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Nonlinear Analysis of Micro-Electromechanical Flow Sensors

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Research Article

Abstract

Micro-Electro-Mechanical (MEM) sensors have been used in various applications such as drug discovery, disease diagnosis, detection and characterization of materials, detection of fluid flow parameters, etc. With these devices, sensing is commonly achieved via detection in change of stress/strain/mass with beam static deflection/frequency. In the present study, the behavior of a MEM flow sensor under electrostatic as well as external mechanical loading due to crossflow of non-viscous fluid is modeled. Hamilton's principle was used to derive the governing equations and the numerical solution was obtained with MATLAB code via finite difference. The model is studied with respect to the basic controllable parameters such as voltage input, initial gap, and maximum flow velocity. The results show that nonlinear relationship exists between the velocity of the moving fluid and the pull-in voltage of cantilever beam. Thus, with proper calibration the velocity of the fluid at any instant can be obtained.

Keywords

Cantilever beam; Electrostatic actuation; Fluid flow; Hamilton's principle; Micro-electro-mechanical sensor.

Article History

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1. Introduction

The use of cantilever-based sensing system became popular in mid-1990's (Boison *et al.*, 2011). Since then, its application in various fields such as drug discovery and disease diagnosis, detection and characterization of materials were reported. Most of the studies were carried out at micro level, and the conventional material of choice were silicon and polymer based, or composites (Boison, *et al.*., 2011). Depending on the problem at hand, cantilever beam sensor (CBS) can operate on different design principles. They could be designed to sense change in their surface stress/strain as function of the beam static deflection (Xinke *et al.*, 2007; Boison *et al.*, 2011), to measure mass changes of very lower magnitude (10-21g order) by detecting the change in the beam fundamental frequency and to detect effect related to bulk stress changes.

The nonlinear dynamics of micro-beams under electrostatic and shocking loading for mechanical switch application was previously investigated (Uncuer, *et al.*, 2007). Solution of the governing equation of the beam were obtained using MATLAB SIMULINK and was compared with results from COMSOL and other experimental data. Most flow sensors were designed to measure flow velocity under mechanical loading. (Bouchala, *et al.*. 2016) demonstrated that the non-linear response of electrostatically activated microelectromechanical (MEM) sensor can be utilized for gas sensing. They used an amplitude-based tracking algorithm to determine the quantity of the captured gas level. (Nguyen, *et al.*, 2015) developed an in-plane MEMS capacitive gas flow sensor for measuring the velocity of the flow of surrounding gas resulting from the dynamic gas pressure. (Wu, *et al.*, 2016) developed a MEMS micromachined thermal flow sensors for measuring liquid flow down to 8.3 x 10-13m^3/s. The sensors are integrated with commercial Teflon tubing for the flow rate measurements. The sensors have demonstrated a flow rate resolution below 8.3 x 10-13m^3/s. In the works of (Wang, *et al.*, 2007), a piezoelectric sensing material is deposited on the beam surface to enable the measurement of change in resistance due to the beam deflection corresponding to a given velocity. Similar techniques were used by (Chen, *et al.*, 2003). Non-linear dynamic analysis of micro cantilever beam was also carried by (Liu, *et al.*, 2012) but electrostatic field force was the only load source. In this paper both mechanical and electrostatic loads are considered and combined.

The objective of this research is to model the behaviour of a micro-cantilever beam under electrostatic as well as external mechanical loading due to flow of non-viscous fluid across the beam. The model will provide a useful insight on to the performance of a controllable micro-electro cantilever beam flow sensor. Voltage input will be a means of varying the initial gap of the CBS, so that the maximum flow velocity could be varied. The voltage flow characteristics curve will serve as a means of measuring the flow velocity at any given voltage.

2. Methodology

2.1 Modeling of Micro-Cantilever System

The governing equations of Micro-Electro-Mechanical Beam under electrostatic and mechanical loading is formulated in this section. The famous Hamilton's principle is usually used to derive the form partial differential equations that describe the beams behavior. The electrostatic force term in the governing equationscan be expanded as a Taylor series and be approximated to sixth-term order as in (Liu *et al*.., 2012), neglecting the higher-order terms. Here the whole expression is utilized without approximation.And for better numerical stability, the governing equationshas been non-dimensionalized. The electrostatic load comes from the presence of an electric field while the mechanical load comes from the presence of a moving fluid.

2.2 Governing Equations

Consider the deflection of a MEM-Beam that is caused by the action of an electrostatic field and that of a moving fluid (figure 1). In order to effectively capture the full presence of the mechanical loading, the beam is made free-standing (Wang*et al*.., 2007) and the fluid is assumed to flow perpendicular to the beam surface.

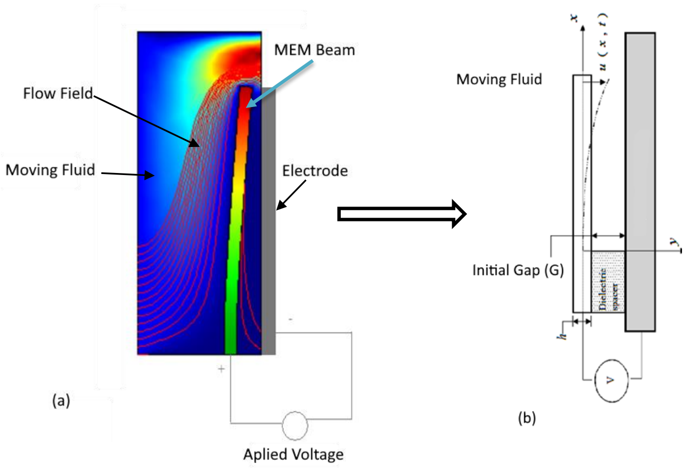

Figure 1: (a) Schematic illustration of the setup, (b) Displacement of MEM beam by action of fluid.

The Hamilton Principle was used to derive the partial differential equations governing the beam's behavior. It states that the sum of the virtual increments in kinetic energy, potential energy and virtual work of a non-conservative force between any two instants (t_1 and t_2) on any continuous deformable body is zero. That is,

$$\delta \int_{t_1}^{t_2} (T - V_P + W_{NC})dt = 0 \qquad (1)$$

The transverse kinetic energy for the deflection of the beam is usually given by:

$$T = \frac{1}{2} \times \int_0^L m(x) \times \left(\frac{\partial u(x,t)}{\partial t}\right)^2 dx \qquad (2)$$

$$\delta T = \frac{1}{2} \times \int_0^L m(x) \times \frac{\partial u(x,t)}{\partial t} \times \delta\left(\frac{\partial u(x,t)}{\partial t}\right) dx \qquad (3)$$

Where:

$m(x) = mass\ per\ unit\ length\ of\ the\ beam,\ u(x,t) = deflection\ of\ the\ beam,\ L = length\ of\ cantilever$

Assuming the cantilever to be one of the terminals of a parallel plate capacitor, we will expect two types of potential energy from the system. One comes from the effect of the beam deflection and the other comes from the effect of electrostatic field. Note that gravitational potential energy is neglected because the weight of the cantilever beam is very small. So, the strain energy or the stored potential energy due to the beam's deflections is given by:

$$V_s = \frac{1}{2} \int_{t_1}^{t_2} \frac{\bar{E}I}{2} \left(\frac{\partial^2 u(x,t)}{\partial x^2}\right)^2 dx, stored\ strain\ energy \qquad (4)$$

Where:

$\bar{E}$ = effective Young's Modulus of the beam

$I = Area\ moment\ of\ inerta\ of\ the\ beam$

The Electrostatic potential energy due to the presence of the electric field is given by:

$$V_e = -\int_0^L \frac{\varepsilon_O w V^2}{2(g-u)^2} dx \qquad (5)$$

Where,

$w = Width\ of\ the\ cantilever$

$\varepsilon_O = permittivity\ of\ free\ space$

$g = gap\ between\ cantilever\ and\ bottom\ electrode$

$V = applied\ potential\ difference$

Then, the total potential energy on the system becomes:

$$V_P = V_s + V_e = \frac{1}{2} \int_{t_1}^{t_2} \frac{\bar{E}I}{2} \left(\frac{\partial^2 u(x,t)}{\partial x^2}\right)^2 - \int_0^L \frac{\varepsilon_O w V^2}{2(g-u)^2} dx \qquad (6)$$

$$\delta V_P = \delta V_s + \delta V_e$$

$$= \frac{1}{2} \int_{t_1}^{t_2} \frac{\bar{E}I}{2} \times \frac{\partial^2 u(x,t)}{\partial x^2} . \delta\left(\frac{\partial^2 u(x,t)}{\partial x^2}\right) - \delta \int_0^L \frac{\varepsilon_O w V^2}{2(g-u)^2} dx \qquad (7)$$

The only non-conservative force acting on the beam comes from the moving fluid. Free standing beam across the fluid flow is the only configuration of beam that can fully sense any slight change in the velocity of the fluid (Wang*et al*.., 2007). Steady, Incompressible flow is assumed for the fluid and the effect of the fluid weight is assumed to be negligible due to the free-standing nature of beam. Going with the same assumption, drag force will be the dynamic force that acts on the cantilever. Unlike other resistive forces which are nearly independent of velocity, drag forces depend on fluid pressure, velocity and viscosity of the fluid. The velocity field that acts perpendicular to the beam constitutes the force per unit length term $f(x,t)$. And $f(x,t)$ can be seen to be changing with time and along the beam as it is bending(as demonstrated in Figure 2 and Figure 8). The velocity distribution is approximately uniform along the beam due to its small size (micro).

Thus, work donedue to non-conservative forces (which comes from a moving fluid in this case) is given by:

$$W_{NC} = \int_0^L f(x,t)\, u(x,t) dx, \qquad (8)$$

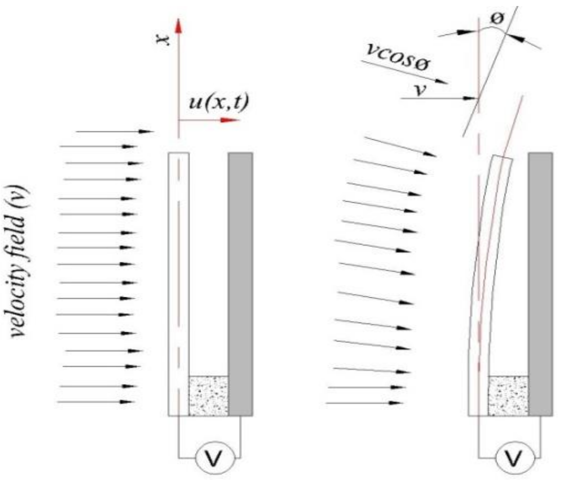

Figure 2: Change in f(x,t) as beam deflects

$$\delta W_{NC} = \int_0^L f(x,t)\,\delta u(x,t)\,dx \tag{9}$$

Where:

$f(x,t)$
$= drag\ force\ per\ unit\ lenght\ due\ to\ fluid\ motion$
$= \frac{1}{2}\rho_m (v\ cos\theta)^2 C_d w$

$$\rightarrow f(x,t) = \frac{\rho_m v^2 cos\theta^2 C_d w}{2}\ , and\ cos^2\theta = \frac{1}{(1+tan^2\theta)} \tag{10}$$

But, $tan\ \theta = \frac{\partial u}{\partial x}$

$$\rightarrow f(x,t) = \frac{\rho_m v^2 C_d w}{2\left(1+\left(\frac{\partial u}{\partial x}\right)^2\right)} \tag{11}$$

$\rho_m = mass\ density\ of\ fluid$
$v = velocity\ of\ flow$
$C_d = Coefficient\ of\ drag$
$\theta = Angle\ suspended\ by\ the\ tangent\ at\ x$

In analyzing the micro cantilever beam system, the fringing field effect is usually taken into account by direct addition on the electrostatic field. It was determined to be,

$$F_f = -0.65 \times \frac{g-u(x,t)}{w} \tag{12}$$

Substituting Equations (3), (7) and (9) into (1) and integrating by parts yield the governing partial differential equation for the cantilever beam as follows.

$$\bar{E}I\frac{\partial^4 u}{\partial x^4} + m_x\frac{\partial^2 u}{\partial t^2} = \frac{\varepsilon_O w V^2}{2(g-u)^2}\left(1 + 0.65 \times \frac{g-u(x,t)}{w}\right) + \frac{\rho_m v^2 C_d w}{2\left(1+\left(\frac{\partial u}{\partial x}\right)^2\right)} \tag{13}$$

The corresponding initial conditions and boundary conditions are given as follows:

$$u(x,0) = \frac{\partial u(x,0)}{\partial x} = 0\ at\ t = 0\ \ \ldots\ldots\ldots\ldots..Initially \tag{14}$$

$$u(x,t) = \frac{\partial u(x,t)}{\partial x} = 0 \qquad at\ x = 0\ \ at\ fixed\ end \tag{15}$$

$$\frac{\partial^2 u(x,t)}{\partial x^2} = \frac{\partial^3 u(x,t)}{\partial x^3} = 0 \quad at\ x = L \quad at\ free\ end \tag{16}$$

Usually, the velocity field should be first obtained by solving the general Navier Stoke's equation of steady incompressible fluid flow depending on the nature of the flow and the corresponding boundary conditions. Hence, fluid flow has been simulated in COMSOL Multiphysics software adopting the actual beam dimensions and estimating the velocity field acting on typical MEMS device. For brevity, only velocity range are stated here (0-3 m/s) and details about such model can be found elsewhere (Liu *et al*.., 2012). In the current research work, the velocity field was assumed values of 0 – 3 m/s (obtained from COMSOL) and flow velocity magnitudes are sequentially coupled to the solution of the governing PDE (Eq. (13)) with the sole aim of studying the sensitivity of the MEMS.

It is very useful to write the governing PDE for the beam in dimensionless units. This is very helpful when investigating which terms are prevalent in a given situation and moreover, numerical convergence could be achieved more easily especially when dealing with extremely small or large dimensions.

The dependent variable, u(x,t) is the unidirectional flow velocity, and the independent variables, (x and t) are non-dimensionalized as follows:

$$\bar{u} = \frac{u}{g}\ ,\quad \bar{x} = \frac{x}{L}\ ,\quad \bar{t} = \frac{t}{T_c},\ \ \bar{W} = \frac{w}{g} \qquad where\ T_c = characteristic\ time \tag{17}$$

By replacing the dimensionalized parameters with the non-dimensionalized ones, and making the resulting coefficients of the PDE to be unity yields:

$$\frac{\partial^4\bar{u}}{\partial\bar{x}^4} + \frac{\partial^2\bar{u}}{\partial\bar{t}^2} = \frac{\bar{V}^2}{(1-\bar{u})^2} + \frac{0.65\times\bar{V}^2}{\bar{w}(1-\bar{u})} + \frac{\bar{v}^2}{\left(1+h\left(\frac{\partial\bar{u}}{\partial\bar{x}}\right)^2\right)} \tag{18}$$

Where,

$$T_c = \sqrt{\frac{m_x L^4}{\bar{E}I}}\ ;\ \bar{V} = \sqrt{\frac{\varepsilon_O w L^4}{2\bar{E}Ig^3}}\ ;\bar{v} = \sqrt{\frac{\rho_{mC_d} W L^4}{2\bar{E}Ig}}\ ;h = \left(\frac{g}{L}\right)^2 \tag{19}$$

Table 1. Microbeam properties & dimensions (Liu *et al* (2012)

Parameter	**Value**
Young's Modulus(E)	155.8 G Pa
Poisson's ratio(v)	0.06
Density($\boldsymbol{\rho}$)	2.33 x 3 Kg/m3
Permittivity of free space($\boldsymbol{\varepsilon_O}$)	8.85 x 10-12 F/m
Width of cantilever beam(w	5000 µ m
Thickness of cantilever beam(t)	57 µ m
Initial gap(g)	92 µ m
Length of cantilever beam(L)	20000 µ m
Density of air($\boldsymbol{\rho_m}$)	1.2 kg/m3
Coefficient of drag($\boldsymbol{C_d}$)	2.0 for fluid flow perpendicular to flat plate

Note: The mass per unit length of the beam is given by: $m_x = \frac{\rho}{wt}$

As determined by Osterberg and Senturia (Osterber *et al.*, 1997), a micro-cantilever beam is considered to be wide when its width is five times its thickness. Since, the cantilever parameters used here satisfy this condition, more accurate result will be obtained by using the effective Young Modulus of a plate in the governing PDE for the beam. The effective Young Modulus is given by:

$$\bar{E} = \frac{E}{(1-\text{v}^2)} \qquad (20)$$

Eq. (18) is solved by adopting the finite difference method. The source term is linearized by adopting the forward difference scheme of first order. The time derivative is discretized with central difference scheme of second order. While, the first and fourth spatial derivative are descritized using the second-order central difference and first-order forward difference schemes respectively.Assuming the spatial independent variable is discretized with equal increments of Δx and time scale by Δt, Eq.(18) becomes:

$$\frac{u_{n-1,i}-2u_{n,i}+u_{n+1,i}}{(\Delta t)^2} + \frac{u_{n,i-2}-4u_{n,i-1}+6u_{n,i}-4u_{n,i-1}+u_{n,i+2}}{(\Delta x)^4} =$$

$$\frac{\bar{V}^2}{(1-u_i)^2} + \frac{0.65\times\bar{V}^2}{\bar{w}(1-u_{n,i})} + \frac{\bar{v}^2}{\left(1+h\left(\frac{u_{n,i}-u_{n,i-1}}{\Delta x}\right)^2\right)} \qquad (21)$$

Eq. (21) is solved iteratively based on the Newton-Raphson scheme. The source term on the RHS of Eq. (21) is computed first before the computation of beam displacement iteratively. It was ensured that the residual of computation does not surpass 1 x 10^{-11}.

3. Results and Discussion

Results obtain from MATLAB finite difference code (based on Equation 7)is shown in Figure 3. The flow velocity has been set to zero and the beam displacement was computed under various voltage. It can be seen that the current result agrees with that reported in previous literature (Liu *et al.*, 2012). Liu *et al.* observed pull-in voltage of 66.7 V, while here the pull-in manifested at 66.6 V which is quite close.

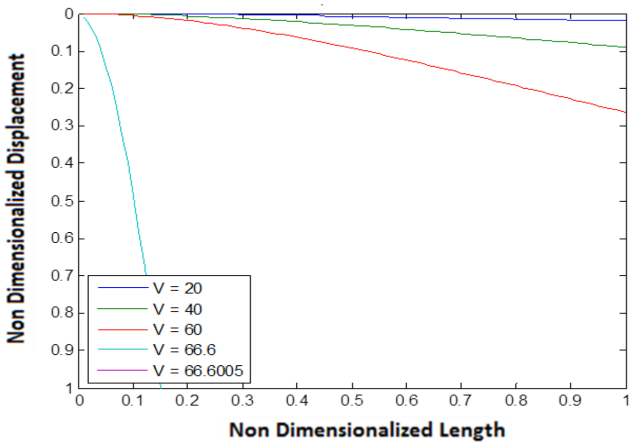

Figure 3: Variation of micro cantilever beam deflection under variable voltage at 0 m/s velocity

For the given material properties and dimensions given in table 1, the velocity and the voltage pushes the beam in the same direction as expected (Fig. 4). It can be seen that, the gap between the beam and electrode reduces as the fluid velocity is increased (the direction of fluid flow being same with that of beam diplacement. Hence, the beam becomes displaced more when the fluid velocity has been increased for a constant voltage. This is because of the exchange of momentum between the flowing fluid and beam structure. At certain velocity, the beam enters the pull-in mode because the gap between the electrode and beam becomes very small. Also, increasing the voltage reduce the pull-in velocity and vice versa. Furthermore, Fig. 4 shows that the relationship between the beam displacement and fluid velocity is non-linear. This is expected, because a square term of the velocity was added in (18). Hence, increase in voltage reduces the initial gap of the beam, thus lowering the pull-in velocity.When the velocity is set to zero and the voltage is varied, a result similar to that obtained in (Liu *et al.*, 2012) is observed as shown in Figure 5 (based on Equation 7). Only that, there is no abrupt pull-in after 67 volts, which might be due to the approximation error resulting from the computation.

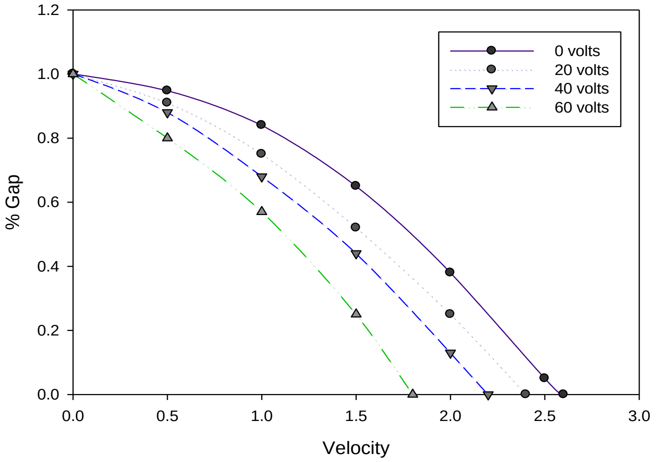

Figure 1: Velocity vs. Gap (constant voltage curves)

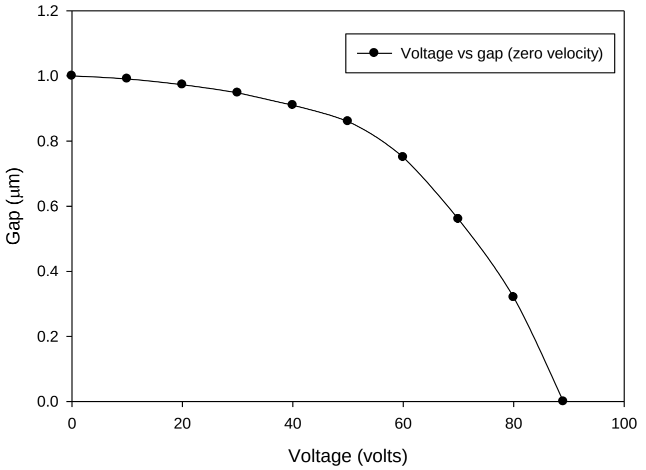

Figure 2: Gap vs. Voltage (zero velocity)

Hence, Figure 5 equally shows that, for a constant velocity, the beam displacement increases with increasing voltage.

Figure 6 (based on Equation 7) shows the state of the beam at various time for 2.2 m/s, upto a steady state. It can be seen that deflection of beam varies from one location to another. The highest deflection is experienced at the tip with the highest non-dimensional value of 0.7. The increase of tip displacement is proportional to time in non-linear fashion. This is because of the non-linear deformation in the beam structure.Also, it can be seen from Figure 7 (based on Equation 7) that the relationship between the velocity of the moving fluid and the voltage of the cantilever is non-linear in nature. Because the interval between the percent pulled contours seem to be decreasing with higher velocity. It can be observed that the maximum velocity used (2.6 m/s) results in pull in of the beam even at 0 Voltage. The plot covers 0 to 60 volts and 0 to 2.6 m/s ranges. More range can be covered by varying the geometry, gap spacing and material property of the beam. And from this, it is obvious very low range change in velocity could be sensed by the sensor. Figure 8 (based on Equation 11) shows the dynamic force due to fluid flow velocity changes along the length of the beam with time.

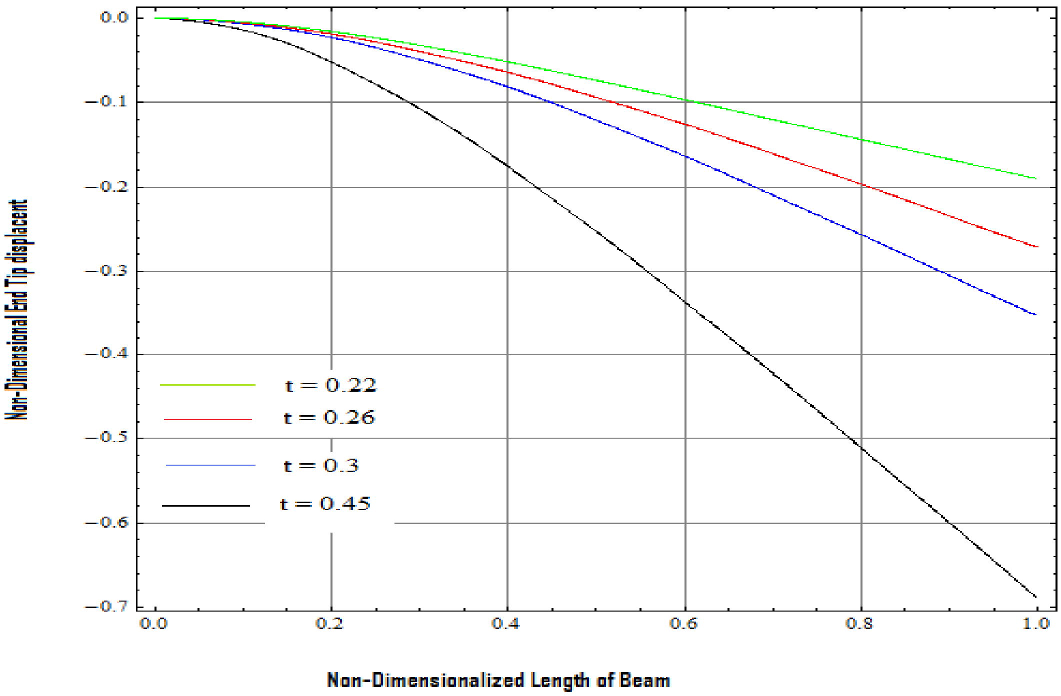

Figure 6: Non-dimensionalized Displacement vs. Length @ 2.2m/s (zero voltage)

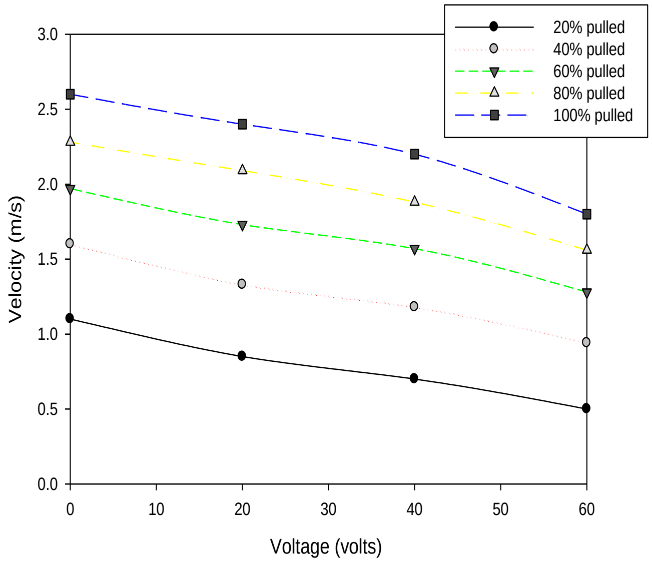

Figure 7: Air Velocity-Voltage Characteristic curve for the CBS

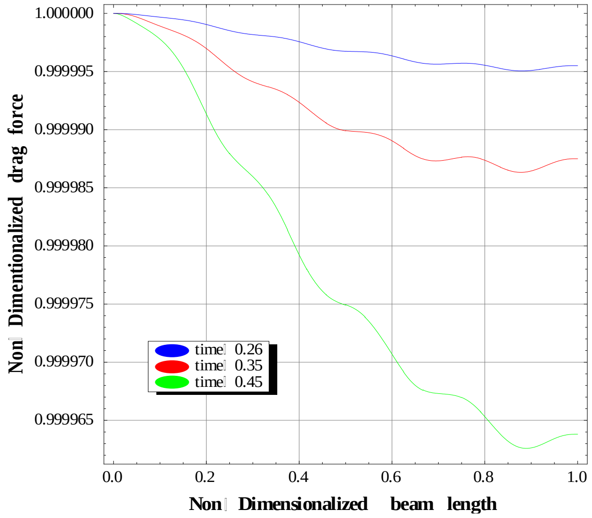

Figure 8: Air Drag Force acting at various points along Beam Length with time

4. Conclusion

Non-linear analysis of a Micro-Electro-Mechanical (MEM) flow sensor has been carried out. The governing partial differential equation was derived using Hamilton's principle. Fluid under steady and incompressible flow was assumed. Air was utilized as the moving fluid. The numerical results obtained were comparable to that of previous work by Liu *et al.* (Liu *et al.*, 2012). The model is studied with respect to the basic controllable parameters of the flow sensor such as voltage input, initial gap, and maximum flow velocity. The results show that nonlinear relationship exists between the velocity of the moving fluid and the pull-in voltage of cantilever beam. The voltage-flow characteristics curves were obtained such that the flow velocity at any given voltage could be determined. Thus, with proper calibration the velocity of the fluid at any instant can be obtained. Hence, the model provides a useful insight on to the performance of a controllable micro-electro cantilever beam flow sensor.

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