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MATHEMATICAL MODELLING OF ALUMINUM SURFACE WHEN DIPPED IN MOLTEN METAL

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Abstract. A mathematical model is presented to describe the undulating surface of aluminium casting during an industrial process involving the dipping of the mould, at a particular velocity, into the molten metal. The problem of air gap formation between the mould and the casting was also considered. Below certain value of the mould velocity the shape of the casting as well as its thickness remain practically unchanged with changes in mould velocities. The undulating surface disappears when the mould temperature is in excess of 120°C.

1. INTRODUCTION.

The process of metal solidification needs to be carefully controlled in order to avoid casting defects that could reduce the quality of such castings [1,2]. One of the ways of obtaining good castings is to reduce the air gap between the mould and the solidified metal [3]. Also, by reducing the absorbed gases in the molten metal, a high quality casting can be produced [4]. Aluminium, for example, is known to absorb hydrogen from the atmosphere thereby leading to the formation of micropores in its castings.

The present work focuses on the problem of surface undulation in aluminum casting during a particular industrial process described in the following. When a flat metal mould was dipped inside a molten aluminium at a velocity, v, and immediately withdrawn, the surface of the solidified layer was found to be undulated. Fig.1a shows a sketch of such process and the resulting undulating surface in Fig.1b. The cause of the surface undulation is still not well known but it may be related to the occurrence of air gaps at the boundary between the mould and the molten metal. The surface undulation may also be due to thermal stresses arising from temperature difference between the mould and the molten metal. The air gap formation at mould wall could serve as barrier to heat transfer across the mould and it will therefore be taken into consideration in the modelling. The aim of the present work is to develop a model predicting the surface undulation during aluminium solidification in relation to the mould velocity and the initial mould temperature.

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1. MATHEMATICAL FORMULATION

Fig.1a shows the mould entering the molten metal at velocity, v, in the x-direction. The moving boundary of the solidifying layer is represented by s(x,t) in Fig.1b. Heat is removed through the wall of the mould. At the boundary between the solidifying metal and the molten metal, the temperature is taken as T_m . Considering a two-dimensional case and taking an element with dimension dx and dz in the already solidified layer, the rate at which heat is removed across the surface of the element must be equal to the rate of decrease of heat in it. Thus,

$$c\rho\frac{dT}{dt}+c\rho v\frac{dT}{dx}=\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial z^2}\right);\ 0<z<s(x,t) \tag{1}$$

The second term, on the left hand side of the equation, represents the rate of decrease of heat due to the introduction of the mould into the molten metal at a velocity, v. The boundary of the solidifying layer is moving and latent heat is generated due to the transformation of liquid to solid state and this can be represented as follows:

$$c\rho\frac{\partial s}{\partial t}+c\rho v\frac{\partial s}{\partial x}=L_h\frac{\partial T}{\partial n};z=s(x,t). \tag{2}$$

where

$$\frac{\partial T}{\partial n}=\left(\frac{-\frac{\partial T}{\partial x}\frac{\partial s}{\partial x}\ \frac{\partial T}{\partial z}}{\sqrt{1+\left(\frac{\partial s}{\partial x}\right)^2}}\right)$$

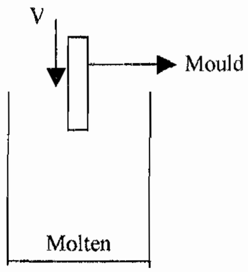

Fig.1a: Introduction of mould into molten aluminum at velocity, v

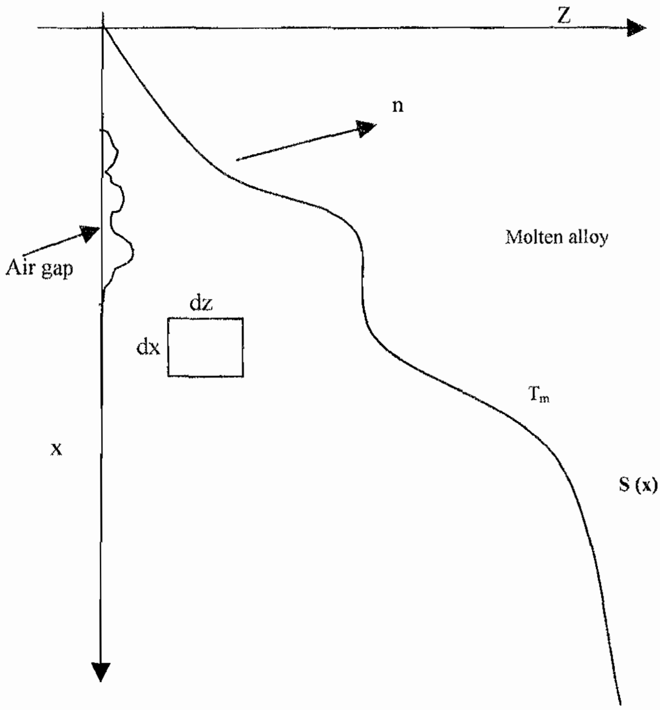

Fig.1b: Advancing boundary of the solidifying metal

L_h is the latent heat of melting and n is the normal to the surface of the solidifying metal as shown in Fig.1b. The meaning of the symbols and their values as relevant to aluminum alloy are presented in Table1.The temperature at the moving boundary is assumed to be equal to that of the melting point of aluminium alloy i.e.

$$T = Tm; \quad z = s(x,t) \tag{3}$$

and the heat flux at the mould wall is given by:

$$K\frac{\partial T}{\partial z} = h(T - Tr); \; z = 0 \tag{4}$$

Considering also that at the initial stage

$$s(x,t) = 0 \text{ at } x = 0 \tag{5}$$

The governing eqns (1) and (2) with the boundary conditions (3) – (5) can be non- dimensionalised to obtain the following eqns (6) to (10):

$$\frac{\partial T}{\partial t} + \frac{\partial T}{\partial x} = a\frac{\partial^2 T}{\partial x^2} + g\frac{\partial^2 T}{\partial z^2}; \; 0 < z < s(x,t) \tag{6}$$

$$\frac{\partial s}{\partial t} + v\frac{\partial s}{\partial x} = \left(-b\frac{\partial T}{\partial x}\frac{\partial s}{\partial x}\;\frac{\partial T}{\partial z}\right)\left(\frac{-b\dfrac{\partial T}{\partial x}\dfrac{\partial s}{\partial x}\;\dfrac{\partial T}{\partial z}}{\sqrt{1 + c\left(\dfrac{\partial s}{\partial x}\right)^2}}\right); \; z = s(x,t) \tag{7}$$

$$T = 1; \; z = s(x,t) \tag{8}$$

$$\frac{\partial T}{\partial z} = h(T - Tr); \; z = 0 \tag{9}$$

$$s = 0; \; z = 0 \tag{10}$$

However when the parameters relevant to the alloy are substituted, the values of a $\approx 10^{-3}$, b $\approx 10^{-3}$ and c $\approx 10^{-3}$, which are small, compared to the value of g. The expression for g is given as:

$$g = \frac{K\kappa\left(T_m - T_r\right)}{L_h \rho v} \tag{11}$$

For this reason the problem can be further simplified by neglecting the terms containing a, b and c.

$$\frac{\partial T}{\partial t}+\frac{\partial T}{\partial x}= g\frac{\partial^2 T}{\partial z^2}; 0<z<s(x,t) \quad (12)$$

$$\frac{\partial s}{\partial t}+\frac{\partial s}{\partial x}=\frac{\partial T}{\partial z};\ z=s(x,t) \quad (13)$$

$$T=1;\ z=s(x,t) \quad (14)$$

$$\frac{\partial T}{\partial z}= h\ (T-Tr);\ z=0 \quad (15)$$

$$s\ =\ 0;\ x=0 \quad (16)$$

The heat transfer coefficient, h, at the mould wall may be taken as constant but if then we consider the presence of air gap we can model it in the form of a sine function:

$$h(x) = 1 + \sin(2n\pi x) \quad (17)$$

n =1 only was considered in our model. The temperature of the mould is assumed not to be constant but to vary in the form

$$T_r = 1 - e^{-\lambda x} \quad (18)$$

Table 1: Definition of symbols and values of parameters used in the model

Thermal conductivity	k =230J/m/s/K
Mould density	ρ = 2650Kg/m^3
Metal thermal diffusivity	K =8.2x10^{5}m^2/s
Specific heat capacity	c= 3x10^{3}J/Kg/K
Latent heat of melting	L_h=3.9x10^{5}J/Kg
Velocity of mould	V = 0-0.25m/s^2
Room temperature	T_r = 25°C
Melting temperature	660°C
Heat transfer coefficient	h= 1100J/s/m^2

The parameter λ is taken as a positive constant. Eqns (12) – (18) can be solved so as to obtain the expression for s(x,t) in terms of x. Attempts were made to solve this problem using integral balance method and the resulting differential equations were solved using Mathematica [5].

1. NUMERICAL RESULTS AND DISCUSSION

In formulating the governing eqns (12 – 18) of the model certain parameters were introduced viz g, λ and h but they are subject to some variations. The physical significance of these parameters needs to be discussed. The parameter, g, is inversely proportional to the velocity of the mould as given by the expression in eqn 11. Table 2 gives the values of g utilized in the modeling and the corresponding values of the mould velocity. It is important to examine how the mould velocity affects the shape and the thickness of the casting. The shape of the casting and its thickness is represented by the curve of s(x) against x. These curves are shown in Fig.2 for eight different values of the mould velocity presented in Table 2. The curves of s(x) against x for velocities below 0.001625m/s, practically coincide and the undulating surface is observed. This implies that neither the thickness of the casting nor the undulating surface is sensitive to changes when the mould velocity is below certain value.

Table 2: Parameter g and the corresponding mould velocities

G	Velocity, v,(m/s)
8	4.06x10^{-4}
4	8.12x10^{-4}
2	1.625x10^{-3}
0.5	6.5x10^{-3}
0.25	1.3x10^{-2}
0.1	3.25x10^{-2}
0.01	3.25x10^{-1}

Table 3: Parameter, λ and and the corresponding initial mould temperature

λ	T_r (°C)
0.1	63.7
0.2	119
0.5	259
0.8	364
1	417
4	570
λ	T_r (°C)

The second parameter, λ, is related to the temperature of the mould before inserting it into the molten metal, given in the expression, $T_r = 1- e^{-\lambda x}$. The values of λ, utilized in the present modeling are presented in Table 3 with the corresponding temperatures of the mould. The effects of the mould temperature on the thickness and shape of the casting are predicted in Fig.3. It can be seen that the surface undulation is remarkable only when the mould temperature is lower than about 120°C. The thickness of the casting is reduced as the mould temperature increases. The results are plausible since lower mould temperature will cause a large thermal difference between the mould and the molten metal so that thermal stresses may provoke surface undulation.

Table 3: Parameter, λ and and the corresponding initial mould temperature

λ	T_r (°C)
0.1	63.7
0.2	119
0.5	259
0.8	364
1	417
4	570

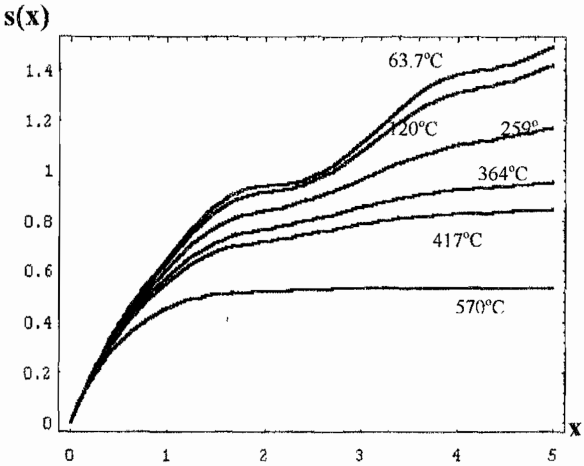

Fig.2: The curve of s (x) against x showing the thickness and shape of the casting at various mould temperatures. No undulation at temperatures above 120°C.

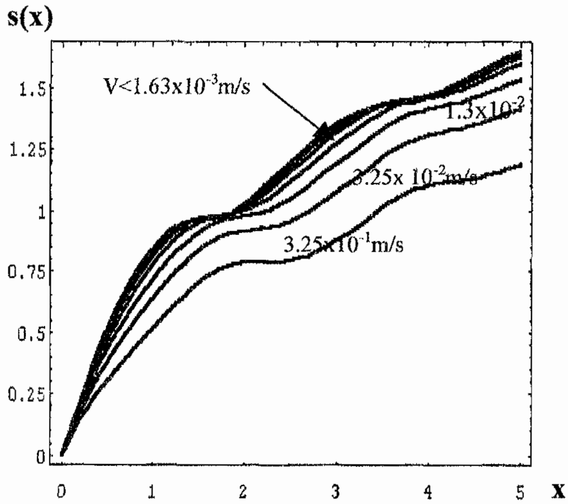

Fig.3: The curve of s (x) against x showing the thickness and shape of the casting at various mould velocities. Thickness and surface undulation remains the same for mould velocities less than 1.63×10^{-3}m/s.

4. CONCLUSION

The surface undulation of the solidified layer was predicted from the model. The prediction shows that neither the thickness of the casting nor its undulating shape is subject to changes when the mould velocity is below certain value. The surface undulation is prominent only when the mould temperature is below about 120°C. It was not possible however to model how the air gap got into the surface of the mould. A main factor affecting the surface undulation is the variation of the heat-transfer coefficient of the mould surface.

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