UNSTEADY FREE CONVECTION FLOW PAST A POROUS PLATE UNDER THERMAL AND MASS CONCENTRATION GRADIENT FOR LARGE SUCTION.

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Abstract

Transient free convection flow past a semi-infinite vertical plate has been considered under temperature and mass concentration gradients in a chemically dissociate fluid. It is found that the flow rate decrease as the porosity parameter increases, with the decrease being more rapid as increase. A flow reversal is also obtained for larger value of free convection parameter Gand G. The mass concentration also falls very rapidly from the plate.

Keywords: *Convection, large suction, mass concentration, semi-infinite and dissociative Fluid.*

Introduction

Flow through porous media is common in nature and has many engineering and scientific applications, including cosmic and geophysical fluid dynamics.

Transient free convection flow past a semi-infinite vertical plate has been investigated by many workers, among whom are Chung and Anderson (1961), Gebhart (1961), Schetz and Echhorn(1962),(Goldstein and Briggs 1964), (Singh and Soundalgekar 1990).Also Jhaet al (1991) studied the transient flow past a vertical plate with variable surface temperature. Tay and Opara (1996) have considered the transient flow past a vertical porous plate under large suction temperature gradient.

Similarly Opara (2007) has extended the work to free convection boundary layer flow with suction in a saturated porous vertical cylindrical tube by applying a non-linear ordinary differential equation of small Reynolds number where the wall temperature is non-uniform. A marginal increase in suction Reynolds number below the initial value R_C was observed for a decrease in skin friction and a slight increase in the rate of heat transfer in the cylindrical walls.

Since thermal buoyancy free convection fluid flow plays a significant role in heat transfer studies when the flow velocity is relatively small and the temperature difference between the surface and the free stream is relatively large (Opara,2007).We seek to analyze the unsteady free convection flow past a porous plate under thermal and mass concentration gradients for large suction.

The present study incorporates mass concentration gradient in a chemical dissociative fluid. In section 2, the governing equations are set up and in section 3 the leading solution and the first order approximations are obtained.

Governing Equations

We consider the unsteady flow of a simple chemically dissociating fluid (A, A) past a vertical semi-infinite porous plate moving in its own plane with constant velocity U_0 in the x-direction. The plate is maintained in at temperature Tw>>1 and fluid concentration

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Such dissociate fluid has been studied by (Wollkind and Frisch, 1971),(Badzil and Frisch,1971. For example, nitrogen gas can dissociate at the gas-solid interface of a space craft as N_2<->2N.

We choose the coordinates such that x-axis is vertically upward along the plate and the y-axis is normal to the plate. The velocity components are (u,v) in the (x,y) directions respectively. Since the flow is direct along the plate, all physical quantities are functions of y and time t only Jha *et al.*, (1991), Singh and Soundalgekar, 1990). Within the Bousinnesg approximation, the governing equations are

$$\frac{\partial v}{\partial y}=0 \qquad (1)$$

$$\frac{\partial u}{\partial t}+v\frac{\partial u}{\partial y}=v\frac{\partial^2 u}{\partial y^2}-\frac{v}{K}u+g\beta(T-T_\infty)+g\beta_c(c-c_\infty) \qquad (2)$$

$$\frac{\partial}{\partial t}(T-T_\infty)+v\frac{\partial}{\partial y}(T-T_\infty)=\kappa\frac{\partial^2}{\partial y^2}(T-T_\infty)+\frac{k_rQ}{\rho_\infty c_p}(c-c_\infty) \qquad (3)$$

$$\frac{\partial}{\partial t}(c-c_\infty)+v\frac{\partial}{\partial y}(c-c_\infty)=D_m\frac{\partial^2}{\partial y^2}(c-c_\infty)+k_r(c-c_\infty) \qquad (4)$$

where Q is the bond dissociation energy that is released when a bond is broken; K_r is temperature independent rate constant; v is kinematics viscosity; k is thermal diffusivity; β, β_c thermal and mass diffusivities, respectively; K is the permeability of the porous plate; D_m is the coefficient of mass diffusivity; denotes ambient conditions far away from the plate, g is the acceleration due to gravity ρ and the density.

$$\frac{\partial v}{\partial y}=0$$

By eqn (1), v is either a constant or a function of time. We follow Hasimoto (1957) and assume a time dependent suction or blowing at the plate.

The boundary and initial conditions are

$u = 0$; $v = 0$; $T= T_\infty$; $c = c_\infty$ for all y and $t\leq 0$.

For $t\geq 0$

$u = U_0$; $v = V_w(t)$; $T = T_w$; $c = c_w$ at $y = 0$

$u\to 0$ $T\to T_\infty$; $c\to c_\infty$ as $y\to\infty$

We now introduce similarity variables and dimensionless quantities as follows:

$$\eta=y(4vt)^{-\frac{1}{2}};\quad u=U_0f'(\eta);$$

$$v=-f_w\left(\frac{v}{t}\right)^{\frac{1}{2}}$$

$$\Theta(\eta)=\frac{T-T_\infty}{T_w-T_\infty};\qquad \varphi(\eta)=\frac{c-c_\infty}{c_w-c_\infty};$$

$$\sigma=\frac{4vt}{Kf_w^2}$$

$$G_r=\frac{4tg\beta(T_w-T_\infty)}{U_0f_w^2};$$

$$G_c=\frac{4tg\beta_c(c_w-c_\infty)}{U_0f_w^2}$$

$$\xi^2=\frac{4tk_rQ(c_w-c_\infty)}{\rho_\infty c_pf_w^2};$$

$$\chi^2=\frac{4tk_r(c_w-c_\infty)}{f_w^2}$$

$$P_r=\frac{v}{\kappa};\qquad S_c=\frac{v}{D_m} \qquad (6)$$

f_w is suction (injection) parameter. For large suction (injection), f_w takes on large (negative) values. Eqns (2) to (4) then becomes With boundary conditions

$$f'''+2(\eta+f_w)f''-f_w^2\sigma f'+f_w^2(G_r\theta+G_c\varphi)=0 \qquad (7)$$

$$\frac{1}{P_r}\Theta''+2(\eta+f_w)\Theta'-f_w^2\xi^2\varphi=0 \qquad (8)$$

$$\frac{1}{S_c}\varphi''+2(\eta+f_w)\varphi'-f_w^2\chi^2\varphi=0 \qquad (9)$$

$f(o)=f_w;\ f'(o)=1;\ f(\infty)=0$ at t=0
$(-)(o)=1=\varphi(o);(-)(\infty)=0=\varphi(\infty)$; at t=0.

(10)

Following Bestman(1990), we introduce new variables and functions

$$\zeta=\eta f_w;\ f(\eta)=f_w F(\zeta);\ \Theta(\eta)=\Theta(\zeta);$$
$$\varphi(\eta)=\Phi(\zeta);\quad \varepsilon=\frac{1}{f_w^2}$$

(11)

Then we obtain

$$F'''+2(\varepsilon\zeta+1)F''-\sigma F'+\varepsilon(G_r\Theta+G_c\Phi)=0$$

(12)

$$\frac{1}{P_r}\Theta''+2(\varepsilon\zeta+1)\Theta'-\xi^2\Phi=0$$

(13)

$$\frac{1}{S_c}\Phi''+2(\varepsilon\zeta+1)\Phi'-\chi^2\Phi=0$$

(14)

The boundary conditions are now

$$F(o)=1;F'(o)=\varepsilon;\quad F'(\infty)=0;$$
$$\Theta(o)=1=\Phi(o);\Theta(\infty)=0=\Phi(\infty)$$

(15)

For large suction (or injection), ε>>1 and to order unity, we can expand the functions in a regular perturbation series in ε, viz

$$F=1+\varepsilon F^{(1)}(\zeta)+$$
$$\Theta=\Theta^{(0)}(\zeta)+\varepsilon(\Theta)^{(1)}(\zeta)+$$
$$\Phi=\Phi^{(0)}(\zeta)+\varepsilon(\Phi)^{(1)}(\zeta)+$$

(16)

We then obtain the following sets of equations

$$\frac{1}{P_r}\Theta^{(0)''}+2\Theta^{(0)'}-\zeta^2\Phi^{(0)}=0$$

(17)

$$\frac{1}{S_c}\Phi^{(0)''}+2\Phi^{(0)'}-\chi^2\Phi^{(0)}=0$$

(18)

with

$$\Phi^{(0)}(o)=1=\Phi^{(0)}(o);$$
$$\Phi^{(0)}(o)=0=\Phi^{(0)}(\infty);$$

For the leading solutions and

$$F^{(1)''}+2F^{(1)''}-\sigma F^{(1)'}=-G_r\Theta^{(0)}-G_c\Phi^{(0)}$$

(19)

$$\frac{1}{P_r}\Theta^{(0)''}+2\Theta^{(0)'}=-2\zeta\Phi^{(0)'}+\zeta^2\Phi^{(0)}$$

(20)

$$\frac{1}{S_c}\Phi^{(0)''}+2\Phi^{(0)'}-\chi^2\Phi^{(1)}=-2\zeta\Phi^{(0)'}$$

(21)

$$F^{(1)}(o)=0;F^{(1)'}(0)=1;F^{(1)'}(\infty)=0;$$
$$\Theta^{(1)}(o)=0=\Phi^{(1)}(o);$$
$$\Theta^{(1)}(\infty)=0=\Phi^{(1)}(\infty)$$

Solutions

We first solve eqn (18) to obtain

$$\Phi^{(0)}=e^{-\lambda\zeta};\quad \lambda=S_c+\sqrt{S_c^2+\chi^2S_c}$$

(22)

Using this in eqn (17), we find

$$\Theta^{(0)}=\frac{1}{\lambda(\lambda-1P_r)}\left[e^{-\lambda\zeta}+\left(\lambda^2-2\lambda P_r-\zeta^2\right)e^{2P_r\zeta}\right]$$

(23)

Similarly, we obtain the first order corrections as

$$\Phi^{(1)}=\frac{2\lambda\zeta}{\lambda^2-2S_c\lambda-\chi^2}e^{-\lambda\zeta}+\frac{4\lambda(\lambda-S_c)}{\left(\lambda^2-2S_c\lambda-\chi^2\right)^2}\left(e^{-}\right.$$

(24)

$$F^{(1)}=\frac{1}{\delta_2}\left(1-\frac{G}{\lambda^2-2\lambda-\sigma}-\frac{bG_r}{4P_r-4P_r-\sigma}\right)\left(1-e^{-\delta_2\zeta}\right)+$$
$$\frac{G}{\lambda\left(\lambda^2-2\lambda-\sigma\right)}\left(1-e^{-\lambda\zeta}\right)+\frac{bG_r}{2P_r\left(4P_r^2-4P_r-\sigma\right.}$$

(25)

where

$$\delta_2=1+\sqrt{1+\sigma};\quad G=aG_r+G_c;$$

$$a=\frac{\zeta^2}{\lambda(\lambda-2P_r)};b=\frac{\lambda^2-2\lambda P_r-\zeta^2}{\lambda(\lambda-2P_r)}$$

(26)

$$\Theta^{(1)} = \frac{2P_r}{\lambda^2(\lambda-2P_r)^2}\left[\zeta e^{\lambda\zeta}+2(\lambda-P_r)\frac{e^{\lambda\zeta}-e^{(\lambda-2P_r)\zeta}}{\lambda(\lambda-2P_r)}\right]$$

$$-\frac{2P_r(\lambda^2-2P_r\lambda-1)}{\lambda(\lambda-2P_r)}e^{-2P_r\zeta}\left(\frac{\zeta^2}{2}+\frac{\zeta}{2P_r}\right)$$

$$+\frac{2P_r\lambda\zeta^2}{\lambda(\lambda-2P_r)(\lambda^2-2S_c\lambda-\chi^2)}\left[\zeta e^{\lambda\zeta}+2(\lambda-P_r)\frac{e^{\lambda\zeta}-e^{(\lambda-2P_r)\zeta}}{\lambda(\lambda-2P_r)}\right]$$

$$+\frac{4\lambda(\lambda-S_c)P_r\zeta^2}{(\lambda^2-2S_c\lambda-\chi^2)^2}\left[\frac{e^{\lambda\zeta}-e^{(\lambda-2P_r)\zeta}}{\lambda(\lambda-2P_r)}-\frac{e^{(\delta_2-2P_r)\zeta}-e^{\delta_2\zeta}}{\delta_2(\delta_2-2P_r)}\right] \quad (27)$$

Heat Transfer Coefficient and Skin Friction

$$Q_w = -k_B\frac{\partial T}{\partial y}\bigg|_{y=0} = \frac{-k_B}{2\sqrt{\upsilon t}}(T_W-T_\infty)\frac{\partial\Theta}{\partial\eta}\bigg|_{\eta=0} \quad (28)$$

where k_B is Boltzman's constant.We introduce the Nusselt number, Nu as

$$Nu = \frac{1}{U_o}\frac{Q\upsilon}{k_B(T_W-T_\infty)} = -\frac{1}{2U_o}\sqrt{\frac{\upsilon}{t}}\frac{\partial\Theta}{\partial\eta}\bigg|_{\eta=0} \quad (29)$$

Defining the unsteady Reynold number Re by:

$$\mathrm{Re} = U_o^2 t/\upsilon \quad (30)$$

we get

$$Nu = -\frac{1}{2}(\mathrm{Re})^{\frac{1}{2}}\frac{\partial\theta}{\partial\eta}\bigg|_{\eta=0} = -\frac{1}{2}(\mathrm{Re})^{\frac{1}{2}}f_w\frac{\partial\Theta}{\partial\zeta}$$

Hence the heat transfer coefficient is

$$h = 2Nu(\mathrm{Re})^{-\frac{1}{2}} = -\frac{\partial\theta}{\partial\eta}\bigg|_{\eta=0} = -f_w\frac{\partial\Theta}{\partial\zeta}$$

$$= f_w(\lambda-2h_1P_r)a_s - \frac{1}{f_w}(a_1-a_2P_r-a_3+a_4P_r^2) \quad (31)$$

where

$$h_1 = \lambda^2-2\lambda P_r-\zeta^2$$

$$a_1 = \frac{2P_r}{\lambda^2(\lambda-2P_r)^2}+\frac{2P_r\zeta^2}{(\lambda-2P_r)(\lambda^2-2\lambda S_c-\chi^2)}$$

$$a_2 = \frac{8P_r(\lambda-P_r)}{\lambda_3(\lambda-2P_r)^3}+\frac{8P_r(\lambda-P_r)\zeta^2}{\lambda(\lambda-2P_r)^2(\lambda^2-2\lambda S_c-\lambda^2)}$$

$$a_3 = \frac{\lambda^2-2\lambda P_r-1}{\lambda(\lambda-2P_r)}$$

$$a_4 = \frac{8\lambda(\lambda-S_c)\zeta^2}{\lambda^2-2\lambda S_c-\chi^2}\left\{\frac{1}{\delta_2(\delta_2-2P_r)}-\frac{1}{\lambda(\lambda-2P_r)}\right\}$$

Similarly, we can define the plate skin friction as τ by

$$\tau = -\mu\frac{\partial u}{\partial y}\bigg|_{y=0} = -\frac{\mu}{2}\frac{U_o}{\sqrt{\upsilon t}}f''(o)$$

and the skin friction coefficient

$$= \tfrac{1}{2}C_f(R_e)^{\frac{1}{2}} = -f''(o)$$

$$C_f = \frac{2\tau}{\rho U_o^2} = -\frac{2\upsilon}{U_o}\frac{1}{\sqrt{\upsilon t}}f''(o) \quad (32)$$

Thus

$$\frac{1}{2}C_f(\mathrm{Re})^{-\frac{1}{2}} = -f''(o) = -f_w^3F''(o)$$

$$= -f_w\left\{\frac{G}{\lambda^2-\lambda-\sigma}(\lambda-\delta_2)+\frac{bG}{4P_r^2-4P_r-\sigma}(2P_r-\delta_2)\right\} \quad (33)$$

Results and Discussion

We apply our analysis to air ($P_r = 0.71$) for three different values of porosity parameter, viz $\delta = 0.5$, and 1.0 and for two sets of the free convection parameters, that is, the local Grash of numbers for thermal and mass diffusion, G_r and G_c respectively. Table 1 shows that as the porosity increases, that is, as the permeability of the plate decreases, the flow rate $f^{(1)}$ decreases. For very low permeability, the flow rate drops sharply in the immediate vicinity of the plate. Table 2 shows that flow reversal occurs for large values of the free convection parameters for all three values of the porosity parameter.

Table 1: Skin friction for $G_r = G_c = 1$

f_w	$\frac{1}{2}C_f(\mathrm{Re})^{-\frac{1}{2}}$	
	$\sigma = 0.5$	$\sigma = 1.0$
± 3.0	∓ 2.0931	∓ 1.5446
± 4.0	∓ 2.7908	∓ 2.0596
± 5.0	∓ 3.4885	∓ 2.5745

Table 2: Skin friction for $G_r = G_c = 5$

f_w	$\frac{1}{2}C_f(\mathrm{Re})^{-\frac{1}{2}}$	
	$\sigma = 0.5$	$\sigma = 1.0$
± 3.0	∓ 10.2576	∓ 9.3918
± 4.0	∓ 13.6768	∓ 12.5224
± 5.0	∓ 17.0960	∓ 15.6530

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