TURBULENT BOUNDARY LAYER APPROACHES TO RESISTANCE COEFFICIENT IN QUASI-ROUGH CIRCULAR PIPES AND GRAVEL BED RIVERS.

Okoli, C.S.
Department of Civil Engineering, Federal University of Technology, Akure, Nigeria.

Abstract: A logarithmic velocity profile has been used, in conjunction with a formulation for the origin of the profile, to study the nature of wall roughness and influence of roughness elements on turbulent flow through circular pipes with part smooth, part rough walls. Experimental data on velocity distribution and frictional head loss have been used to derive expressions for surface characteristics of pipes with uniform and non-uniform wall roughness surface characteristics equations representative wall roughness have been developed for use in the hydraulic design of pipes in the transition zone. In this paper also an experimental investigation is carried out in order to study the effect of concentration of coarse roughness elements on the friction factor. The investigation has shown that it is possible to represent indirectly the effects of concentration by using d_{84} or d_{90} as characteristic diameter on gravel bed rivers.
Keywords: velocity profile, frictional head loss, resistance coefficient.

1. INTRODUCTION

Schlichting [1] experiment demonstrates that three regimes can exist for flow characteristics in a roughened pipe. These are smooth turbulent, transitional and rough turbulent conditions. For low flow velocities smooth turbulent conditions prevail changing to transitional and then into fully developed rough turbulent flow (FDRT) at high velocities (Reynolds numbers). For the transitional regime flow, Nikuradse has given data on pipes with varying degrees of uniform wall roughness while the frictional characteristics used in engineering practice have been reported by various investigators Smart [2], Song and Graf [3].

2. THEORETICAL ANALYSIS

2.1. Velocity distribution and friction factors for smooth and rough pipes.

The starting point is the argument attributed originally to Prandt [4] and presented typically by Yuan [5] which leads to the statement that, moderately near to the pipe wall in a region where turbulent effects predominate

$$\tau_o \approx \tau \approx \tau_t \approx \rho l_m^{\,2}\left(\frac{d\bar{u}}{dy}\right)^2 \text{......} (1)$$

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where τ_o is the wall shear stress: τ is the viscous plus Reynolds stress; τ_t is the Reynolds stress, lm is the mixing length, ρ is the density $\overline{U}$ the time averaged velocity at a distance y from the pipe wall.

The mixing length l_m is defined in the following way. When a fluid lump travelling at its original mean velocity is displaced due to the turbulent motion in the transverse direction from y_1 to y_2, its velocity differs from the surrounding mean velocity verse fluctuation velocity is the mixing length l_m with this relation, the eddy viscosity can now be expressed as (assuming the proportionality constant to be unity).

$$\text{Vt} = l_m^{\,2}\left|\frac{du}{dy}\right| \text{...... ....}(2)$$

This is the Prandtl mixing length hypothesis, it relates to the eddy viscosity to the local mean velocity gradient and involves as single unknown parameter the mixing length l_m.

The second essential introduction is the argument present typically by Massey [6] and based on conventional dimensional analysis that, in a pipe of radius R, whose surface is characterized by a roughness height k, the mean velocity distribution must have the functional form

$$\frac{\overline{U}}{V_*} = f_1\left(\frac{V_* y}{v}, \frac{y}{k}, \frac{y}{R}\right) \text{............} (3)$$

where $V_* = \sqrt{\left(\tau_o / p\right)}$ is the shear friction velocity and v is the kinematic viscosity. In a region moderately close to the pipe wall, the dependence of y/R should become insignificant, hence

$$\frac{\overline{U}}{V_*} = f_2(\frac{V_* y}{v}, \frac{y}{k},) \text{ .... (4)}$$

where f_2 is a simplified version of the function f_1.

2.2. Derivation

By combining Eqs. (1) and (2) and using the definition of friction velocity.

$$\frac{\overline{U}}{V_*}\frac{d\overline{u}}{dy} = \frac{1}{ay} \text{ ....(5)}$$

On integration, this gives

$$\frac{\overline{U}}{V_*} = \frac{1}{a}\ln\frac{y}{y} \text{ ....(6)}$$

It should be noted, at this stage that Colebrook White [7] departed from the traditional method of writing this, namely

$$\frac{\overline{U}}{V_*} = \frac{1}{a}\ln y_1 + b \text{ ... .(7)}$$

and concentrate attention on the constant of integration y_1, which for obvious reasons of dimensional homogeneity must have the dimensions of length, and be characteristic of the problem and its parameters.

If equation (4) is examined, it can be seen that two different lengths, v/v_* and k team up with y to create dimensionless ratios. In a perfectly smooth pipe, for which k= 0, only viscous effects prevail, so that y, can be proportional only to v/v_*. At the other extreme, where viscous effects are dwarfed by the effect of turbulence even close to the wall of "fully" rough pipe, y_1 can be proportional only to k. Between these two asymptotic curves y and y_1 and these two lengths $\frac{v}{v_*}$ and k Colebrook [7], however, in acknowledging an inspired suggestion by White [7], proposed the simplest possible hypothesis, namely that y_1, is a linear function of the two lengths i.e. $y_1 = \alpha k + \beta v/V_*$ ....(8).

Where α and β are numerically constants to be determined empirically or gleaned from the earlier work of Prandtt, VonKarman [4] and Schlichting [1].

Therefore, over at least a portion of pipe section, near but not too near the wall, the mean velocity distribution is predicted to be.

$$\frac{\overline{U}}{V} = -1\frac{1}{a}\ln\left(\frac{\alpha k}{y} + \frac{\beta v}{V_* y}\right) \text{ ......(9)}$$

A friction factor relationship is obtained next by rehearsing the argument that this distribution can be applied with reasonable accuracy over the complete cross-section of the pipe. The mean velocity of flow is given by

$$\overline{V} = \frac{Q}{\pi R^2} = \frac{1}{\pi R^2}\int_o^R \overline{U}.2\pi(R-y)dy = \frac{2}{R^2}\int_o^R \overline{U}(R-y)dy \text{ .... (10)}$$

By using the device of integration by parts, the following equation is obtained.

$$\overline{V} = \frac{2}{R^2}\left\{\left[\overline{U}(Ry - \frac{y^2}{2}\right]_{y=0}^{R} \int_o^R \frac{d\overline{u}}{dy}(Ry - \frac{y^2}{2})\right\}dy \text{ ....(11)}$$

On substitution of $d\overline{u}/dy$ from equation (5) integration and simplification.

$$\overline{V} = \overline{U}_{\max} - (3V_*/2a) \text{......... (12)}$$

Now tentatively applying equation (6) on the pipe axis

$$\overline{U}_{\max} = \frac{V_*}{a}\ln\frac{R}{y_1} \text{ .... (13)}$$

so that

$$\frac{V}{V_*} = \frac{1}{a}(\ln\frac{R}{y_1} - \frac{3}{2}) \text{ ... . (14)}$$

At this point, hypothesis (8) is introduced together with the identities.

$$\frac{V}{V_*} = \sqrt{\frac{P\overline{V}^2}{\tau_o}} - \sqrt{\frac{2}{\lambda}} \text{ ... . (15)}$$

and

$$\frac{V_* R}{V} - \frac{V_*\overline{V}D}{\overline{V}.2v} = \text{Re}\sqrt{\frac{2}{g}} \text{ ... ..(16)}$$

Hence

$$\sqrt{\frac{2}{\lambda}} = \frac{1}{a}\left[\ln\left(\frac{2\alpha K}{D} + \frac{\beta\sqrt{8}}{\text{Re}\sqrt{\lambda}}\right) + \frac{3}{2}\right] \text{... ..(17)}$$

or

$$\frac{1}{\sqrt{\lambda}} = -\frac{1}{2(a)}\ln\left[e^{3/2}\left(\frac{2\alpha K}{D} + \frac{\beta\sqrt{8}}{\text{Re}\sqrt{\lambda}}\right)\right] \text{... (18)}$$

This can be written more simply in the form

$$\frac{1}{\sqrt{\lambda}} = \frac{1}{\sqrt{(2)a}} \ln\left[\frac{\alpha^1 K}{D} + \frac{\beta^1}{\text{Re}\sqrt{\lambda}}\right] \ldots(19)$$

where $\alpha^1 = 2e^{3/2}\alpha = 8.963\alpha$ and $\beta^1 = \sqrt{8e^{3/2}}\beta = 12.68\beta$

Alternatively, we write in the form

$$\frac{1}{\sqrt{\lambda}} = \frac{1.628}{a}\log_{10}\left(\frac{\alpha^1 K}{D} + \frac{\beta^1}{\text{Re}\sqrt{\lambda}}\right) \ldots.(20)$$

which is the COLEBROOK-WHITE equation in use for engineering design and research purposes today.

If α = 1/3.11, β^1 = 1.26 and a = 0.407. (essentially a value of the Von Karman constant) then we obtain the equation of form

$$\frac{1}{\sqrt{\lambda}} = -4\log\left(\frac{K}{3.71d} + \frac{1.26}{\text{Re}\sqrt{\lambda}}\right) \ldots\ldots..(21)$$

which is the popular COLEBROOK-WHITE [7] equation in use for engineering design and research purpose today.

With reference to equation (19), the corresponding constants in the velocity distribution are α =0.0301 and β = 0.0994 which allows the velocity profiles to be expressed (approximately) in the form

$$\frac{\overline{U}}{V_*} = 2.5\ln\left(\frac{\hat{y}}{0.0994 + 0.00301K}\right) \ldots..(22)$$

where $\hat{y} = \frac{V_* y}{v}$ *and* $\hat{k} = \frac{V_*}{v}k$

The smooth and rough pipe expressions can simply be extracted as special cases from these wide results, as follows:

The smooth turbulent law can be arranged as

$$\frac{1}{\sqrt{\lambda}} = 2\log\frac{(\text{Re}\sqrt{\lambda})}{2.51} = -2\log\frac{(2.51)}{(\text{Re}\sqrt{\lambda})} \ldots(23)$$

$$\frac{1}{\sqrt{\lambda}} = 2\log\frac{(3.71d)}{k} = -2\log\frac{1}{(3.71d/k)}$$

.............................(24)

2.3. Hypothetical origin of velocity profile

Assuming the viscous effect decreases as the transition progresses, that inertial effects due to roughness elements in a surface vary during the transition and that the velocity distribution of equation (6) is applicable for flow over a quasi-rough wall, y, may be expressed as

$$y_l = \frac{v}{9V_*} + \frac{K\alpha}{30} \ldots\ldots\ldots\ldots\ldots\ldots..(25)$$

where α is the non-dimensional scale factor. α is defined as the ratio of the equivalent sand roughness projection length of the surface at a stage in the transition to its equivalent sand roughness projection length K in the fully developed rough zone (FDRT). It is expected that α = f(Re) for a given surface.

9. Hence for a Quasi-rough surface

$$\frac{U}{V_*} = 5.75\log\frac{yV_*}{v} + 5.5 - G(\text{Re}) \ldots\ldots (26)$$

$$\frac{V}{V_*} = 5.75\log\frac{dV_*}{2v} + 1.75 - G(\text{Re}) \ldots\ldots.(27)$$

where

$$G(\text{Re}) = 5.75\log\left(1 + \frac{\text{Re}}{3.3}\alpha\right) \ldots\ldots\ldots\ldots(28)$$

Also with similar numerical adjustments to those necessary in the statement of the equation (22) and (23).

$$\frac{1}{\lambda} = -2\log\left(\frac{2.51}{\text{Re}\sqrt{\lambda}} + \frac{1}{3.71}\frac{K\alpha}{d}\right) \ldots(29)$$

This is the COLEBOOK-WHITE transition function for Quasi circular pipes.

Further Advances Equation (28) is not explicit in λ, as λ appears on both sides of the equation and suffers the same disadvantage as equation (23). Churchill in 1973 proposed the following direct solution equation.

$$\frac{1}{\sqrt{\lambda}} = -2\log\left(\left(\frac{7}{R_e}\right)^{0.9} + \frac{k}{3.7d}\right) \ldots\ldots (30)$$

which was attributed later to Jain [11]. It should however be pointed out that Barr (1972), gave equation (30) which is very similar to (29) as

$$\frac{1}{\sqrt{\lambda}} = -2\log\left(\frac{5.15}{R_e^{0.892}} + \frac{k}{3.7d}\right) \ldots\ldots\ldots\ldots\ldots.(31)$$

Equation (30) was later modified by Barr in 1975 to give

$$\frac{1}{\sqrt{\lambda}} = -2\log\left(\frac{5.1286}{R_e^{0.892}} + \frac{k}{3.7d}\right) \ldots\ldots.(32)$$

Barr [1] therefore, preceded Churchill [10] in arriving at this form of equation. The advantages of such direct solution equations are obvious-. They are yet to come into general use.

Transformation of Colebrook-white pipe equation to ASCE Task Force [13] channel equation.

Henderson [14] quoting ASCE Task Force [13], suggested the following more general resistance equation for channels this covering transition flow

$$\frac{1}{\sqrt{\lambda}} = -2\log\left(\frac{k}{12R} + \frac{2.51}{R_e\sqrt{\lambda}}\right) \ldots \ldots(33)$$

where, k = Nikuradse equivalent sand roughness,

as would be expected, comparison of equation (23) and (32) show a lot of similarity as k tends to zero, with coefficient in equation (32) falling close to the range specified . The ASCE equation [34] showed marked similarity with the Colebrook-white equation for flow in pipes.

$$\frac{1}{\sqrt{\lambda}} = -2\log\left(\frac{k}{3.71d} + \frac{2.51}{R_e\sqrt{\lambda}}\right) \ldots \ldots(34)$$

The combination of Darcty-Weibach and Colebrook –white equations yields an expression for velocity V.

$$V = -2\sqrt{2gds}\,\log_{10}\left[\frac{k}{3.71d} + \frac{2.51}{d\sqrt{2gds}}\right] \ldots (35)$$

These equations (33) and (34) form the basis of the charts for the hydraulic design of channels and pipes produced by Hydraulic Research Walling ford. These equations (33) and (34) are readily transformed from dimensionless physical parameters into the usual engineering terminology by replacing the friction factors by its defined equivalent $2gds/V^2$. The next step is to reintroduce the hydraulic radius in place of pipe diameter (R = d/4) to give the transformed equation suggested by Crump [16] in equations (35) and (36) and (37) for channels.

$$\frac{1}{\sqrt{\lambda}} = \frac{C}{\sqrt{8g}} = 2\log_{0}\left[\frac{k}{1483R} + \frac{2.51}{R_e\sqrt{\lambda}}\right] \ldots\ldots(36)$$

$$V = \sqrt{32gRS} = \log_{g_0}\left[\frac{k}{1483R} + \frac{1.25\nu}{R_e\sqrt{32gRS}}\right] \ldots(37)$$

$$Q = A\,A\sqrt{32gRS} = \log_{g_0}\left[\frac{k}{1483R} + \frac{1.25\nu}{R_e\sqrt{32gRS}}\right] \ldots(38).$$

3. MATERIALS AND METHODS

The experiment has been carried out using the flume of William Frasier Laboratory at the University of Strathclyde, Glasgow. The experimental installation is composed of a flume, three small tanks, two pipes and a pump. The flume is divided into two parts: the first part has iron sides and the second part has glass sides. The flume is 4.70m long and 0.3m wide. The water is raised by pump form the reservoir tank to the feeding tank. The settling tank set at the end of the flume, is connected to the reservoir by two pipes. The water discharge was measure by venturimeter. The water depth measured by pezometer, was calculated by mean value of ten measurements. The velocity of flow was measured by the use of pitot tube. The reference level for evaluating the water depth has been chosen in order to consider the influence of bed elements. The reference level was taken as the one obtained by replacing the roughness bed elements with an equivalent bed layer having the same volume and constant thickness. The equivalent bed layer has been calculated for the ground layer for the experiments carried out with different concentrations of coarser elements. The size of coarser elements have been chosen in the range from 18mm to 25mm. In each experiment, the flume has been divided in reference area 0.3 x $0.3m^2$, and in each area a fixed number of coarser elements has been arranged.

The experiments were carried out with bed slopes ranging from 0.5% to 8%. The water temperature was measured with standard thermometer. The grain distribution of the ground bed (bedshape 1) and the grain distributions corresponding to different concentrations of coarser elements (bed shapes II, III, IV, V) were obtained by the Wolman's sampling method. In table 1, the number n of coarser elements arranged in reference area (0.3 x $0.3m^2$) together with the corresponding concentration (C) is expressed as percentage of maximum of coarser elements, which is possible to arrange I the reference area.

Table 1. Number of Coarse Elements and Concentration in Reference Area.

BED GRAIN DISTRIBUTION (1)	n (2)	C % (3)
I	0.0	0.0
II	5.0	3.68
III	10.0	7.50
IV	20.0	15.00
V	30.0	22.00

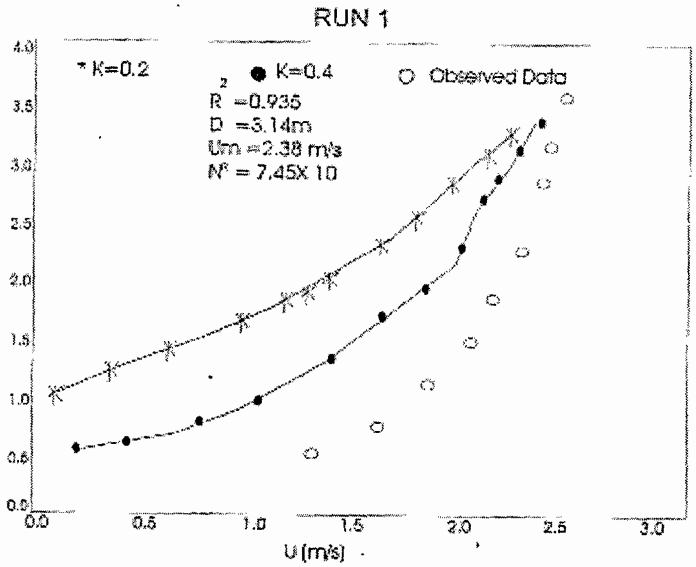

FIG. 1 Prandtl- von Karman velocity Distribution Plotted Against Observed Profile (Run 1)

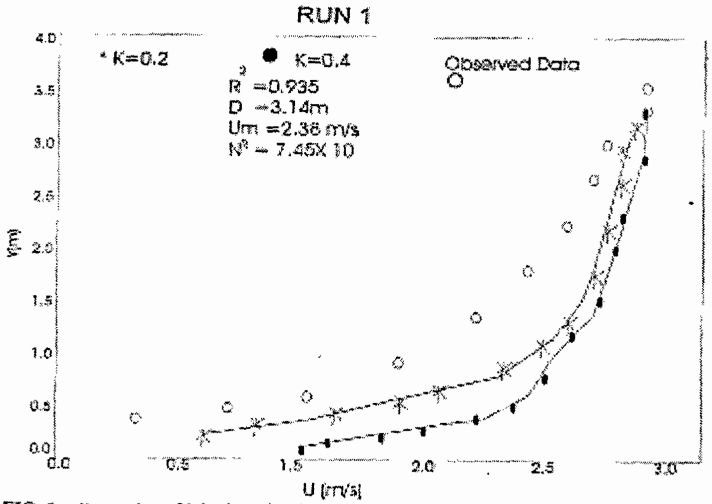

FIG. 2 Power Law Velocity Distribution Plotted Against Observed Profile (Run 1)

4. EXPERIMENTAL RESULTS

Using the measurements of discharge and water depth, corrected for the side-wall effect, the friction factor parameter X/g ½ was calculated. The semilogarithmic equation (39) has given the best fit of the experimental points:
follows as given by (Daugherty and Franzini 17):

$$\frac{X}{g^{1/2}} = C_1 \log \frac{h}{d_{xx}} + C_2 \qquad \text{.........39}$$

In Figs 1(a) and 1(b) the experimental points are respectively compared with power and semilogarithmic equations; comparing the two figures shows the best fit of (39).

Further remarks can be made if we examined all the experimental results concerning different experimental series (bed shapes I, II, III, IV, V). in fact, the relationships that, give the best fit to the experimental points (h/d_{50}, $X/g^{1/2}$) are straight lines when plotted in the semilogarithmic distinguished from the other ones and is characterized by a fixed concentration Fig 2.

If we add to the ground layer (represented by the grain distribution on type I of fig 2 some coarse elements,

Bed grain distribution I	D_{50}		D_{84}		D_{90}	
	C_1 (2)	C_2 (3)	C_1 (4)	C_2 (5)	C_1 (6)	C_2 (7)
I	8.60	0.59	8.60	1.32	8.60	1.48
II	7.93	0.43	7.93	1.35	7.95	1.61
III	7.08	0.64	7.08	1.36	7.08	1.64

IV	7.14	0.06	7.14	1.94	7.14	2.17
V	7.13	-0.37	7.13	1.42	7.13	1.60
VI	8.34	-1.19	8.34	0.81	8.34	0.94

We obtain the grain distribution of type 2. The increment of the diameter Δd increases when the frequency F (the percentage of bed particles having diameter less or equal to d) increases (Daugherty and Franzini [17]).

The results suggest that the increments of (Δ_{84}) or d_{90} (Δ_{90}) are representative, unlike the increment of d_{50}; of the corresponding increments of concentrations. Therefore, if d_{84} or d_{90} are used as characteristic diameter, it is not necessary to introduce the concentration in the flow resistance law. This statement is confirmed by Deigaard. R and Summer B.M. (18).

The relationships of α against RE. for roughnesses 2,3 and 4 are found to be the same type as that for roughness 5. α decreases in the earlier part of the transition and increases with unity at the end of the transition for roughness 2,3 and 4.

It is observed that the scale factor α increases with increasing discharge and approaches unity at the end of transition for pipes with uniform sand grain roughness and for some pipes used in engineering practice. The scale fact or decreases with increasing discharge and approaches unity for pipes with non-uniform roughness and for some pipes used in engineering practice.

5. CONCLUSION

For a fully developed turbulent flow and condition of small scale resistance, the resistance law can be deduced by applying the Prandtl-von Karman theory. In a gravel bed river, when a hydraulic condition of large-scale resistance equation can be applied only if it results from an empirical correlation. In the commonly accepted equations for evaluating friction factor parameter, hydraulic radius, or water dept and a characteristic diameter (d_{50}, d_{84}., d_{90}) are generally included. These equations, empirically deduced, have a power or semilogarithmic form. In the paper an experimental study for evaluating the influence of concentration of coarse bed elements on river and pipes was carried out. The result has confirmed that the semilogarithmic equations gives the best fit to the experimental data and that the use of d_{84} or d_{90} as the characteristic diameter implicitly includes the effect of the particle concentration. It could be observed from this study that both the quasi-rough pipes and gravel bed rivers follow the semilogarithmic equations.

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