Nig. J. of Pure & Appl. Physics Vol.3 pages 21-27, 2004

FINITE ELEMENT MODELLING OF SOLIDIFICATION OF ZINC ALLOY

G.A.Osinkolu[1], S. O. O. Olusunle[2]
[1]Division of Materials & Electronics, Centre for Energy Research & Development, Obafemi Awolowo University, Ile-Ife, Nigeria
[2]School of Metallurgy & Mineral Resources, Kogi State Polytechnic (Lokoja)
Osara Campus, Kogi State, Nigeria

Abstract: The solidification process of Zinc alloy is modelled by solving heat transfer equations with the aid of finite element method (FEM) using appropriate boundary conditions at the mould walls. The commercial software, Matlab, has been used to model the solidification process. The temperature profiles for each casting condition were determined. The model demonstrated capability in predicting accurately the location as well as the shapes of heat centres in any casting. The calculated shapes of the heat centers are in good agreement with experimental results for the various casting conditions. The FEM modelling also serves as useful tool for designing feeders for the heat centres.

Keywords: finite element, modelling, zinc alloy, heat transfer process

1. INTRODUCTION

Most metallic components are produced in their final form by casting techniques or initially in form of ingots by pouring the molten metal into a mould where solidification takes place. The control of solidification rates is very essential if the casting must be sound and meet the required mechanical properties. A number of problems are associated with the solidification processes. In a binary or multi-component alloy micro-segregation of the elements occurs and the final microstrcuture is determined by the solidification rates. Macro-defects such as cracks, micropores, and hot tears are usually observed in some castings or ingots and these defects concentrate mainly in the last region to solidify or the so-called heat centres due to shrinkage upon solidification [1]. The shrinkage can be avoided by introducing feeders at the heat centres. The identification of the heat centre is therefore crucial because it can be used in the design of feeder and in calculating the value of feed metal. The usual method for predicting the location of heat centres is by modulus concept[2,3] but the calculation can be in error because the value of the modulus depends on the location as well as the cross section of the casting. The solidification process is essentially heat transfer problem, and so the solidification rate and the temperature distribution can be calculated by solving appropriate heat conduction equations [4-8].

The aim of the present work is to carry out heat transfer analysis of castings of different shapes by finite element methods so that the heat centres can be identified and the shape of the shrinkage region predicted.

2. FINITE ELEMENT FORMULATION

The finite element method (FEM) is a versatile numerical tool for solving differential equations arising from physical problems and has the capability to handle complicated boundary conditions. In the present work, the heat conduction equation in two-dimension is solved but the basic mathematics involved as well as the finite element technique is well known; so only a brief description would be undertaken. The mould is assumed to be in two-dimension and the governing heat conduction equation can be expressed as:

$$\rho c\frac{\partial T}{\partial x}=\frac{\partial}{\partial x}\left(k_x\frac{\partial T}{\partial x}\right)+\frac{\partial}{\partial y}\left(k_y\frac{\partial T}{\partial y}\right)+Q \tag{1}$$

where ρc is the thermal capacity and k_x, k_y are the thermal conductivities in x, y directions. Q is the heat source. The numerical approach of FEM considered in the work consists of drawing the shape of the casting to be modeled and then discretising the shape into triangular elements so that temperature at each node of an element is approximated as:

$$T=\sum N_i T(t) \tag{2}$$

Nig. J. Pure Appl. Phys. (2004) Vol. 3 pp 20-26 Printed in Nigeria, ISSN 1596-0862. E-mail: njpapfuta@yahoo.com. For rights of reproduction and web access, contact African Journals Online, UK, www.inasp.ajol/journals/njpap

where T_i (t) is the time dependent temperature at node i and N_i is the interpolation function. The approach of FEM is to obtain the best approximation to the temperature, T, and thus we test the heat conduction equation for T against all possible shape functions. This implies that the residual of equation (1) is multiplied by the shape function and then integrated i.e:

$$\int \left(\frac{\partial}{\partial x}\left(k_x \frac{\partial T}{\partial x} \right) + \frac{\partial}{\partial y}\left(k_y \frac{\partial T}{\partial y} \right) + Q - \rho c \frac{\partial T}{\partial t} \right) N_i dxdy = 0 \qquad (3)$$

For a specified domain, either the temperature, T, is prescribed on a part of the boundary or the heat flux is prescribed on the remaining part of the boundary. The heat flux between the outer wall of the mould and the surroundings can be written as:

$$q = h\left(T_{md} - T_\infty\right) \qquad (4)$$

where h is the heat transfer coefficient, T_{md} and T_α are the mould temperature and surroundings respectively.

After integration by parts we obtain a linear system of equations, which can be expressed in matrix form as [9]

$$KT + \mathrm{CT} = \mathrm{F} \qquad (5)$$

The matrix elements can be obtained from:

$$K_{ij} = \sum_e \int \left(K_x \frac{\partial N_i}{\partial x}\frac{\partial N_j}{\partial x} + K_y \frac{\partial N_i}{\partial y}\frac{\partial N_j}{\partial y} \right) \mathrm{dxdy} + \sum_e \int_\Gamma h N_i N_j dxdy \qquad (6)$$

$$C_{ij} = \sum_e \int_\Omega \rho c N_i N_j dxdy \qquad (7)$$

$$F_i = \sum_e \int_\Gamma N_i h T_\infty d\Gamma \qquad (8)$$

shrinkage is due mainly to volume contraction in the region that solidified last. Since heat extraction starts from the wall of the mould

the summation is carried over every element. A detailed procedure for the transformation of equation (1) into a finite element formulation is well documented [9,10]. A typical discretisation of an L-shape is shown in Fig.1 and the temperature for each node is determined by solving a large number of linear equations resulting from equation (5). A number of commercial software is available for mesh generation and for solving the linear equations. For this particular work we have made use of the FEM package available on Matlab version 6.1.

3. MODEL CASTINGS

The aim of this experiment is to reveal the actual location of heat centres or the shrinkage regions in castings of some specific shapes as examples.

Wooden patterns, having a L-shape and a circular-shape, were cut and zinc castings of these shapes were produced by sand casting technique (synthetic sand). However since the occurrence of shrinkage is purely a solidification problem, the locations of the heat centres do not depend on the alloy used in the casting. The photographs of the castings were taken to show in particular the shrinkage regions.

4. RESULTS AND DISCUSSION

The photographs of the zinc castings, having a L-shape and a circular shape, are presented in Fig.2. For the L-shape casting, the shrinkage [illegible] has an appearance of a half-moon and it is 22 [illegible] in the bend. In the same figure the [illegible]ge shape is circular for the circular casting and it is located at its centre. The volume contraction would obviously occur in the bulk of the casting, away from the mould wall. However, previous calculation of heat centers based on modulus is not capable of determining the shape of the heat center.

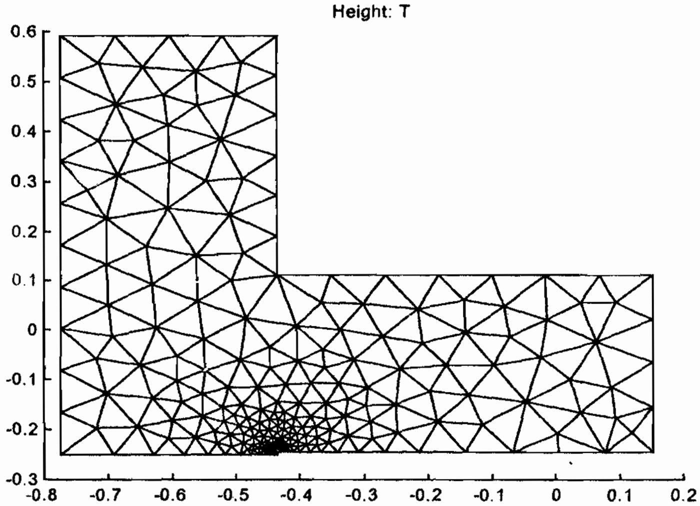

Fig 1. Typical mesh generated for the L-shape geometry. Size is non-dimensionalised

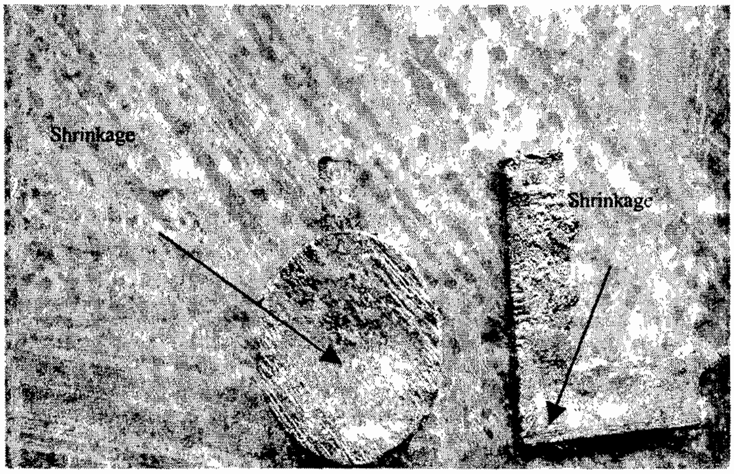

Fig.2: Photographs of model castings showing the location of shrinkages.

The heat conduction equation is then solved for any desired shape and the boundary condition is imposed either in form of temperature on a part of the boundary or as heat flux which is written in the form:

$$n^*k^*\nabla T + h^*T = q \qquad (9)$$

where h controls the degree to which heat is lost from the casting during solidification or cooling, q is the heat flux and n is the normal to the surface.

The corresponding FEM calculations for L-shape and circular shape castings are shown in Fig.3 and Fig. 4 respectively. The initial temperature of the melt was specified at the boundaries as 420°C and the temperature of the surrounding as 25°C. The contours of the temperature distribution actually give exact similarity with the shape of the shrinkage region obtained from the actual casting. The innermost contour actually depicts shape of the last region to solidify or the heat centre, a half-moon for L-shape casting and circular for the circular casting. The FEM calculations are in accord with similar calculations based on modulus concept of determining the heat centres[2] but provide better details on the shape of the heat centres. Thus the FEM modelling can be used to design appropriate feeders for the different heat centres.

Due to the success of these calculations in determining the temperature distributions of well-known shapes, we have also applied the FEM technique to model the solidification process of ingots, under certain peculiar conditions. In the solidification of ingots certain defects such as blowholes and piping are common at the heat centres [11]. One way of reducing the cropping losses is to control the solidification rates by covering the top of the mould with insulating materials.

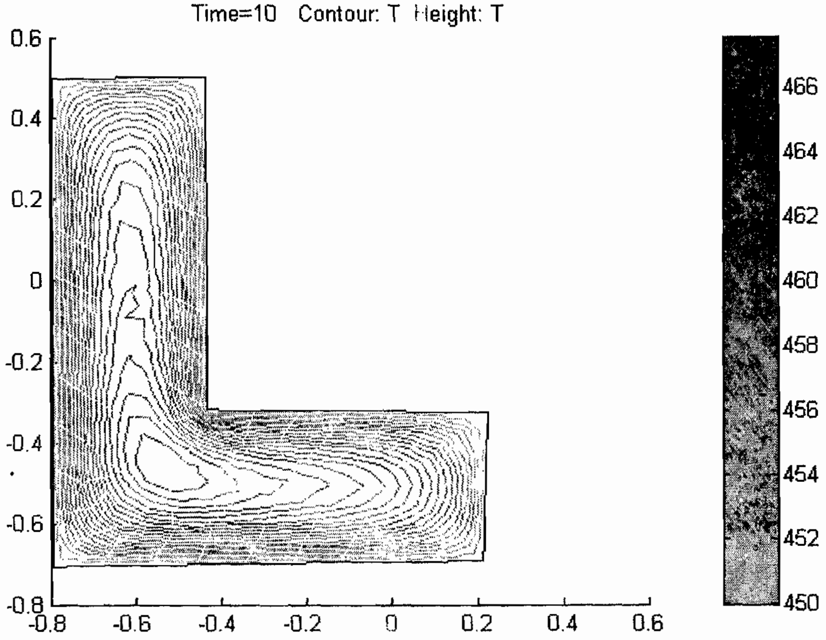

Fig. 3: Temperature profile in an L-shaped object showing the shape of heat centre

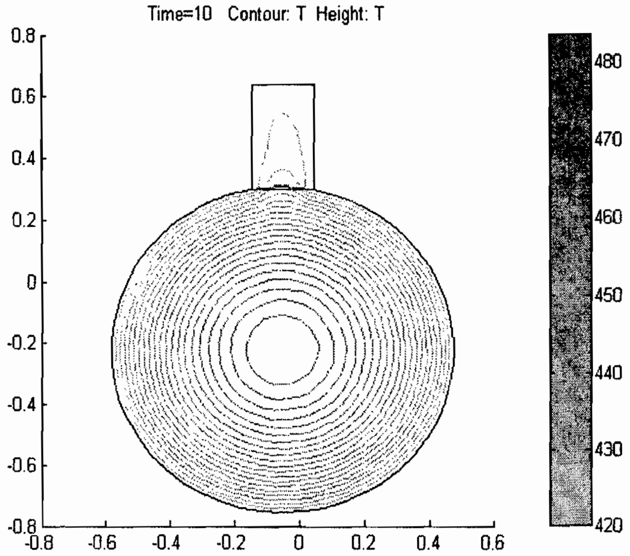

Fig.4: temperature profile in a circular-shape object showing the shape of heat centre

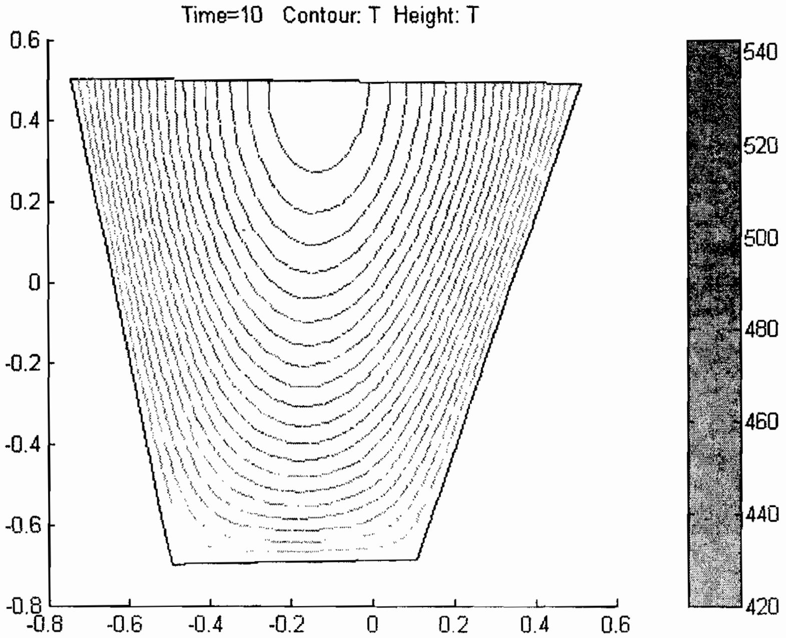

Fig.5: Temperature profile of an ingot with the top covered with insulating material during solidification

A typical shape of an ingot is modeled in Fig.5 where the top is insulated as it is sometimes done in practice. For this case the boundary condition at the top is prescribed in form of Neumann condition and the heat flux is set to zero. The temperature profile (Fig.5) predicts the shape of the shrinkage region usually observed in practice when the top of the mould is insulated [11]. Another way of reducing piping in an ingot is to cover the top of the mould with an exothermic mixture commonly referred to as hot tops. Theoretically it is the boundary conditions that are altered in various parts of the casting. The solidification process in this case is modelled by maintaining the cover of the mould at a temperature (600^0C) higher than the molten metal (420^0C). The FEM prediction is shown in Fig.6 in form of temperature contour and it is again in excellent agreement with the situation encountered in practice [11].

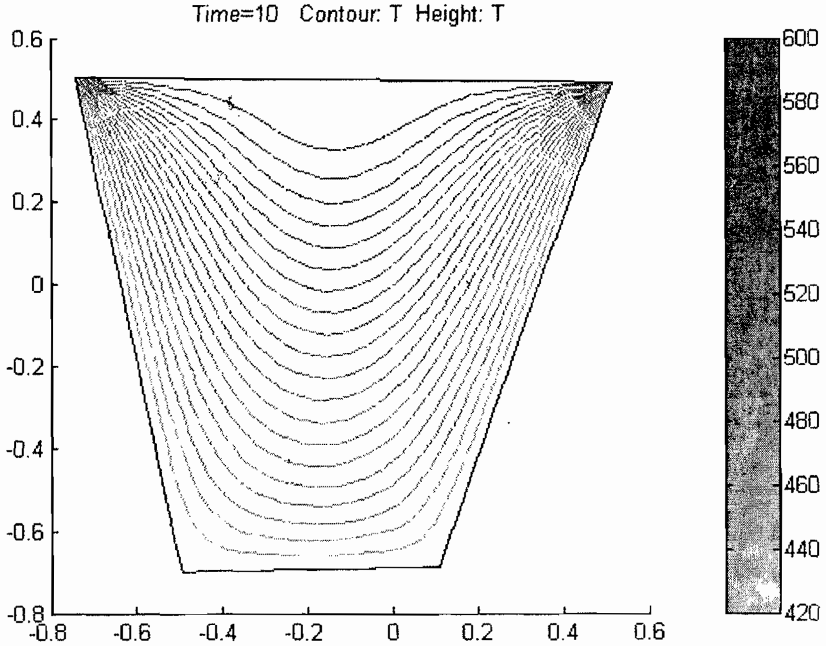

Fig.5: Temperature profile of a casting with the top covered with exothermic mixture

5. CONCLUDING REMARKS

The solidification process is modelled by carrying- out heat transfer analysis, using finite element method, for different casting geometries. The modelling predicted the location of heat centres and the shape of shrinkage regions in different type of castings in excellent agreement with situation encountered in practice. The FEM modelling of the solidification process is therefore a useful tool that can be applied in the design of feeders.

REFERENCES

1 Chandra V. (1995), Computer predictions of hot tears, hot cracks, residual stresses and

distortion in precision castings Basic concepts and approach. Proc. 124 TMS Annual meeting, Warandale, P. A. pp. 107-117,.

2. Sirilertworakul, W, Webster , P. D., Dean T. A. The Foundryman, (1993).
3. Bishop H. F., Myskowski E. T., Pellini W.S, (1995), Transactions of the American Foundrymen Society, Vol. 63, p.273.
4. Jaine Alvares Spim Jr., Amauri Garcia, (2000), Numerical analysis of solidification of complex shaped bodies: coupling of mesh elements of different geometries, Matls Sc. & Eng, A277, 198- 206.
5. Gandin Ch. A., Desbiolles J.L., Rappaz M. Thevoz Ph. (1999), A three-dimensional cellular automaton- finite element model for the prediction of solidification grain structures, Metall. & Mat. Transc. A, vol. 30A, 3153-3165.
6. Brown S.G.R., Spittle J.A. (1990) finite element simulation of solidification of aluminum casting alloy LM 25, Matls Sc. & Techn., Vol. 6, 543-547.
7. Clyne T.W. (1982), The use of heat flow modeling to explore solidification phenomena, vol. 13B, 471-478.
8. Sung P.K., Poirier D.R., Felicelli S.D., Poirier E.J., Ahmed A. (2001), Simulation of porosity in IN718 equiaxed investment castings, J.of crystal growth, 363-377.
9. Lewis R.W., Morgan K, Thomas H.R., Seetharamu K.W. (1996), The finite element method in Heat Transfer analysis, New York: John Wiley.
10. Morgan K., Lewis R.W., Seetharamu, K.N.(1981), Modelling heat flow and thermal stress in ingot casting: Simulation, 55-63.
11. Serope Kalpakjian (1985), Manufacturing processes for engineering materials, Chapter 5, Addison-Wesley Publishing Company.