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STABILITY OF A BINARY MIXTURE OF A CHEMICALLY REACTING INCOMPRESSIBLE FLUID IN PLANE COUETTE FLOW

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Abstract

The stability of a binary mixture of a chemically reacting incompressible fluid in plane couette flow is considered with respect to a two-dimensional infinitesimal disturbance. The eighenvalues are found with the aid of microcomputer. Neutral stability curves for various Schmidt numbers at Reynolds numbers up to 150 are obtained by Gallagher method. It is found that the principle of the exchange of stabilities holds for this problem. Thus, using the basic orthogonal functions, the differential equations are reduced to a set of paired matrix equations with the conclusion that all fluids are unstable at different values of stability parameter $C_1 = 1.40 \times 10^4$ to 2.0×10^4 for wave number $(a) = 0.2$ to 2 (based on half-channel width and half the relative velocity of the plates).

Keywords; *Incompressible fluid, binary mixture, wave number, couette flow and stability parameter.*

1. Introduction

The study of the physics of the stability of couette flow of fluid has become the basis of many scientific and engineering applications. In two previous papers, Gallagher and Mercer (1962 and 1964) considered the problem of plane couette flow of a viscous and incompressible fluid with respect to infinitesimal disturbances. Chandrasekhar (1961) has compiled bibliographies of the couette flow problems.

The present paper investigates the stability of couette flow through a binary mixture of a chemically reacting incompressible fluid. This type of flow is of importance to the petroleum engineer who is concerned with the movement of oil, gas or waters through the reservoir of an oil or gas field. In section two the problem is formulated by assuming that the velocity of the disturbance, for example, in the transverse direction is obtained by a superposition of functions of the form $W(z)\exp\{(ia(x-\eta)\}$. The factor is the wave number and is taken to be real and positive.

In section three, the reductions of the differential equations to a set of algebraic equations are carried out. The effect of the chemical reaction rate constant on the stability parameter is discussed in section four using the d-method (Gallagher and Mercer, 1964).

2. Model Formulation

The geometric model of the flow consists of two axes 0_x, and 0_z, in the plane of flow sections of the plates forming the lines $z =$. We consider the plates to move with equal and opposite velocity parallel to the x axis and to be held at constant concentrations C_0 and C_1. It is easy to verify that in non-dimensional form the steady state solution is given as:

$$\bar{\omega}=0, \bar{V}=x, P'\tfrac{1}{2}GR^{-2}\left(z-\tfrac{1}{2}z^2\right), \bar{C}=\tfrac{1}{2}(1-z)(-1<z<1) \quad (1)$$

If we use v, 1, and $\delta C = C_1 - C_0$ as representing the velocity, length and concentration, respectively, and on applying the usual perturbation to velocity, pressure and concentration as given by:

$$\mathbf{W=W`+W(x, z, t), v = v` + v(x, z, t), p = p`+ p(x, z, t),}$$
$$\mathbf{C = C` + C(x, z, t)} \quad (2)$$

We then substitute these values in the general equations. If we neglect second-order terms and on separating the variables by the scheme

$$W=W(z)\exp\{ia(x-\eta t)\}, v=v(z)\exp\{ia(x-\eta t)\}, p=p(z)\exp\{ia(x-\eta t)\}$$
$$C=C(z)\exp\{ia(x-\eta t)\} \quad (3)$$

The equation governing motions in a binary mixture in a chemically reacting incompressible fluid are (Gallagher and Mercer, 1964)

$$(D^2 - a^2)^2 W = iaR\,\{z-\eta\}\,(D^2-a^2)\}W + k_r C/R \qquad (4)$$

$$\{D^2 - a^2\ iaRS\,\{z-\eta\}\}C\} = -\tfrac{1}{2}RSW \qquad (5)$$

where the following non-dimensional parameters are defined by

$$D \equiv \frac{d}{dz},\ k_r = \frac{g\alpha l^4 C}{\gamma},\ S = \frac{\gamma}{D_m},\ R = \frac{v_0 l}{\gamma}$$

W is the velocity whereas g is the gravitational acceleration and a is the wave number. In addition, C is the mass concentration of a depleting species, p is the pressure, S is the Schmidt number, is the coefficient of volume expansion of the fluid, D_m is the mass diffusivity, k_r is the chemical reaction rate constant and is the kinematics viscosity.

The stability or otherwise of the velocity component a (wave number) depends on the sign of the imaginary part of the time constant (η) which appears as the eigenvalue in the pair differential eqns (4) and (5) as well as depending on the wave number a. η also depends on R (Reynolds number), the Schmidt number and the reaction rate constant k_r. Although, it is difficult to maintain the concentration at a fixed value at a boundary, we assume the boundary conditions as

$$W = D_w = C = 0 \text{ at } z = 1 \qquad (6)$$

for a chemically reacting fluid. However, the criterion for the stability of the flow with respect to this particular mode is that if fη<0 the flow is stable, while if fη>0 the flow is unstable. If fη=0 the flow is said to be critically or neutrally stable. We find it convenient to transform the independent variable z by putting $z_1 = \tfrac{1}{2}(z+1)$ (7)

So that the arrange of integration is now (0,) and at the same time we defined C_1 by

$$C_1 = -\frac{\pi^2 C}{2RS}$$

On dropping the subscripts from z_1 and c_1, equation (1) can now be written as

$$(D^2 + X + imz)(D^2 + n)W = dC \qquad (9a)$$

$$(D^2 + X + iSmz)C = W \qquad (9b)$$

where

$$m\ a = \frac{-8aR}{\pi^2},\ n = \frac{-4a^2}{\pi^2},\ d = \frac{-32a^2}{\pi^6}Cl$$

where Cl is the stability parameter, while the new form of the eigenvalue is now

$$X = n - \tfrac{1}{2}\pi mi\,(1+\eta) \qquad (10)$$

3. Differential Equations: Reduction to a Set of Algebraic Equations.

Equation (9) can be replaced by a pair of matrix equations by assuming the expansions:

$$W(z) = \sum_{r=1}^{\infty} q_r\, Y_r \qquad (11a)$$

$$C(z) = \sum_{r=1}^{\infty} b_r\, Sin(rz) \qquad (11b)$$

where q_r and b_r may be complex. The basic orthogonal functions Y_k (k=1, 2, 3,…….) and their integrals have been fully described by Gallagher and Mercer (1962 and 1964). If eqn. (9a) is multiplied by Y_k and equation (9b) is multiplied by Sin (kz) and each is integrated over the interval (0,) we obtain after some calculations the equations;

$$\sum_{r=1}^{\infty}\{(-\xi_k^4 + x_c)I_{kr}^{(0)} - (X+n)I_{kr}^{(2)} + im(I_{kr}^{(1)} - J_{kr}^{(2)} + nJ_{kr}^{(0)})\}q_r = d\sum_{r=1}^{\infty} L_{ik}b_r \qquad (12a)$$

$$\sum_{r=1}^{\infty}\{-k^2\tfrac{\pi}{2}X\tfrac{\pi}{2})\delta_{kr} + iSm\varepsilon_{kr}\}b_r = \sum_{r=1}^{\infty} L_{ik}q_r \qquad (12b)$$

δ_{kr} is the Kronecker delta symbol. To ensure the convergence of the infinite determinant involved we divide equation (12a) by ξ_k^4 and equation (12a) by k^2. On employing matrix notation, the two sets of eqn (12) can now be written as

$$(\vec{A_1} + i\vec{A_2} - X'B)\vec{a} = d\vec{F}\,\vec{b} \qquad (13a)$$

$$(\vec{C_1} + i\vec{C_2} - X'\vec{D})\vec{b} = \vec{E}\,\vec{a} \qquad (13b)$$

The eigenvalue has been changed to so as to make the elements A_2 and C_2 zero wherever $(k+r)^2$ is even. $X' = X + \dfrac{i\pi m}{2}$ All the above matrices are real and are given by

$$\vec{A}_1 := \frac{\left(\xi_k^4 I_{kr}^{(0)} - n I_{kr}^{(2)}\right)}{\xi_k^4}; \; \overline{A}_2 = \frac{m\left(I_{kr}^{(0)} - J_{kr}^{(2)} + \frac{\pi}{2} I_{kr}^{(2)} + mn\left(J_{kr}^{(0)} - \frac{\pi}{2} I_{kr}^{(0)}\right)\right)I}{\xi_k^4} \quad (14)$$

$$\vec{B} = \frac{\left(I_{kr}^{(2)} - n I_{kr}^{(0)}\right)}{\xi_k^4}; \; \vec{C}_2 = Sm\left(\varepsilon_{kr} - \frac{\pi^2}{4}\delta_{kr}\right); \vec{D} = \frac{-\pi}{2}\left(\delta_{kr}/k^2\right),$$

In eqn. (13), we find that the matrix $\vec{A} + i\vec{A}_2$ is complex. Hence we rewrite eqn. (13) so that the only possible complex quantities are X', $\bar{a} \, and \, \bar{b}$

This technique was used successfully by Gallangher et al (1964). Thus, following this technique we let stand for the diagonal matrix in which the element in the (k,k)th position be 1 or i according as k is odd or even, and we let $\vec{F} = -\vec{T}^2 \cdot \vec{F}$ is of course a real matrix. Since $\vec{A}_1, \vec{C}_1, \vec{B}, \vec{D}, \vec{E}$ and $\vec{F}$ have zero elements when k+r is odd and A_2 and C_2 have zero elements when k+r is even the transformation $\vec{a} = \vec{T}\vec{P} \, and \, \vec{b} = \vec{T}\vec{Q}$.

Equation (13) then reduces to

$$\left(\vec{A} - X'\vec{B}\right)\vec{P} = d\vec{F}\vec{Q} \quad (15a)$$

$$\left(\vec{C} - X'\vec{D}\right)\vec{Q} = \vec{E}\vec{P} \quad (15b)$$

where

$$\vec{A} = \vec{A}_1 + \vec{F}\vec{A}_2 \; and \; \vec{C} = \vec{C}_1 + \vec{F}\vec{C}_2.$$

Equation (15) may be regarded as a coupled set of matrix equations in which A is the eigen value while b, c, and d are parameters. We now apply the d-method technique in the calculation of the neutral stability curves. This occurs when the eigenvalue corresponding to the neutral stability mode has the value zero. This determines the range of aR that could be considered. The results obtained by this method are shown in Figs (1 to 5).

4. Results and Discussion

The results obtained by the d-method are indicated in Figs (1 to 5). Figs 1 to 3 give several neutral stability curves for the Schmidt numbers s=0 and 1 (i.e the case of one gas mixed with another) and s=6 (such as aqueous solutions of electrolytes) respectively. The numerical values are used in plotting Fig (1to 5). The dotted line in each figure joins the minimum points of the neutral stability curve. These points were obtained using a (wave numbers) ranging from 0.2 to 2 at intervals of 0.2. The resulting minimum values of the stability parameters and the corresponding wave numbers (a) are plotted in Fig 4. In each of Figs (1 to 5) the

curve R=0 is almost the same, showing the curve for the Bernard problem (Opara et al, 1990). In this case the minimum stability parameter for a= 1.56. In addition, Fig. 5 shows neutral stability curves for the value of the Schmidt number 1. In these the quantity d is plotted against R for various values of wave number (a). For a = 2 the neutral stability curves corresponds to the first two modes which shows a likely form of unstable disturbance.

Finally, from Fig 5 it can be seen that the d-values are dependent on the stability parameter. Hence there are different values of the stability parameter for different fluids say one gas mixed in another or aqueous solutions of electrolytes. Thus, all fluids become unstable at different values of the stability parameter, C_1
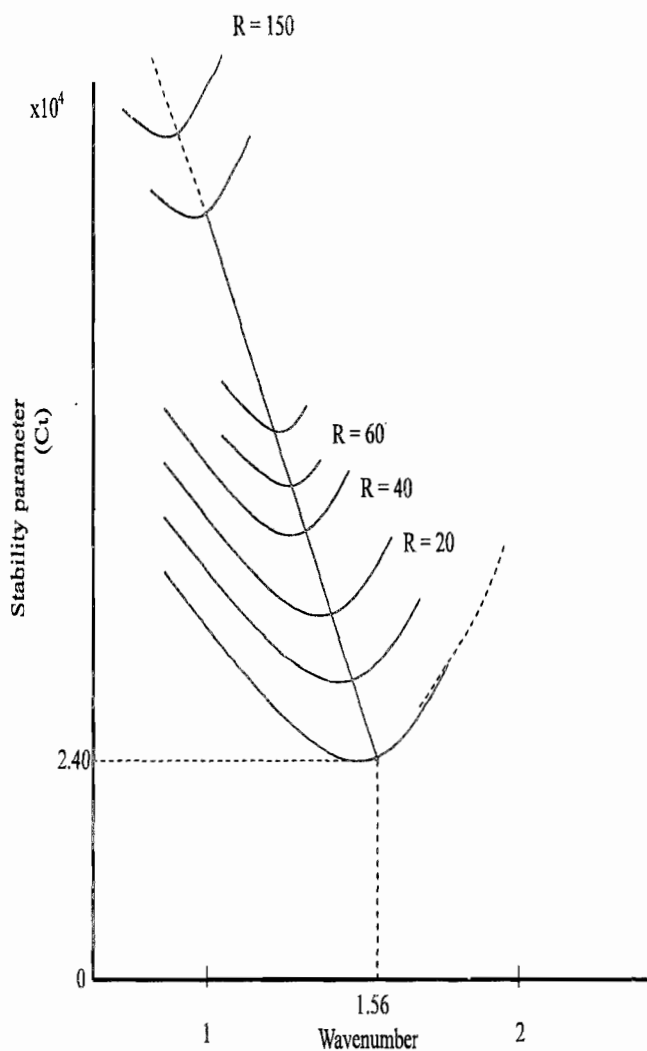

Fig. 1: Neutral stability curves for the case
S = 0, for various Reynolds numbers.
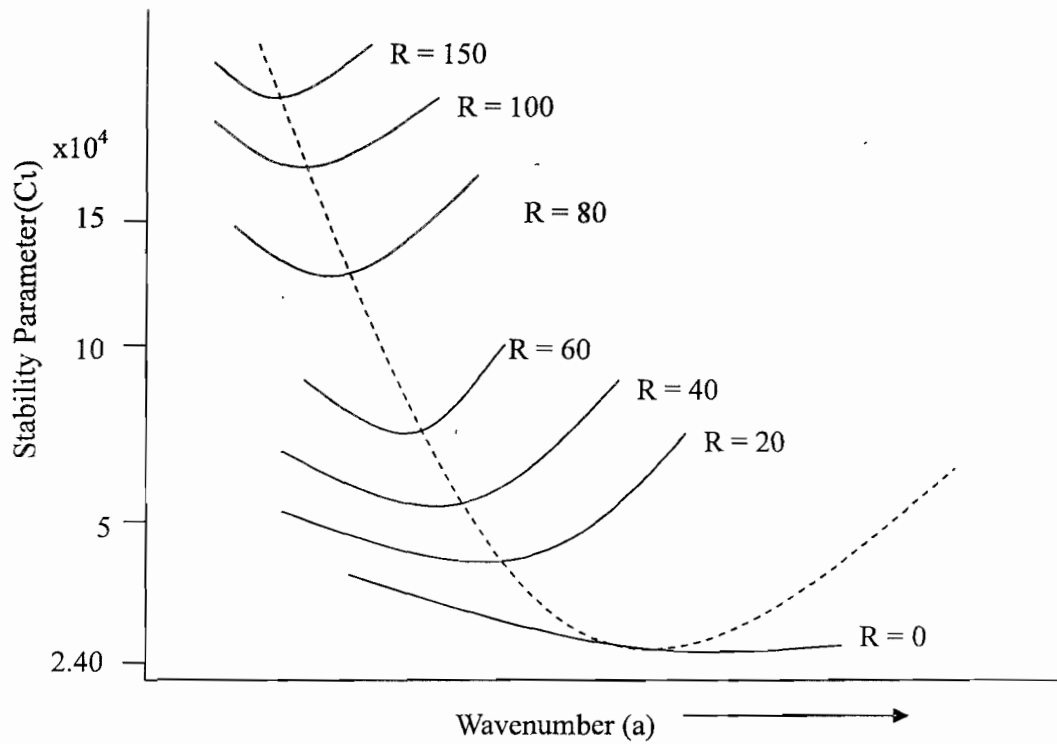----------------, Curve of minimum stability parameter (C_i).

Fig. 2:Stability curve for the case $S = 1$, for various Reynolds numbers
----------- shows curve minimum stability parameter.
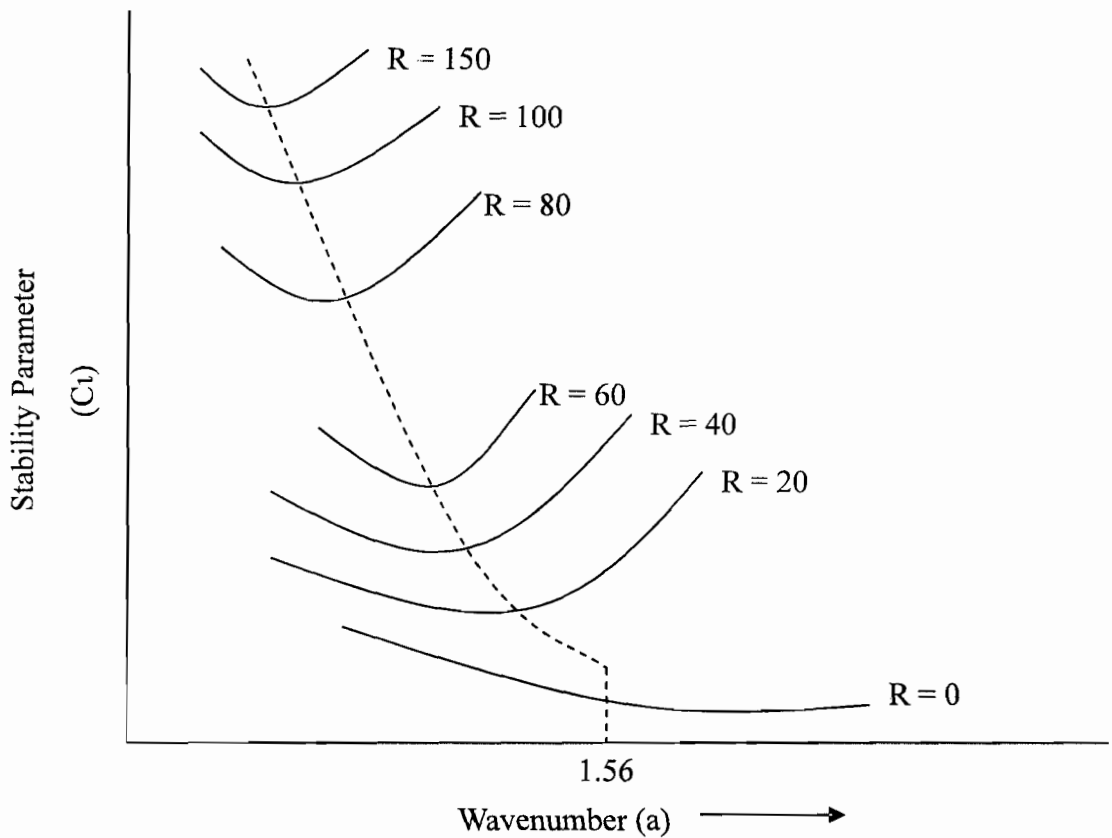

Fig. 3:Neutral stability curves for the case

$S = 6$ (acqueos solution of electrolyte), for various Reynolds numbers.

------------- shows curve for minimum stability parameter (C_i).
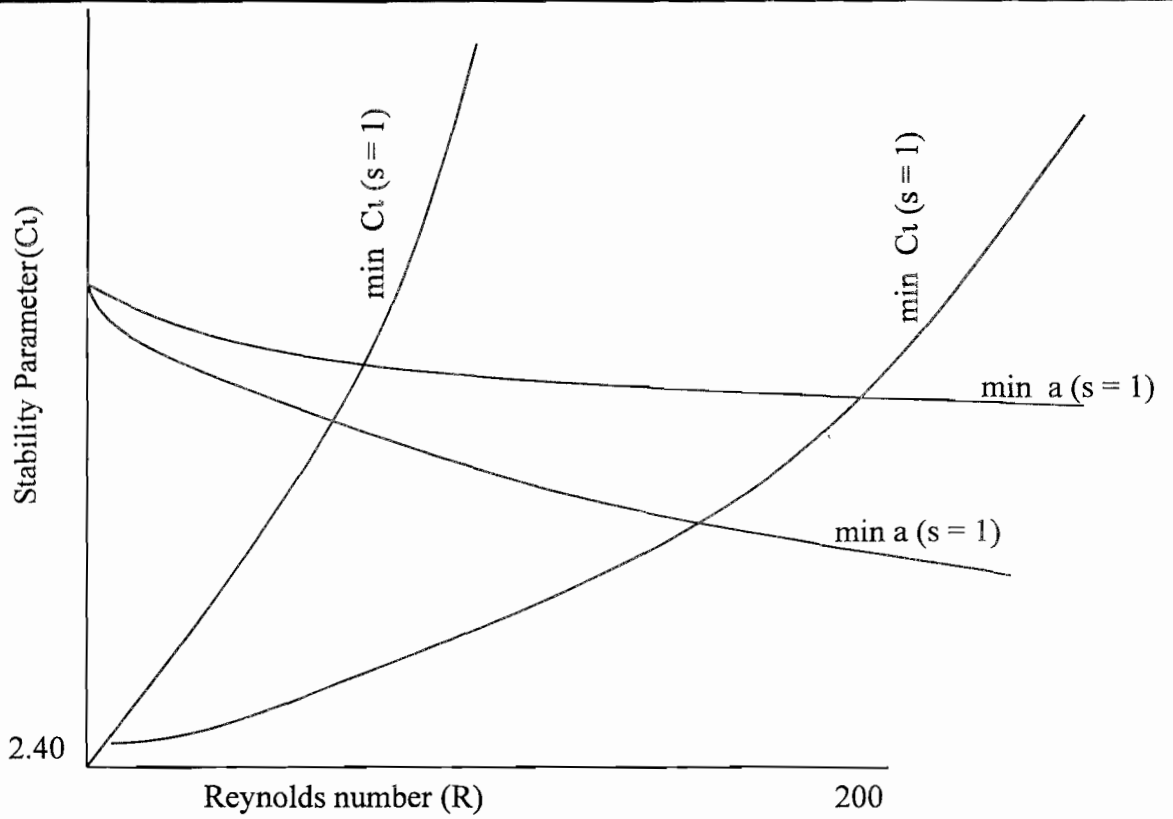
Fig. 4:Curves of minimum stability parameter (C) for neutral stability for the cases S = 0, 1 and 6.

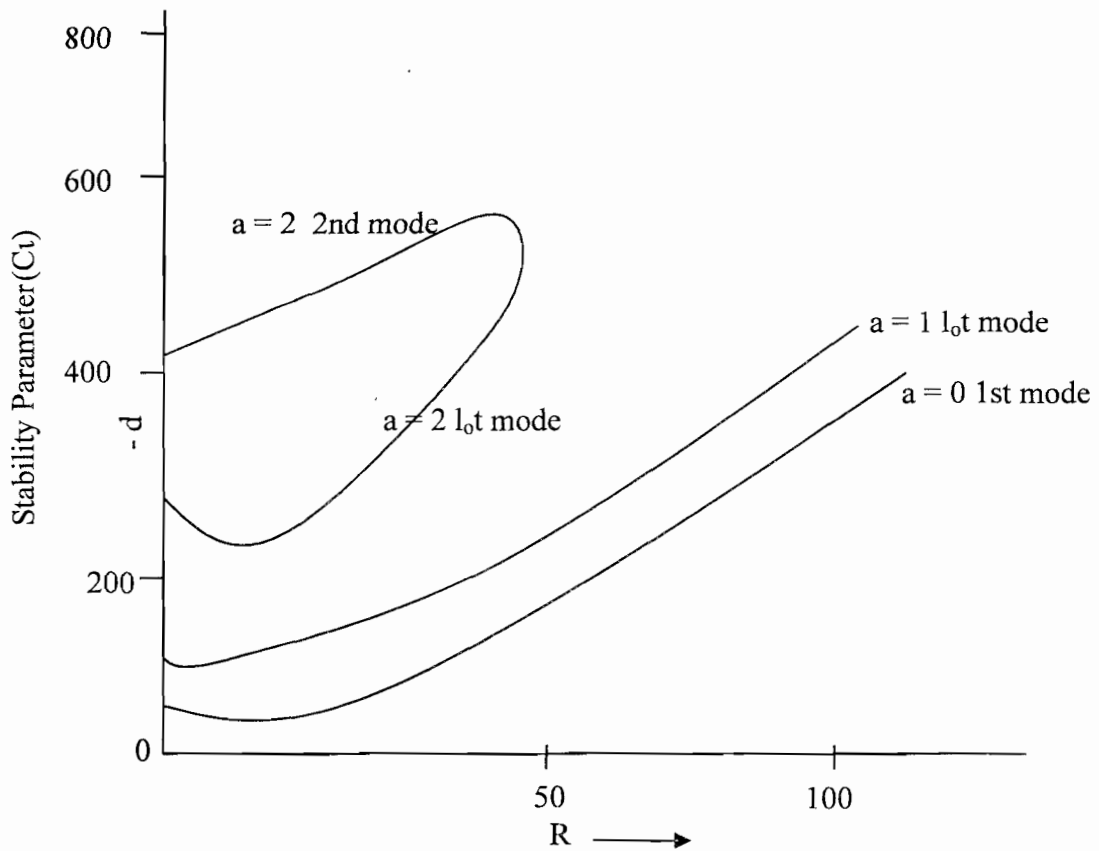Fig. 5:Neutral stability curves for the cases S = 1,a = 0, 1, 2

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