but rather they rise progressively to a mean depth of about 300 km (Figs. 2,3,4, and 5). This feature of these profiles is in agreement with those of other regional profiles such as those of Campbell and Stiffmacher (1988), Arora *et al.* (1995) and Cambell *et al.* (1998).

The low velocity layer between 80 km (i.e base of the lithosphere) and 220 km (i.e base of the asthenosphere) show a very good correspondence with the high conductivity zones indicated by the present study in L'Aquila. The general correspondence observed between high

conductivity zones and low velocity zones here is in agreement with the global results of Tarrits (1992), even though processes giving rise to this correlation is yet to be discovered. It can be stated that the rapid rise in conductivity in all the profiles from a mean depth of 380 km to 800km is a cumulative effect of the distinct sharp increases in conductivity at the world-wide discontinuities of 410 km, 670 km and 770 km. In terms of the rapid increase in conductivity in the depth interval of 380 - 800 km, the present conductivity models are similar to the global models such as Parker (1971), Bott (1982), Schmucker (1985) and Stacey (1992) which show a rapid rise below 350-500 km depth (Schutz and Larsen, 1990).

5. Conclusion

Considering all the profiles obtained in the study, we conclude that the average conductivity of the upper mantle in L'Aquila rises from a low sub-crustal value of 0.07 S/m to a value of about 0.17 S/m at a mean depth of about 290 km; fall to a value of about 0.16 S/m at a mean depth of approximately 380 km and rises sharply to a value of about 0.28 S/m at 750 km depth with no indication of leveling off. Comparing the results of this study with those of similar works carried out at other regions such as the work of Arora *et al* (1995) in the Himalayan region and that of Campbell

et al. (1998) in the Australian region, it is obvious that the results are reasonable. Nevertheless, the conductivity values of this present study are noted to be slightly higher than those of previous research works at comparable depths.

The cause of the enhanced conductivity results obtained in L'Aquila may be generally due to the complicated structure of the crust and upper mantle which results in large lateral variations in electrical conductivity (Lowrie, 1997). It is therefore recommended that further research work be carried out in the study area perhaps with multiple observatory stations to confirm these findings

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References

- Arora, B.R., Campbell, W.H. and Schiffmacher, E. R. (1995): Upper Mantle Electrical Conductivity in the Himalayan Region, *J. Geophys Geoelect.* **47**, 653-665.
- Bott, M. H. P. (1982): *The Interior of the Earth: its Structure, Constitution and Evolution*, 403pp. Edward Arnold company, London.
- Campbell, W. H, Schiffmacher, E. R. and

- B. R. Arora (1992): Quiet Geomagnetic Field Representation for all Days and Latitudes. *J. Geomagnetism and Geoelectric.*, **44**, 459-480
- Campbell, W. H., Arora, B. R and Schiffmacher, E. R (1993): External Sq Currents in the India-Siberia Region, *J. Geophys. Res.* **98**, 3741-3752
- Campbell, W. H., Barton, C. E., Chamalaun, F. H and Welsh, W.(1998); Quiet- Day Ionospheric Currents and their Application to Upper Mantle Conductivity in Australia, *Journal of Geomagnetism and Space Science*, **50**, 347-360
- Campbell, W. H. (1989): The Regular Geomagnetic Field Variations During Quiet Solar Conditions, Chap. 6 in Geomagnetism, vol. 3 edited by J. A. Jacobs, PP. 385-460, academic press, London.
- Campbell, W. H and Stiffmacher, E. R. (1987): Quiet Ionospheric Currents and Earth Conductivity Profile Computed from Quiet-Time Geomagnetic Field Changes in the Region of Australia, *Aust. J. Phys.*, **40**, 73-87.
- Campbell W. H. and E. R. Stiffmacher (1988b): Upper Mantle Electrical Conductivity for Seven Sub-continental Regions of the Earth, *J. Geomagn. Geoelectric.* **40**, 1387-1406.
- Chapman, S (1919): The Solar and Lunar Diurnal Variations of the Earth's Magnetism, *Phil. Trans. Roy. Soc. London*, **A218**, 1-118.
- Cleveland, W .S. (1979): Robust Locally Weighted Regression and Smoothing Scatter Plots, *J. Amer. Statistical Assn.*, **74**, 829-833.
- Dziewonski, A.M. and D.L. Anderson (1981): Preliminary Reference Earth Model, *Phys Earth Planet Inter.* **25**, 297-356.
- Gauss, C. F. (1838): *Allegemeine Theories des Erdmagnetismus Resultate Magnetischen Vereins* (reprinted in Werke, Bd 5, S. 121.
- Kenneth, B.L.N. and E.R. Engdahl (1991): Travel Times for Global Earthquake Location and Phase Identification. *Geophys. J. Int.* **105**, 429-465.
- Lowrie, W. (1997): *Fundamentals of geophysics*, Cambridge University press, New York, 346

- Maxwell, J. C (1873): Treatise on Electricity and Magnetism, 566 pp., Cambridge University press, Cambridge.
- Okeke, F. N. (2000): Ionospheric Quiet Day Current and their Effects on the Interpretation of Upper Mantle Conductivity, *Surveys in Geophysics* 21: 487-498.
- Parker R.L. (1971): The Inverse Problem of Electrical Conductivity in the Mantle, *Geophys. Roy. Astr. Soc.*, **22**, 121- 138.
- Schmucker, U. (1970): An Introduction to Induction Anomalies, *J. Geomagn. Geoelectr.*, **22**, 9-33.
- Schuster, A. (1889): The Diurnal Variation of Terrestrial Magnetism, *Phil, Trans. R. Soc.* **180**, 467-518.
- Schuster, A. (1908): The Diurnal Variation of Terrestrial Magnetism, *Phil. Trans. Roy. Soc. London*, A208, 163-202
- Schultz A. and J. Larsen (1990): On the Electrical Conductivity of the Mid-Mantle, *Geophys. J. Int.* **101**, 565- 580.
- Stacey, F.D. (1992): Physics of the Earth 3rd edition, 513 pp., Brookfield press, Brisbane.
- Tarits, P. (1992): Electromagnetic Studies of Global Geodynamic Processes 11th Workshop on Electromagnetic Induction in the Earth, Victorian University of Wellington, 26 August- 2 September 1992, 7.1.-7.21