CLIMATIC AND SEASONAL VARIATIONS OF EFFECTIVE-EARTH-RADIUS FACTOR AND SCALE HEIGHT IN THREE METEOROLOGICAL STATIONS IN WEST AFRICA

O.D. Oyedum
Department of Physics Federal University of Technology, Minna, Nigeria.

(Submitted: 12 April, 2008; Accepted 16 June, 2008)

Abstract

The degree of reliability of terrestrial microwave links generally depends on proper assessment of the controlling influence of refractive index variations in the troposphere. Two important radiometeorological parameters in this regard are the effective-Earth radius factor k, and the scale height H. In particular, terrestrial line-of-sight links must be planned on the basis of time-averaged k-values to ensure continuous clearance of the link from the first Fresnel zone. Based on radiosonde data from three West African meteorological stations of Lagos, Kano and Niamey, values of k and H were determined and compared. The results show significant seasonal and climatic variations which must be taken into consideration for reliable terrestrial microwave links in the region.

Keywords: *Microwave links, effective-earth radius, scale height and line -of-sight.*

Introduction

Refractive index variations of the atmosphere affect radio frequencies above 30 MHz, although these effects become significant only at frequencies greater than about 100 MHz. Generally, variations of refractivity gradients are not strong enough to cause significant effects at lower radio frequencies for which the ground wave and ionospheric propagation mechanisms dominate at transhorizon ranges.

The radio refractive index, n, of the troposphere deviates slightly from unity due to (1) polarisability of the constituent molecules by incident EM field, and (2) quantum mechanical resonances at certain frequency bands. Whereas molecular polarisability is independent of frequency up to millimetre waves, molecular resonance is totally frequency dependent, and n tends to be dispersive above ~50 Ghz.

Radio refractivity N is a measure of deviation of refractive index from unity scaled-up in parts per million. N is defined as (Bean *et al.*, 1967)

$$N = (n-1) \times 10^6 \qquad (1)$$

and depends on meteorological factors of pressure P (hPa), temperature T (K) and water vapour pressure e (hPa), as given by the Smith and Weintraub (1953)

$$N = 77.6/T\,(P + 4810e/T^2) \qquad (2)$$

P and e decrease rapidly with height while T decreases slowly with height.

Horizontal variation of refractive index is generally negligible in the lower troposphere compared to the large-scale vertical variation which has a median gradient of about -40 N/km above the surface in midlatitude and most temperate regions (Bean *et al.*, 1966). However, significant deviations can arise from local or mesoscale meteorological factors, especially in the tropics (Owonubi, 1982). Decrease of n with height causes radiowaves to curve downwards, and to a degree which depends on the gradient dn/dz. Refractive bending causes extension of the radio horizon beyond the optical horizon. The curvature of a radio path is generally less than the curvature of the Earth; and there exists a distribution of n (z), which could produce a ray curvature equal to that of the earth, and consequently no horizon as the ray remains parallel to the surface. This atmosphere model which allows straight-line propagation of tropospheric waves is normally preferred in plotting to scale the progress of a radio ray from transmitter to receiver in the troposphere. This gives rise to the concept of 'effective-Earth-radius'.

Effective Earth Radius Factor, k

The decrease of N with height, which causes bending of a radio ray towards the Earth, is normally not sufficient to overcome the curvature of the Earth. For a vertical gradient of refraction dn/dz, the rays are refracted towards the region of higher refractive index (i.e. towards the Earth's surface) with a radius of curvature r, such that the ray curvature may be given by (Bean and Dutton, 1968; Hall, 1979; Sizun, 2005)

$$\frac{1}{r} = -\frac{1}{n}\frac{dn}{dz}\cos\alpha \qquad (3)$$

Where α (z) *is* the ray angle with respect to the horizontal at height h above the Earth's surface.

At low elevation ($\alpha \approx 0$) and with $n \approx 1$, the ray curvature is

$$\frac{1}{r} = -\frac{dn}{dz} \qquad (4)$$

Thus, the curvature of a ray depends on the refractive index gradient dn/dz, not on the absolute value of n. For the Earth's radius a, the curvature of the radio ray relative to Earth's curvature 1/a is

$$\frac{1}{a} - \frac{1}{r} = \frac{1}{a} + \frac{dn}{dz} = \frac{1}{a_e} = \frac{1}{ka} \qquad (5)$$

where k is the effective Earth's radius factor and a_e is the radius of the equivalent Earth for which the ray now appears straight, with relative curvature to that of the equivalent Earth as $1/a_e$. Equation (5) is derived from the fact that the effective ray curvature relative to the equivalent Earth is $1/a_e$-$1/$=$1/a_e$. Using median midlatitude dn/dz value of $-40\text{x}10^{-6}$ km^{-1} and Earth's radius of 6378 km gives a value of $ka=a_e=8500$ km or k=4/3. Equation (5) can also be represented by an alternative model of a flat earth with curved ray of radius of curvature r_e such that its relative curvature with the earth is preserved. Clearly,

$$\frac{1}{a_e} = \frac{1}{r_e} = \left(157 + \frac{dN}{dZ} \times 10^{-6}\right)^{-1} \qquad (6)$$

And the effective Earth's radius factor is

$$k = \frac{a_e}{a} = \frac{r_e}{r} = \left(1 + a\frac{dN}{dZ} \times 10^{-6}\right)^{-1} \qquad (7)$$

Equation (7) applies to a well-mixed atmosphere where dN/dZ is constant, with constant lapse rate, e.g. after turbulent convection. From eqns (5) and (7), when the ray curvature equals the Earth's curvature, $dN/dZ = -(1/a)\ \text{x}10^{6} = -157$ N/km, k = and the ray, if initially horizontal, runs parallel to the Earth's surface. If dN/dZ <-157, k < 0 and the ray is bent more steeply towards the surface; the Earth appears flatter and the radio horizon becomes extended. When dN/dZ > -40 N/km, 0 < k < 4/3, and the ray is subrefracted; the Earth appears more curved while the radio horizon is reduced. A mean dN/dZ of -40 N/km (typical of temperate climates) is normally assumed near the surface, for which ray bending towards the ground is more gentle with k = 4/3. Table 1 shows the different types of atmospheric refraction with the associated refractive index gradients and k-factor values.

Table 1: Types of refraction with corresponding refractivity gradient and k values (IEEE Std 211, 1997)

Refraction Types	Refractive Index Gradient		
	dN/dh (N-Units)	dM/dh (M-Units)	K-Factor
Homogeneous	0	157	1
Adiabatic	-23	134	1.2
Standard	-39.2	118	4/3
Subrefractive	>-39.2	>118	<4/3
Extreme Subrefractive	>0	>157	<1
Superrefractive	<-39.2	<118	>4/3
Ducting Threshold	-157	0	∞
Ducting	<-157	<0	∞

Variations in k-value can cause significant degradation of line-of-sight (LOS) paths. A temporary subrefraction due to k variation has the same effect as raising the height of potentially obstructing obstacles. Normally k values are higher in hot and wet climates, but lower in dry climates. High k variability occurs in certain climates such as tropical continental Africa (Owonubi, 1982). In practice LOS links must be designed to accommodate a range of k variability and ensure at least 60% Fresnel zone clearance at the least value of k expected. It is also better to plan a link based on path-averaged k exceeded for 99.9% of time. The local value of k is usually preferable for radiometeorological work as it gives results that are more reliable. Kolawole (1981) obtained the mean values of k in Nigeria for Dry and Wet Seasons. Terrestrial LOS link reliability however depends also on other factors such as accurate determination of the scale height.

Scale Height, H

Decrease of N in the troposphere due variations in P, T, and e may be represented by an exponential model given by (Craig, 1996)

$$N = N_s \exp(-\frac{z}{H}) \qquad (8)$$

where N_s is the surface value of refractivity, z is the height in kilometres above the surface and H is the scale height defined as the height at which the upward decrease of the refractivity reaches e^{-1} of the surface value N_s. The refractivity change between the surface and height z is

$$\Delta N = N(z) - N_s = N_s \exp\left(-\frac{z}{H}\right) - N_s$$

or

$$\mathrm{In}(N_s + \Delta N) = \mathrm{In}\, N_s + \mathrm{In}\, \exp(-bz) \qquad (9)$$

where b=1/H. Thus, from refractivity gradient N in the first kilometre of height (z=1)

$$b = \mathrm{In}\left(\frac{N_s}{N_s + \Delta N}\right) \qquad (10)$$

Equation (10) readily gives the scale height H=1/b in terms of N_s and ΔN which is normally used for refractivity gradient within the first kilometre of height. H is associated with the vertical distribution of atmospheric gases and can also be empirically derived from available vertical profiles of N. It is an important radiometeorological parameter in determination refractivity-reduced-to-sea-level N_0 which gives the station refractivity values 4-5 times more accurate than N_s values.

Higher values of H (~8 km) are associated with dry atmosphere while lower values (~6.5 km) are associated with saturated atmosphere. H=7 km is accepted as a compromise value for different atmospheric conditions. However, more appropriate H-values derived from prevailing local conditions are generally preferable. Kolawole (1980) obtained average value of H= 7.0 km for Nigeria.

Data and Method of Analysis

Daily summaries of radiosonde observations on air pressure P (hPa), air temperature T (K) and water vapour pressure e (hPa) from two Nigerian meteorological stations of Lagos (06° 27'N, 03° 24') and Kano (12°03'N, 08°32'E) for the years 1991-1999 and Niamey (13°29'N, 02° 10'E) in Niger Republic for the years 1994-1999 are used with equation (2) to obtain vertical profiles of radio refractivity N. The scale height H is empirically determined from the N versus Height curve of each profile. Derived daily H-values are further statistically analyzed to obtain the monthly medians and corresponding maximum and minimum values for the three stations. Secondly, derived daily vertical N-profiles are used to compute refractivity gradient dN/dZ (between available isobaric levels up to 300 hPa level), which are used with equation (7) to obtain corresponding daily vertical profiles of k. The k-values obtained are statistically analyzed to obtain the monthly maximum, minimum and median values for each of the stations. Values of H and k derived for the stations are also compared to explore their seasonality and climatic tendencies.

Results and Discussion

Sample daily refractivity profiles derived from eqn. (2) are shown in Fig. 1 for a typical Dry Season month (January) and for a typical Wet Season month (July) in the three climatic regions. The Figure shows that refractivity values are generally higher in Wet Season, although the difference between the two seasons is reduced in Lagos compared to Kano and Niamey. Progressive decrease of N values from

Lagos through Kano to Niamey is also observed, with Dry Season values for Kano and Niamey relatively close, peaking at 250-300 N-units, as for Wet Season values for Kano and Lagos with peaks around 350-400 N-units. Similar observations were also made by Kolawole (1981) and may largely be attributed to the north-south migration of the Inter-Tropical Discontinuity (ITD), as further explained below. The variability of the radiometeorological parameters (k and H) presented above also derives directly or

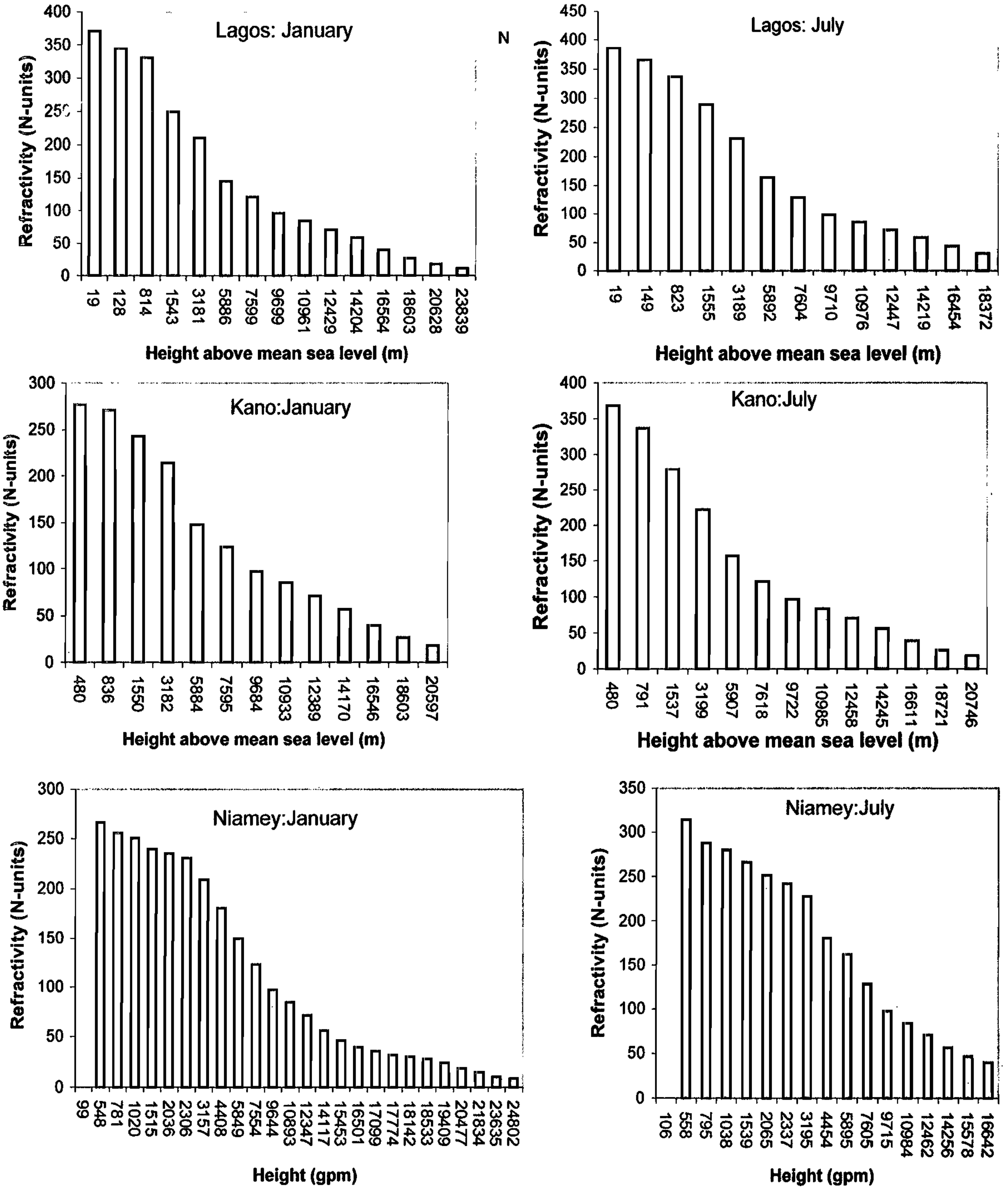


Fig. 1: Typical daily vertical refractivity profiles for Lagos, Kano and Niamey

variability of k

Indirectly from the observed vertical N - profiles.

Values of effective Earth radius factor obtained in the three stations for the months of January and September are shown in Figure 2, while monthly median values of k derived from surface refractivity gradients are shown in Figure 3. These Figures show that:

∞

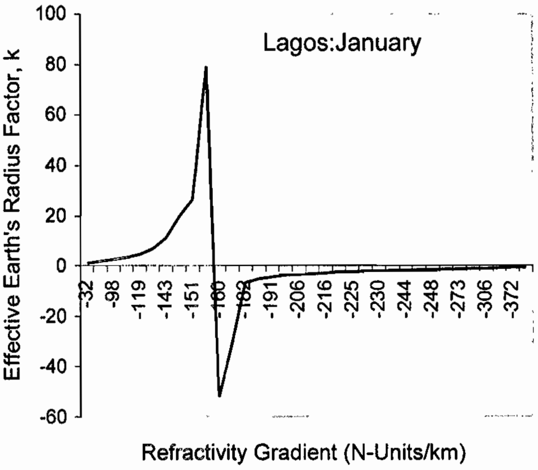
i. In Lagos, k-value rises from 1.3 to 80 before crossing the ducting threshold to settle at -6.5-0.6 in Dry Season; in the Wet Season the values are between 0.4-12.4 before the ducting values which lie in the range -8.5 to -0.4. Thus, Lagos is characterized by ducting k values in both seasons with greater variation and potential for subrefraction in Wet Season.

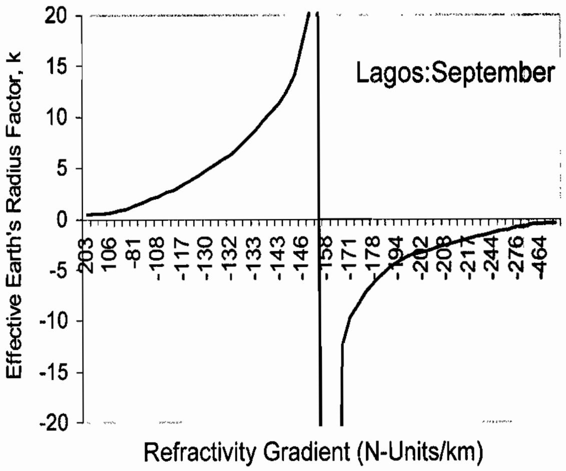
ii. In Kano, values of k are in the range 1.07-1.4 in Dry Season, or 1.3-4.0 in the Wet Season. A few ducting values are observed. Subrefractive k-values are more probable in Dry Season than in Wet Season. These values compare favourably with 1.12-1.38 and 1.30-1.50 obtained for Nigeria by Kolawole (1981), for Dry Season and Wet Season respectively.

iii. Subrefractive k values are probable in both seasons in Niamey, with a range of 0.7-1.4 in Dry Season and 0.9-5.0 in Wet Season. A few negative (ducting) values also occur in the Wet Season.

iv. There is a seasonal tendency of higher monthly k-values in Wet Season compared to Dry Season, with a corresponding reverse trend of surface refractivity gradients (Fig. 3). Substantial climatic trend is also noticeable for both k and dN/dZ values, except for the months of August and September in Lagos.

v. Monthly median k-values vary between -2.0 and 22.0 in Lagos, between 1.0 and 1.5 in Kano and between 1.0 and 2.0 in Niamey. Corresponding dN/dZ values are 137-270 N-units/km in Lagos, 12-85 N-Units/km in Kano and 21-53 N-units/km in Niamey.





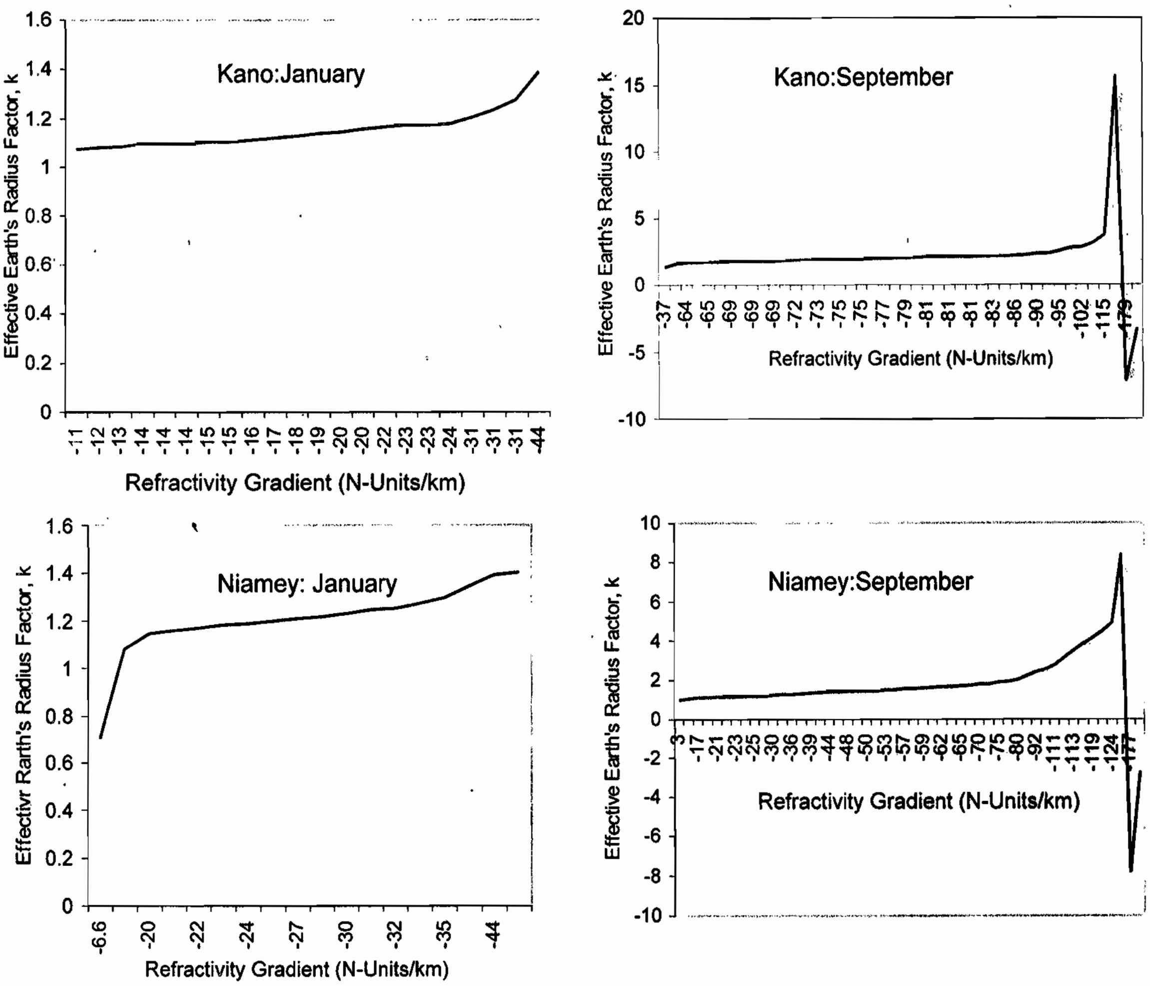


Fig. 2 :Refractivity gradients and corresponding earth's radius factor for January (typical dry season month) and September (typical rainy season month) in Lagos, Kano and Niamey.

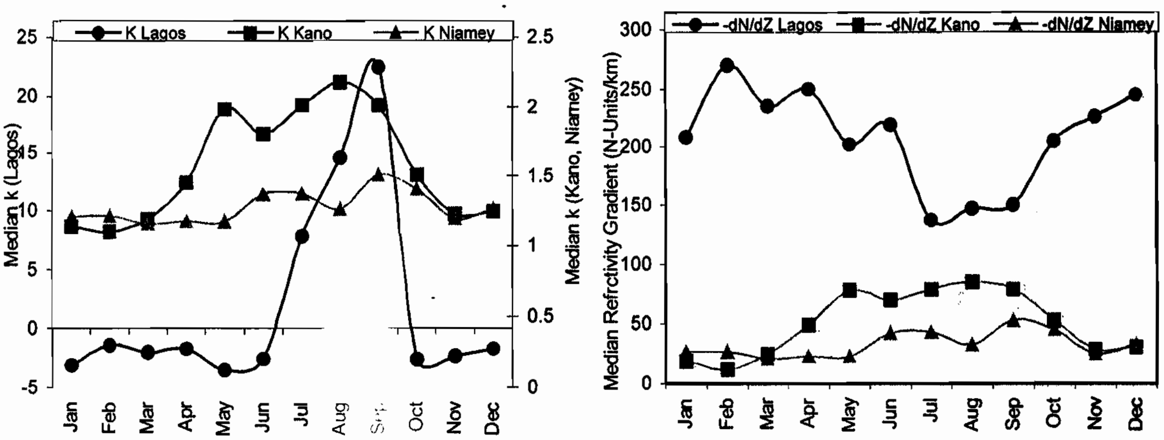


Fig.3 : Monthly median surface refractivity gradients and corresponding k values for Lagos, Kano and Niamey.

Variability of H

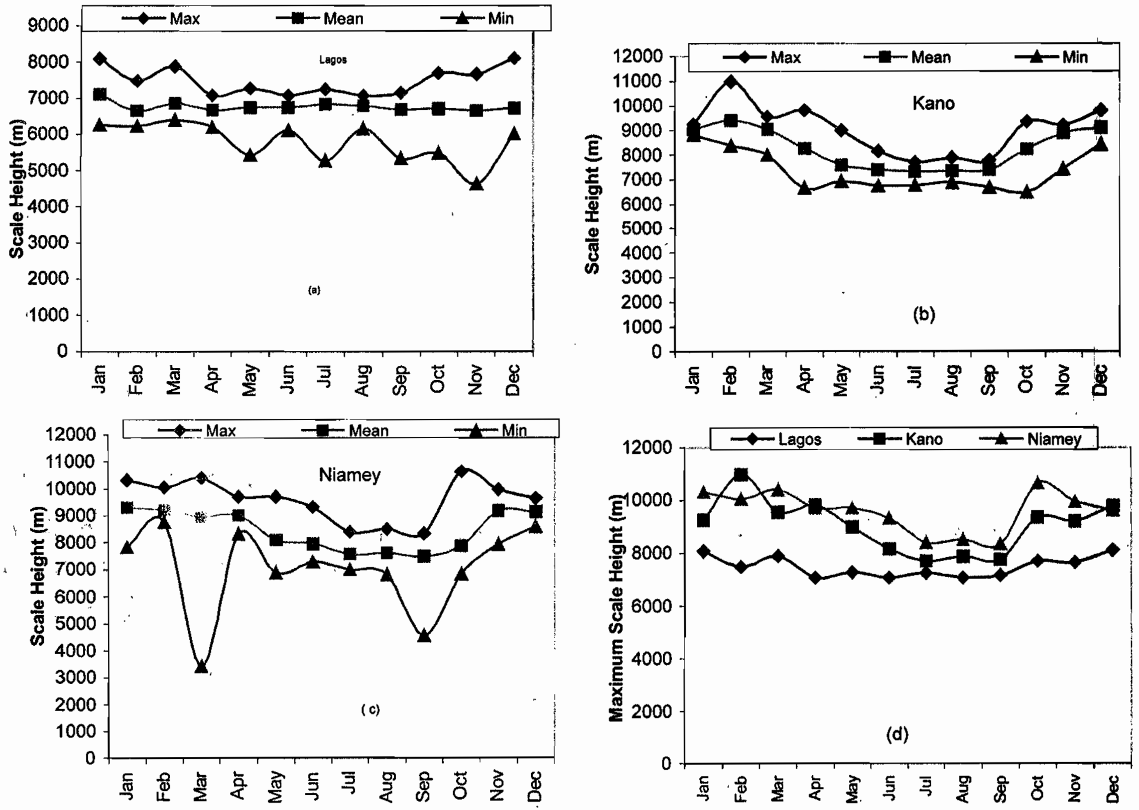
The seasonal variability of scale height H is presented in Figure 4 a-c while the climatic tendency is shown in 4d-f . The Figures shows that:

i. Maximum value of H is ~7.0 km between April and September in Lagos; and may reach 8.0 km in Dry Season months (Fig. 4a). Mean H values are between 6.0 and 7.0 km with no significant seasonality observed. Minimum values are in the range 4.0-6.0 km and are more variable in the Wet Season.

ii. In Kano, maximum H-values are between 7.0 and 11.0 km with lower values in Wet Season (Fig. 4b). Mean H values are between 7.0 and 9.0 km with lower values in Wet Season; while minimum values are 6.0-9.0 km, also with similar seasonal tendency.

iii. In Niamey the maximum, mean and minimum H-values are respectively 8.0-11.0 km, 7.0-9.0 km and 3.0-9.0 km with more variability in seasonal transition months of February-March-April and August-September-October.

iv. Generally maximum values are highest in Niamey, followed by Kano and Lagos respectively, reflecting a climatic trend (Fig. 4d). Least mean H-values are observed in Lagos while values for Kano and Niamey are relatively close with strong seasonality (Fig. 4e). Some degree of climatic influence is also seen between minimum H-values in Lagos and Kano; but no definite trend is observed either between Niamey and Lagos or between Niamey and Kano (Fig. 4f).



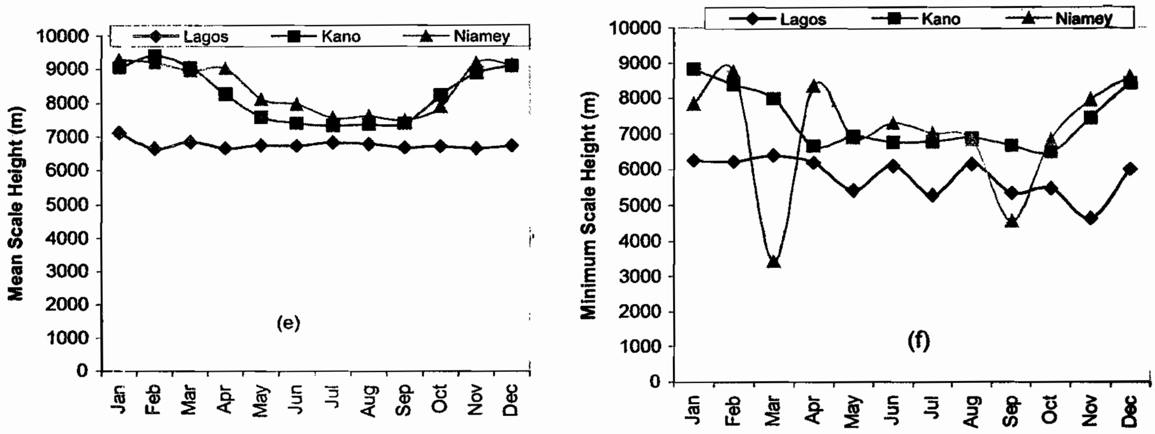


Fig. 4: Seasonal and climatic variation of radio refractivity scale height in Lagos, Kano and Niamey.

Summary

The effective Earth radius factor k and the scale height H are two important radiometeorological factors much in use for planning terrestrial radio links. In particular reliability of terrestrial line-of-sight links is ensured if clearance of first Fresnel zone is based on the least value of path-averaged k which increases with length of the link (Boithias and Battesti, 1967). Although it is recommended to use a lower limit of k=1 for a wet climate and k=0.6 for a desert climate (Picquenard, 1974; Hall, 1979), this study shows that considerable variability of both k and H may be expected in a tropical environment, with significant seasonal and climatic influences. In particular, negative ducting k-values occur for a good percentage of the time in a tropical maritime environment like Lagos. Values of k and H observed in Lagos, Kano and Niamey clearly reflect seasonal and climatic differences, which may largely be attributed to the meteorological north-south migration of the Inter-Tropical Discontinuity (ITD), the Sub-Tropical High pressure belt and the increasing northward influences of the Sahara Desert. A close study of the refractivity profiles and measured radiometeorological parameters shows that dN/dZ variations are strongly tied to changes in mixing ratio, humidity lapse rates and temperature inversions. However, dN/dZ variations are generally well behaved above the 300 hPa level (~9 km above the surface). Acknowledging the spatial and temporal limitations of radiosonde data used in this study, the need to obtain long-term values of k and H through improved and state-of the-art methods of determining refractivity gradients such as reported by Falodun and Kolawole (2006) cannot be over stressed.

References

Bean, B.R., Cahoon, B.A., Samson, C.A., and Thayer, G.D. (1967): A World Atlas of Atmospheric Radio Refractivity, (ESSA Monograph No. 1, [Access No. AD648-805], NTIS, Springfield, VA, USA

Bean, B. R., Dutton, E. J. (1968): Radio Meteorology. Dover, 1-22

Boithias, L. and Battesti, J. (1967): Protection Contre les évanouissements sur les faisceaux Hertziens en visibilité (Protection against Fading on Line-of-Sight Radio Relay Systems), Ann. Telecommun., 22, 230-242

Craig, K.H. (1996): Clear-Air Characteristics of the Troposphere. In Hall *et al* (Eds): Propagation of Radio Waves. IEE, London, 105-130

Falodun, S.E. and Kolawole, L.B. (2006): VHF and Microwave Propagation in Tropical Environment: (A Case Study of Akure, South-Western Nigeria), The Nigerian Journal of Space Research, Vol. 2, 95-104

Hall, M. P. M. (1979): Effects of the Troposphere on Radio Communications. IEEE Peter Peregrinus Ltd., London and New York.

IEEE Std211 (1997): IEEE Standard Definition of Terms for Radio Wave Propagation. The Institute of Electrical and Electronics Engineers, Inc. New York, USA.

Kolawole, L.B. (1980): Climatological Variations of Surface Radio Refractivity in Nigeria. Bull. Nig. Inst. Phys. 4, 97-117.

Kolawole, L. B. (1981): Vertical Profiles of Refractivity over Nigeria. Journ. West Africa Science Association, Vol. 28, 41 59.

Owonubi, J.J. (1982): The Tropospheric Radio Refractivity over Africa. M.Sc Thesis, Department of Physics, University of Ife, Ile-Ife, Nigeria.

Piquenard, A. (1974): Radio Wave Propagation, Macmillan Press Ltd., London

Sizun, H. (2005): Radio Propagation for Telecommunication Applications. Springer-Verlag, Berlin Heidelberg, Germany.

Smith, E.K. and S. Weintraub (1953): The Constants in the equation for Atmospheric Refractive Index at Radio Frequencies, Proc. IRE 41, 1035-1037.