BOUGUER CORRECTION DENSITY DETERMINED FROM FRACTAL ANALYSIS USING DATA FROM PART OF THE NIGERIAN BASEMENT COMPLEX

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(Submitted: 20 October, 2005: Accepted: 18 August, 2006)

Abstract

In this work, Bouguer density is determined using the fractal approach. This technique was applied to the gravity data of the Kwello area of the Basement Complex, north-western Nigeria. The density obtained using the fractal approach is 2500 kgm^{-3} which is lower than the conventional value of 2670 kgm^{-3} used for average crustal density. Further analysis showed that the range of the topographic features within the Kwello area supported by crust without isostatic compensation is not greater than 8.5 km.

Keywords: *fractal, density, dimension, spectra, Bouguer anomaly*

Introduction

A wide range of geophysical processes and rock properties has been described in fractal terms (Turcotte, 1992; Tatiana *et al.*, 2000; Sunmonu and Dimri, 2001; Bansal and Dimri 2005). A fractal is any entity that is scale invariant. This means that it looks similar at a greater variety of scales. See floor and mountain topography for example show fractal behaviour (Mandelbrot, 1983). These phenomena are characterized by a power-law distribution in that their power spectra **P** are proportional to some power (known as the scaling exponent) of frequency **f**. that is. $P \propto f^{\alpha}$. Plotting power versus frequency on log-log paper produces a straight line with slope α. Pilkington and Todoeschuck (1990) found scaling exponents of -0.7 to -1.7 for density, -0.8 to -0.9 for resistivity and -0.9 to -1.5 for porosity, based on well-log measurements. Gregotski *et al.*, (1991) analyzed the power spectra of aeromagnetic surveys at various scales and sampling intervals over the North American continents and found that a scaling exponent of approximately -3 occurs in all areas.

In order to calculate the Bouguer gravity anomaly the average density (Bouguer density) for the topography whose gravitational influence is to be removed must first be computed. The traditional approach is to estimate this density by minimizing the resulting correlation of the Bouguer gravity anomaly with the topography. The assumption is that the topography is supported by the lithosphere without deflection. However, as the size of the topographic anomalies increases, the lithosphere is deflected and the anomalous mass must be supported by isostatic compensation. In this work a suitable crustal density for the Kwello area of the Nigeria Basement Complex is derived using the fractal technique.

Density Determination using Fractal Technique

Fractals are generally treated quantitatively using spectral techniques. Considering a single valued function x(t) that is random but has a specified spectrum, can be either represented in the physical domain as x(t) or in the frequency domain in terms of the amplitude X(f,T) where f is frequency and T is period. The quantity X(f,T) is generally a complex number indicating the phase of the signal. The amplitude in the frequency domain, X(f,T), is obtained using the Fourier transform of x(t) in the interval $0 < t < T$. It is given by

$$X(f,T) = \int_0^T X(t) e^{2\pi i f t} dt \quad \text{where } i = \sqrt{-1} \quad (1)$$

The complementary equation relating x(t) to X(f,t) is the inverse Fourier transform

$$x(t)=\int_0^T X(f,T)e^{-2\pi i f t}df \qquad (2)$$

The quantity $|X(f,T)|^2 df$ is the contribution to the total energy of x(t) from those components with frequencies between f and f + df. The vertical bars in | X | refer to the absolute value of the complex quantity. The power is obtained by dividing by T. Therefore the spectral power density of x(t). in the limit T is defined as

$$S(f)=\frac{1}{T}|X(f,T)|^2 \qquad (3)$$

The product S(f)df is the power in the time series associated with the frequency range between f and f + df. Turcotte (1992) showed that for a time series to be fractal the power spectral density has a power law dependence of frequency;

$S(f) \propto f^{-\alpha}$ where α is the scaling exponent.(4)

For the two dimensional case we consider an N x N equally spaced data point in a square with linear size L. The N^2 data point are denoted by h(n,m) specifying the position in the x and y directions respectively.

The two dimensional discrete Fourier transform on the N^2 set of data points h(n,m) with N x N complex coefficients H(s,t) is given by

$$H(s,t)=\left(\frac{L}{N}\right)^2 \sum_{n=0}^{N-1}\sum_{m=o}^{N-1} h(m,n)e^{-\frac{2\pi i}{N}(sn+tm)} \qquad (5)$$

S = 0,1,2..........N-1
t = 0,1,2,.........N-1

Where s and t denotes the transform in the x and y direction respectively.

Each transform coefficient H(s,t) is assigned an equivalent radial number using the relation

$$r = (s^2 + t^2)^{1/2}$$

the two dimensional mean spectral density for each radial wave number k_j

$$S_j = \frac{1}{L^2 N_j}\sum_{n=1}^{N_j}|H(s,t)|^2 \qquad (6)$$

where S_j is the number of coefficient that satisfy the condition $j < r < j+1$ and the summation is carried over the coefficient H(s,t) in this range.

The dependence of the mean power spectral density on the radial wave number k_j for fractal distribution is given by Voss (1988) is

$$S_j \propto k_j^{-\alpha+1} \qquad (7)$$

Fractals can be identified and measured by making log-log plots with the physical phenomenon's attributes as one axis and the measurement scale on the other axis. If the physical phenomenon is fractal, it will plot as a straight line over a variety of scales and the slope is therefore proportional to the fractal dimension. Turcotte (1992) gave the relationship between α the scaling exponent and the fractal dimension as

$\alpha = 5 - 2D$ (for one dimensional case)
$\alpha = 7 - 2D$ (for two dimensional case)

For a two dimensional case the fractal dimension is therefore related to the scaling exponent as:

$$D = \frac{7-\alpha}{2} \qquad (8)$$

The fractal dimension can be obtained following the technique used by Thorarinsson and Magnusson (1990) and Mumtaz and Naci (2002), by first plotting the power spectra of the gravity data on a log-log scale and the scaling exponent α is determined from the slope of the line of best fit for the region where the scaling exists. The fractal dimension is then obtained using Equation 8. The fractal derived density is obtained by computing the Bouguer anomaly for varying values of density, and in each case, the fractal dimension is calculated. The computed fractal dimension is plotted as a function of density, and the density at which the fractal dimension is minimum is chosen as the required Bouguer density (Chapin, 1996).

Application to the Kwello Gravity Anomaly

The Kwello area of the Basement Complex of North-Western Nigeria lies between latitude 10.94° N to latitude 11.54° N and longitude 6.7° E to longitude 7.5° E. Figure 1. Two hundred and sixty seven gravity stations were taken at 2 km intervals using a La Coste and Romberg Model of gravimeter. Results of the gravity measurements showed that the area is characterised by negative

Bouguer anomaly values ranging in amplitude from 30 mGal to 58 mGal.

The data obtained were interpolated on to a regular gride of 16 x 21 at 2 km interval, and then Fourier transformed into frequency domain. The Fourier coefficients were squared and radially averaged to produce a power spectrum profile. Ten Bouguer gravity data sets were computed for densities ranging from 2.0 g/cm^3 to 3.0 g/cm^3 at 0.1 g/cm^3 intervals. The result of the power spectra for the first four sets of these data are shown in Figure 2, indicating the value of the scaling exponent α and the fractal dimension D. The fractal dimension obtained were plotted against the density variations and the result is shown in Figure 3 and it shows decreasing fractal dimension with increasing density. However, removal of a least-squares linear fit to the graph in Figure 3 yields a result that shows a unique density that minimizes the topographic effect (Figure 4). The curve in Figure 4 has a minimum at a density value of 2.50 ± 0.02 g/cm^3, which corresponds to the best density value to use for the Bouguer computation, and this is also different from 2.67 g/cm^3 which has been traditionally used. Akaolisa (1997) carried out density measurements of the rocks around the area of study (predominantly schist and granite) and obtained an average value of 2.53 g/cm^3, which is closer to 2.50 g/cm^3 obtained from the fractal technique. The power spectrum of the Bouguer gravity data computed with a density value of 2.50 g/cm^3 shown in Figure 5, and this indicates that the topographic features within the area of study supported by curst without isostatic compensation have horizontal dimensions not greater than 8.5 km. This is obtained by observing the region at which the plotted points begin to deviate from the linear trend, which is at a frequency of about -1.3 rad/km in Figure 5.

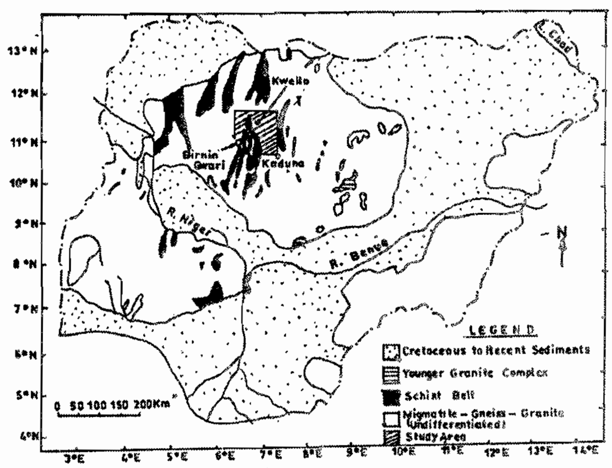

Fig. 1: Simplified geological map of Nigeria showing the study are

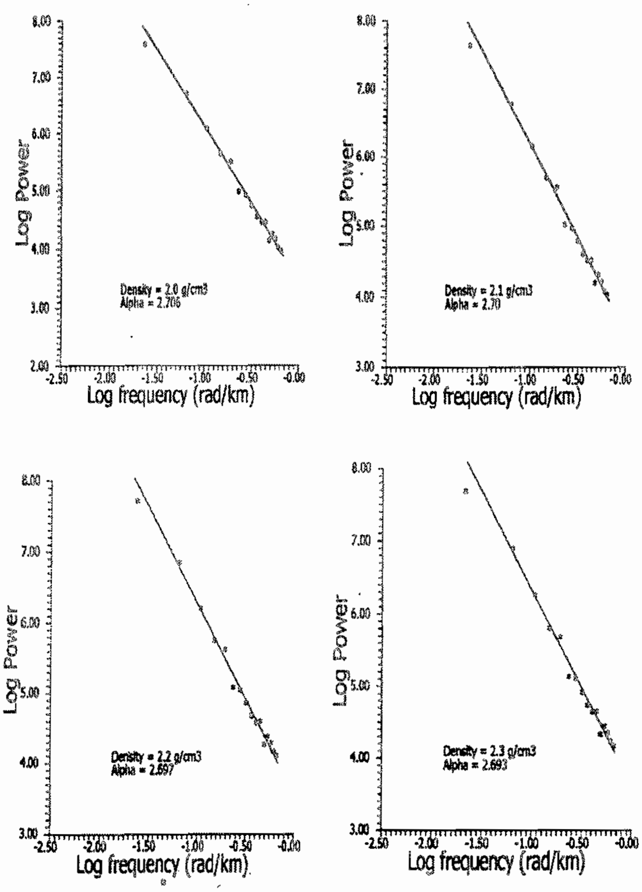

Fig. 2: Power spectra computed for the first four data sets of varying density values

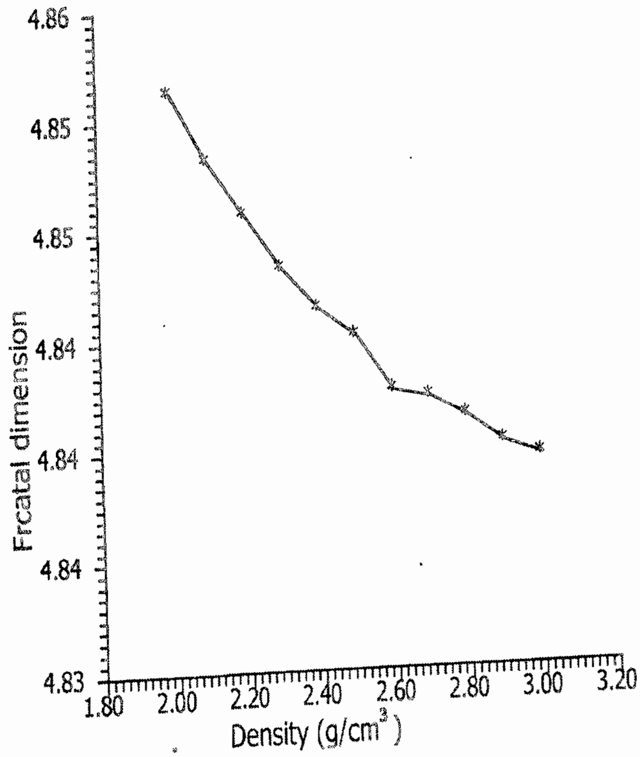

Fig. 3: A plot of fractal dimensions against density variations

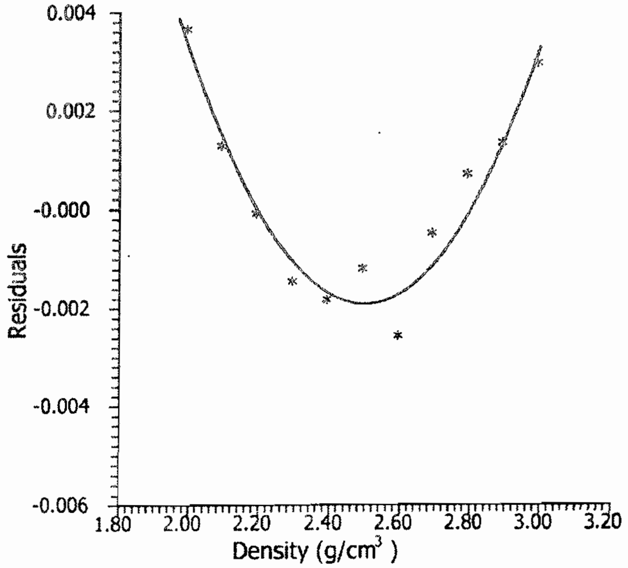

Fig. 4: A plot of the least square residuals fit of Fig. 3 against density variations

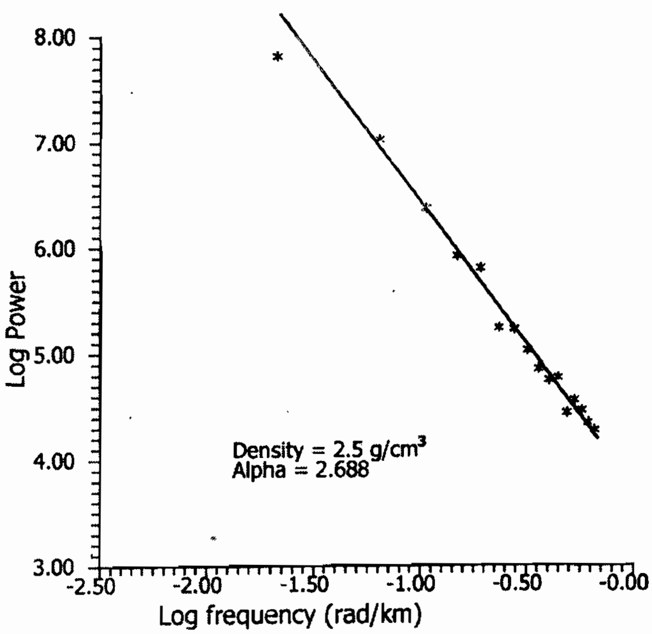

Fig. 5: The power spectrum of the Bouguer gravity data computed with a denisty value of 2.50g/cm^3

Conclusion

It has been a common practice in the process of calculating Bouguer gravity anomaly to use an average crustal density of 2.67 g/cm^3. This value can produce erroneously high or low Bouguer values in areas where the actual crustal density deviates substantially from this value. Application of the fractal technique for the determination of the crustal density at the Kwello area of the Nigeria Basement Complex produced a value of 2.50 ± 0.02 g/cm^3 as crustal density.

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