

ANALYTICAL DERIVATION OF AND SIMULATIONS ON CURRENT VOLTAGE CHARACTERISTICS EQUATION OF A SOLAR CELL

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Abstract: A quantitative approach was employed with the aim of reviewing and elucidating the development of Current Density-Voltage (J-V) characteristics of solar cell with a single dominant current transport from transport and continuity equation involving drift and diffusion terms with the application of Fermi-Dirac statistical distribution. A number of assumptions were made on unidirectional flow of minority- current carrier outside and within the depletion layer. The simulations of saturation current density with forward bias voltage and diode factors show similar curves but with increase in total current densities with a lower diode factor. Temperature increment was however found not to favour the increase of the current density. Suggestions were also given on experimental works on the photovoltaic cell based on the simulations.

Keywords: Current Density, Recombination, Diffusion, Concentration.

1. INTRODUCTION

In a semiconductor –a material for which the allowed energy band of highest energy for electrons is totally occupied, at 0 K the number of charge carriers depend on temperature and on the presence of impurities [1]. Various authors have set out to describe the experimental and theoretical behaviours of solar cells with little consideration of the derivation and the assumptions underlying their behaviours [2, 3]. This paper provides intensive review, derivation, computation and simulation of the J-V characteristics for typical crystalline silicon photovoltaic cell. The various electronic processes (Generation, Diffusion, Drift and Recombination of charge carriers) involved in the photovoltaic semiconductors can be described to a good extent by Continuity equations, Fermi-Dirac distribution and Current transport equations, under varied assumptions and boundary conditions which are stated and are noticed to favour past experimental works [4].

These equations can be generalized and applied to single or polycrystalline semiconductors (with each specific conditions considered) and are used in characterizing solar cells [1, 4]. Continuity equation was applied from the onset in addition to thermodynamic equilibrium assumption within the

conduction and valence bands and a constant current density, commonly used instead of current within the bands and throughout the solar cells [1, 5], was used throughout the homojunction photovoltaic cell.

2. CONTINUITY EQUATION

The rate of outward flow of particle across a surface of area S is given as

$$= \sum_{all\ dA} \bar{n}V \cdot dA = \int_S \bar{n}V \cdot dA$$

The number of particle inside this surface is $\bar{n}V$

Where \bar{n} is the average numerical density of particles and V is the volume enclosed by surface S.

For a case where there is no carrier generation and recombination,

$$\frac{\partial \bar{n}V}{\partial t} = - \int_S \bar{n}V \cdot dA$$

Negative sign is due to the decrease of right hand side when the left hand side increases. In a case where there are generation and recombination of particles, the net rate of generation is taken as $\bar{n}V$ while that of

recombination is $-\bar{n}V$. For the general case in which V is infinitesimally small, the rate of change of number of particles inside the enclosure is

$$\Delta V \frac{\partial n}{\partial t} = - \int_S nV \cdot dA + c\Delta V$$

$$\frac{\partial n}{\partial t} = - \frac{1}{\Delta V} \int_S nV \cdot dA + c$$

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Hence

$$\frac{1}{\Delta V} \int_s nV.dA = \lim \frac{nV(r+\Delta r) - nV(r)}{\Delta V(r)} = \nabla.(nV)$$

for $r(x, y, z)$

$$\frac{\partial n}{\partial t} = -\nabla.(nV) + c \dots\dots\dots 1$$

This PDE known as continuity equation is a combination of geometrical and kinetic concept. It has application in fields of flow, ionization of gas, production and motion of electron and hole in semiconductors, flow of electron in vacuum tubes, production and flow of neutrons in a reactor etc.

3. HOLE AND ELECTRON FLOW IN A PN JUNCTION SEMICONDUCTOR

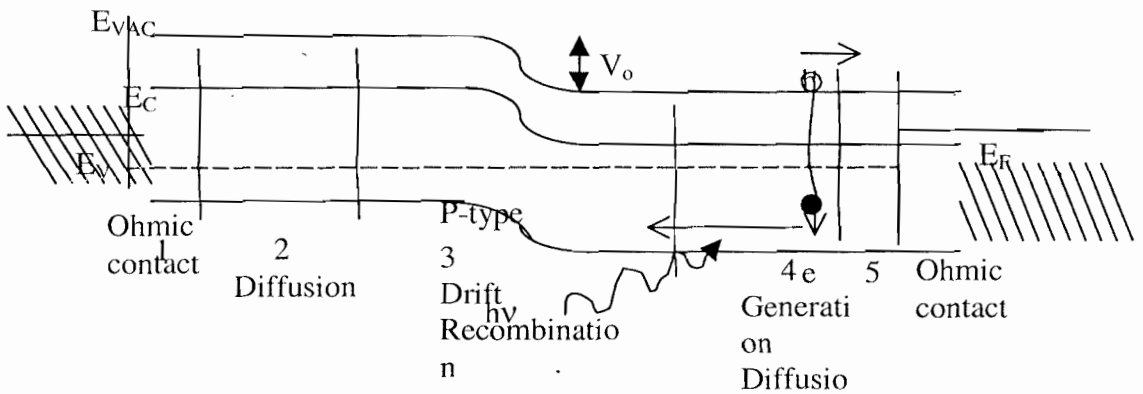


Fig 1. Electronic Processes involved in the photovoltaic cell.

The continuity equation in 1 dimension for hole and electron with hole and electron flux J_h/e and J_e/e respectively using equation 1 are:

$$\frac{\partial p}{\partial t} = -\frac{1}{e} \frac{\partial J_h}{\partial x} - R \dots\dots\dots 2$$

for positively charged holes.

$$\frac{\partial n}{\partial t} = \frac{1}{e} \frac{\partial J_e}{\partial x} - R \dots\dots\dots 3$$

for negatively charged electrons. Where R stands for the net recombination rate per unit volume, this is assumed in this case equal and opposite of the net generation rate per unit volume [6, 1] corresponding to c in equation 1.

For steady-state behaviour of the particles in a pn junction semiconductor

$$\frac{\partial n}{\partial t} = 0 = \frac{\partial p}{\partial t}$$

$$0 = \frac{1}{e} \frac{\partial J_h}{\partial x} + R \dots\dots\dots 4$$

the respective continuity equations 2 and 3 become:

$$0 = \frac{1}{e} \frac{\partial J_e}{\partial x} - R \dots\dots\dots 5$$

and

The current densities for hole and electron are composed of the drift and diffusion terms [1, 6]

$$J_h = pqU_h \varepsilon - eD_h \nabla p \dots\dots\dots 6$$

and

$$J_e = nqU_e \varepsilon + eD_e \nabla n \dots\dots\dots 7$$

The recombination rate are expressed in terms of holes and electron concentration as

$$R_h = \frac{p - p(0)}{\tau_h} \quad \text{and} \quad R_e = \frac{n - n(0)}{\tau_e} \quad \text{where}$$

τ_h and τ_e

are hole and electron lifetime

$p(0)$ and $n(0)$ are the thermal equilibrium values of hole and electron concentration in the p-type and n-type materials respectively. The total current

$$J = J_n + J_p$$

must be continuous throughout the solar cell. From equation 7 and using the value of R_e

$$\frac{1}{e} \frac{d}{dx} (ne\mu_n \mathcal{E} + eD_e \frac{dn}{dx}) - \frac{n - n(0)}{\tau_e} = 0 \dots\dots\dots 8$$

$$\frac{d^2 n}{dx^2} - (n - n(0))(D_e \tau_e)^{-1} = 0 \dots\dots\dots 9a$$

let $A^2 = (D_e \tau_e)^{-1}$

hence

$$n = n(0)e^{Ax} \text{ for } e^{Ax} \gg 1 \dots\dots\dots 9b$$

A solution $n(x) = n(0)e^{Ax}$ with

$$A = (D_e \tau_e)^{-\frac{1}{2}}$$

and $n(0)$ is the thermal equilibrium of excess electron concentration at the p edge of depletion layer where x is taken as zero.

The electron and holes obey Pauli exclusion principle thus the most probable number of electron per level or energy state obey Fermi-Dirac statistical distribution [7],

$$P_{FD} = (1 + e^{\alpha + \beta})^{-1} \text{ with } \alpha = \frac{E}{kT} \text{ and}$$

$$\beta = -\frac{E_F}{kT}$$

the Fermi-Dirac distribution becomes

$$P_{FD} = \frac{1}{(1 + e^{\frac{E - E_F}{kT}})}$$

Since the exponential term in the equation is very large and introducing the constant A for diode factor,

$$P_{FD} = e^{\frac{E_F - E}{kT}} = e^{\frac{qV}{kT}}$$

The concentration of electrons adjacent to the junction on the n-type side varies exponentially with the height of the barrier. Hence

$$n = n_{po} \exp\left(\frac{qV}{kT}\right)$$

The excess electron concentration at the p-side of the junction with $n = n(0) + n_{po}$ is then

$$n(o) = n_{po} (\exp\left(\frac{qV}{kT}\right) - 1) \dots\dots\dots 10$$

The minority-carrier diffusion density can be calculated from the equation 7 and using the definition of $n(x)$ from the solution of equation 8.

$$J_e = \frac{eD_e}{\sqrt{D_e \tau_e}} n(o) \exp\left(-\frac{x}{\sqrt{D_e \tau_e}}\right) \dots\dots\dots 11$$

The quantity $\sqrt{D_e \tau_e}$ is known as the diffusion length L_e for electron. From equations 9 and 10 it is obvious that,

$$J_e = \frac{eD_e}{L_e} n_{po} (e^{qV/kT} - 1) e^{-x/L_e} \dots\dots\dots 12$$

Similar equation obtainable for holes is of the type

$$J_h = \frac{eD_h}{L_h} p_{no} (e^{qV/kT} - 1) e^{-x/L_h} \dots\dots\dots 13$$

The total current density is the sum of electron and hole current densities. At $x=0$, the total current density under light generation is

$$J_T = J + J_L = \left[\frac{eD_e n_{po}}{L_e} + \frac{eD_h p_{no}}{L_h} \right] \left[e^{\frac{qV}{kT}} - 1 \right] \dots\dots\dots 14$$

If J_0 is the non-temperature dependent part, then equation 13 becomes,

$$J_T = J_0 \left[e^{\frac{qV}{kT}} - 1 \right] - J_L \dots\dots\dots 15$$

which is the current density -voltage characteristics equation of solar cell with a single dominant diffusion transport mechanism. In the dark J_L vanishes and the characteristics equation is

$$J = J_0 \left[e^{\frac{qV}{kT}} - 1 \right] \dots\dots\dots 16.$$

The diode factor A depends on the conditions predominating at the junction ($A=1$ for injection dominated transport mechanism, while $A=2$ for recombination based mechanism).

If equation 14 is transformed as

$$[J_T + J_0] e^{\frac{E_g}{kT}} = J_0 \dots\dots\dots 17$$

$$[J_T + J_0] = J_0 e^{\frac{qV}{kT}} \dots\dots\dots 18$$

The convergent value of J_0 can be found from Newton-Raphson iteration [8] of equation 16. this

can be used with equation 18 to get the J-V characteristics data and curves.

Tables 1 and 2 give the computed values of total current densities for two values of saturated current densities $J_0 = 2.4 \times 10^{-15}$ and $J_0 = 1.2 \times 10^{-12}$ for two separate diode factors $A=1$ and 2 . The first computation was to simulate voltage variation effect on the J_T while the other is simulation of the effect of temperature variation on the total current density. Figures 1 and 4 drawn on log-linear graph made it possible for comparison of 2 graphs with large scale difference on the y-axis and both have similar variation of total saturated current densities with bias voltages and the effect of diode factors is to lower the current densities, since the specific processes (i.e. carrier separation, diffusion, recombination) occurring at the junction influence diode factor values the ones that produce smaller

diode factor values should be enhanced. Figure 2, 3, 5 and 6 showed the effect of temperature variation on the total current densities. The increase bias voltage produced increase current density while increasing temperature causes decreasing current density. The voltage-current density characteristics of solar cells are of importance for investigation of the performance of solar cells. The optimum temperature is not within the normal environmental range as the highest total current obtained in this simulation is at 260 and 280 K; this information however suggests that light concentration and not increasing of temperature is necessary for efficient solar cells. It will not be possible to maintain external temperature where solar cells are exposed but the condition for conduction and diffusion of current within the solar cells can be optimized to produce more efficient solar cells.

Tables 1 and 2. Computed Total Current Density Variation with Voltage and Temperature for Two Saturation current densities

Table 1.

J ₀ = 2.40E-15 (mA/cm ²)					
Bias Voltage(V)	JT(mA/cm ²) T= 300K A=1	JT (mA/cm ²) T=300K A=2	Ambient temperature (K)	JT (mA/cm ²) A=1	JT (mA/cm ²) A=2
0.4	1.27E-08	6.06E-12	273	5.95E+16	1.20E+01
0.5	6.07E-07	4.29E-11	278	8.11E+15	6.24E+00
0.6	2.91E-05	3.04E-10	283	2.31E+15	3.33E+00
0.7	1.39E-03	2.16E-09	288	6.90E+14	1.82E+00
0.8	6.68E-02	1.53E-08	293	2.14E+14	1.01E+00
0.9	3.20E+00	1.08E-07	298	6.92E+13	5.76E-01
1	1.53E+02	7.68E-07	303	2.32E+13	3.34E-01
1.1	7.35E+03	5.45E-06	308	8.06E+12	1.97E-01
1.2	3.52E+05	3.86E-05	313	2.89E+12	1.18E-01
1.3	1.69E+07	2.74E-04	318	1.07E+12	7.18E-02
1.4	8.08E+08	1.94E-03	323	4.11E+11	4.44E-02
1.5	3.87E+10	1.37E-02	328	1.62E+11	2.79E-02
1.6	1.86E+12	9.75E-02	333	6.56E+10	1.77E-02
1.7	8.90E+13	6.91E-01	338	2.73E+10	1.14E-02
2	9.79E+18	2.46E+02	343	1.17E+10	7.48E-03
2.3	1.08E+24	8.76E+04	348	5.10E+09	4.95E-03
3.2	1.44E+39	3.96E+12	353	2.28E+09	3.31E-03
3.7	3.63E+47	7.08E+16	358	1.05E+09	2.24E-03
4	3.99E+52	2.52E+19	363	4.90E+08	1.53E-03
4.7	2.32E+64	2.27E+25	368	2.34E+08	1.06E-03

Table 2.

$J_0 = 2.4 \text{ E-12 (mA/cm}^2\text{)}$

Bias Voltage(V)	JT (mA/cm ²) A=1 T=300K	JT (mA/cm ²) A=2 T=300K	Ambient temperature (K)	JT (mA/cm ²) A=1	JT (mA/cm ²) A=2
0.4	1.27E-05	6.06E-09	273	5.95E+19	1.20E+04
0.5	6.07E-04	4.29E-08	278	1.62E+19	6.24E+03
0.6	2.91E-02	3.04E-07	283	4.63E+18	3.33E+03
0.7	1.39E+00	2.16E-06	288	1.38E+18	1.82E+03
0.8	6.68E+01	1.53E-05	293	4.28E+17	1.01E+03
0.9	3.20E+03	1.08E-04	298	1.38E+17	5.76E+02
1	1.53E+05	7.68E-04	303	4.64E+16	3.34E+02
1.1	7.35E+06	5.45E-03	308	1.61E+16	1.97E+02
1.2	3.52E+08	3.86E-02	313	5.79E+15	1.18E+02
1.3	1.69E+10	2.74E-01	318	2.15E+15	7.18E+01
1.4	8.08E+11	1.94E+00	323	8.22E+14	4.44E+01
1.5	3.87E+13	1.37E+01	328	3.24E+14	2.79E+01
1.6	1.86E+15	9.75E+01	333	1.31E+14	1.77E+01
1.7	8.90E+16	6.91E+02	338	5.46E+13	1.14E+01
2	9.79E+21	2.46E+05	343	2.33E+13	7.48E+00
2.3	1.08E+27	8.76E+07	348	1.02E+13	4.95E+00
3.2	1.44E+42	3.96E+15	353	4.57E+12	3.31E+00
3.7	3.63E+50	7.08E+19	358	2.09E+12	2.24E+00
4	3.99E+55	2.52E+22	363	9.79E+11	1.53E+00
4.7	2.32E+67	2.27E+28	368	4.68E+11	1.06E+00
5.2	5.86E+75	4.06E+32	373	2.28E+11	7.40E-01
5.4	1.35E+79	2.04E+34	378	1.13E+11	5.21E-01

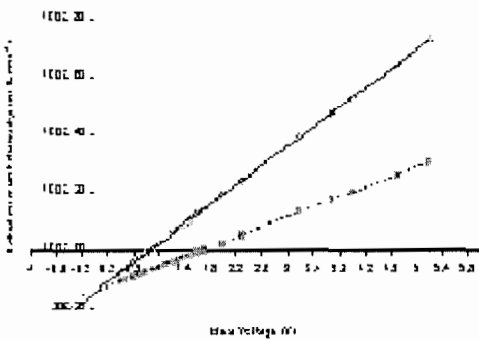


Fig. 1. Current Density-voltage Characteristic for solar cell A=1 and A=2 $J_0=2.4 \text{ E-12}$

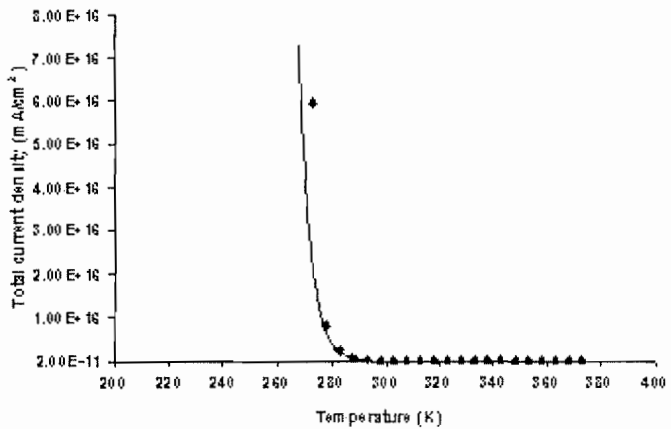


Fig. 2. Current Density-Temperature Variation for Solar cell A=1 $J_0=2.4 \text{ E-12}$

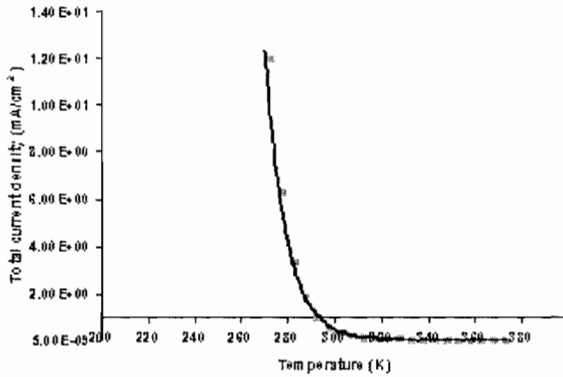


Fig. 3. Current Density-Temperature Variation for Solar cell A=2 $J_0=2.4 \times 10^{-15}$

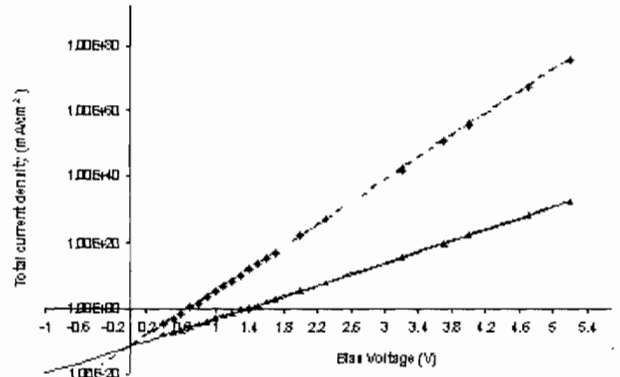


Fig. 4. Current Density-Voltage Characteristics for Solar cell A=1 and A=2 $J_0=2.4 \times 10^{-12}$

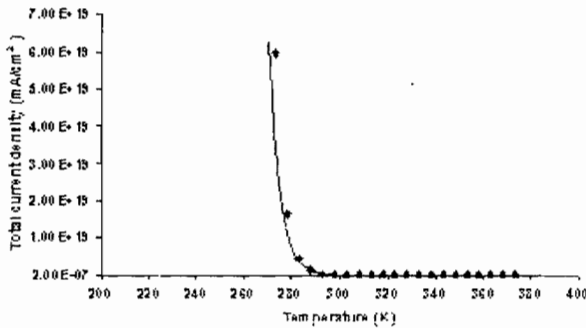


Fig. 5. Current Density-Temperature Variation for Solar cell A=1 $J_0=2.4 \times 10^{-12}$

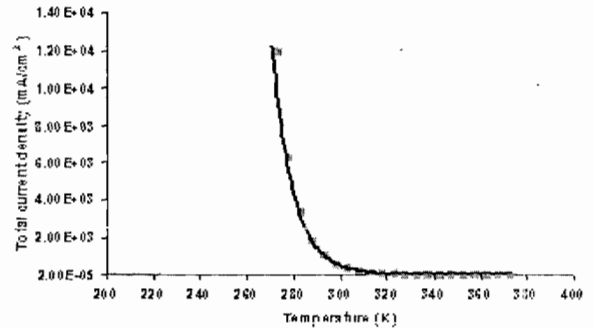


Fig. 6. Current Density-Temperature Variation for Solar cell A=2 $J_0=2.4 \times 10^{-12}$

4. CONCLUSION

From the above theoretical studies, the behaviours of total current densities so derived enhanced our appreciation of factors that enhance photovoltaic conversion, it is noteworthy that further experiments should be done on variations of total current density and maximum power output with various dominant transport mechanisms in different junction solar cells as it was found that diode factors are important for producing increasing total current density. More experimental check should be done on various solar cells to investigate their break down voltages and the effect of the near or post avalanche breakdown on the current density and maximum lifetime of the photovoltaic cells.

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