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PROBABILITY DISTRIBUTION FOR EXTREME RAINFALL IN MAKURDI METROPOLIS, BENUE STATE, NIGERIA

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ABSTRACT
Statistical analysis of rainfall events and clustering of extreme values are important for risk assessment of floods and decision-making amidst other hydrologic studies. Annual and partial series rainfall data from 1968 to 2015 in Makurdi metropolis was subjected to Frequency analysis in order to determine the best fit probability distribution function (PDF) for the area, to allow for better estimation and prediction of extreme rainfall associated with the area, due to its general low relief that makes it liable to flooding during heavy rain storm. Five commonly used rainfall frequency distributions, namely; Generalized extreme value (GEV); Gumbel (EV1); Generalized Pareto (GPA); Generalized Logistic (GLO) and three parameter Log Normal (LN3) were adopted for the study. The L-Moments method of parameter estimation and a combination of the Chi-squared (χ^2), Anderson-Darling (AD) and Kolmogorov-Smirnov (KS) goodness of fit tests were used for the analysis. The result of the goodness of fit tests showed the best fit distribution for annual series is generalized extreme value (GEV) distribution while the best fit for partial series is Generalized Pareto (GPA) distribution. However, water budget analysis is recommended to identify periods of excess and deficit precipitations.

Keywords: *Rainfall, Frequency Analysis, probability distribution function, Parameter Estimation, Goodness of fit.*

INTRODUCTION
Flooding is one phenomenon that has received quite a lot of attention all over the world, this is evident by innumerable works and studies, both published and unpublished that abounds on it. The reason for its worldwide attention is not far-fetched from its devastating, disastrous and deleterious effects to the environment. The negative impacts of flood cannot be over emphasized, as the Disaster Management Support Group averred flood to be the most frequent and widely distributed disaster in the world, leading to significant economic and social damages than any other natural disaster (Odunuga *et al.*, 2015).

In Makurdi, flooding has almost become an annual event leading to loss of lives and destruction of properties worth millions of naira (Ocheri and Okele, 2012). According to Aderogba (2012) records from the Federal Ministry of Environment shows the occurrence of flood in different parts of the town in the years, 1996, 2000, 2005, 2007, 2008 and 2012. Also, owing to her geographical location, there have been predictions of higher flooding frequency and magnitude (Odunuga *et al.*, 2015).

Several recent studies have attributed extreme rainfall to be the major cause of flood w orldwide (Kunkel *et al.*, 1999; Easterling *et al.*, 2000; Groisman *et al.*, 2001; Koutsoyiannis, 2004; Ologunorisa and Tersoo, 2006). The major cause of flooding in Makurdi has also been reported to be due to heavy rainfall (Shabu and Tyonum, 2013). Hence, the need for thorough analysis of rainfall events and clustering of extreme values to enable effective management of flood, reduction of its associated dangers, decision-making, hydraulic structure design, planning, and other hydrologic studies (Aucoin *et al.*, 2011; Izinyon and Ajumuka, 2013).

Hydrologic frequency analysis is the method used for evaluation of the probability of the hydrologic events which are averaged out in statistical viewpoints, either greater than or equal to a specified magnitude within a certain area, that will occur within a certain period (Lee, 2005). According to Jeb and Aggarwal (2008) the objective of frequency analysis of hydrologic data is to relate the magnitude of extreme events to their frequency of occurrence using probability distributions.

Presently, there is no universally accepted probability distribution model for rainfall frequency analysis. Rather, a whole group of models such as Gumbel (EV1), Generalized extreme value (GEV), Log Pearson type three (LP3), three parameter Log Normal (LN3) etc. have been suggested in literatures (Izinyon and Ajumuka, 2013; Lee, 2005; Jeb and Aggarwal, 2008; Topaloglu, 2002; Galoie *et al.* 2013;Win and Win, 2014). Although some distributions have been adopted by certain regions, while some others are peculiar to the type of hydrologic investigation, for instance the Gumbel (EV1) distribution is the currently recommended distribution for use in Canada, while the Log Pearson type three (LP3) is routinely used in the U.S. (Das *et al.*, 2013); review of literatures indicates that, it is best to use more than one distribution and afterwards select the appropriate model based on its fitness. Five probability distributions functions are therefore investigated in this study to establish the best fit PDF to the annual and partial series rainfall data of the study area.

THEORETICAL BACKGROUND
The theory of some elected distributions and their fitting technique is presented thus:

Generalized extreme value (GEV) distribution
The Generalized extreme value (GEV) distribution is a family of continuous probability distributions that combines

the Gumbel, Frechet and Weibull distributions. It makes use of 3 parameters namely; location (ξ), scale (α) and shape (κ) (Izinyon and Ehiorobo, 2015). ξ describes the shift of a distribution in a given direction on the horizontal axis; α describes how spread out the distribution is, and defines where the bulk of the distribution lies; and κ affects the shape of the distribution, and governs the tail of the distribution.

GEV is a flexible distribution which has been found to fit flood and rainfall extremes in a variety of environments (Izinyon and Ehiorobo 2015). Nadarajah and Choi (2007) studied annual maxima of daily rainfall of 41 years for five locations in South Korea and found that GEV is well fitted to data from each location to describe the extremes of rainfall. Similarly, Shabri *et al.* (2011) identified GEV and GLO distributions as the most suitable distributions for representing the statistical properties of extreme rainfall in Selangor in Malaysia.

The Cumulative distribution function (CDF) and Probability density function (PDF) for GEV as defined in Izinyon and Ehiorobo (2015) are as given in Equations1 and 2. A probability density function (PDF) is a continuous mathematical expression that determines the probability of a particular event (Izinyon and Ajumuka, 2013). The cumulative distribution function (CDF) represent flows of different recurrence intervals (Aucoin *et al.*, 2011) quantile function is given by Equation 3.

$$(CDF)\ F(x) = exp\left\{-\left(1-\frac{\kappa(x-\xi)}{\alpha}\right)^{\frac{1}{\kappa}}\right\} \quad (1)$$

$$(PDF)\ f(x) = \alpha^{-1}exp[-(1-\kappa)y - exp(-y)] \quad (2)$$

$$Q_t = \xi + \left(\frac{\alpha}{\kappa}\right)\left\{1-\left(-LN\left(\frac{T-1}{T}\right)\right)^{\kappa}\right\} \quad (3)$$

Where: T is the desired return period, and

$$y = -\kappa^{-1}\ LN\left[1-\frac{\kappa(x-\xi)}{\alpha}\right],$$

$when\ \neq 0$

Gumbel distribution (EV1)

Gumbel (EV1) distribution is a particular case of GEV which only uses two parameters, namely location (ξ) and scale (α). In hydrology, it is used to analyze such variables as monthly and annual maximum values of daily rainfall, river discharge volumes, flood, and droughts (Galoie *et al.*, 2013).

Over the years, Gumbel has been prevalent for quantifying risk associated with extreme rainfall. Several research works have recommended EV1 for extreme rainfall frequency analysis. An archetype is Galoie *et al.* (2013) who compared four commonly used rainfall frequency distributions, consisting of GEV, EV1, Log Pearson type three and LN3 and concluded that the Gumbel distribution exhibited the best fit.

Despite the prevalence of EV1, it experiences a lot of criticism. Some studies claim that it seriously underestimates the largest extreme rainfall amounts (Coles and Pericchi, 2003). Koutsoyiannis (2004) extended the skepticism for EV1 distribution to the case of rainfall extremes, showing that it is inappropriate for rainfall extremes. He based his argument on the fact that EV1 yields the smallest possible quantiles for small probabilities of exceedence (large return periods); which means that, it results in the highest possible risk for engineering structures. Another point against the EV1 is that, it is a two parameter distribution, which are assumed to be less flexible when compared to three parameter distributions and thus possess less potential (in comparison with three parameter distributions) for describing the underlying magnitude-frequency relationship (Izinyon and Ehiorobo, 2015).

The CDF, PDF and quantile function for EV1 as defined in Galoie *et al.* (2013), are as expressed in Equations 4 – 6.

$$(CDF)\ F(x) = exp\left[-exp\left(-\frac{x-\xi}{\alpha}\right)\right] \quad (4)$$

$$(PDF)\ f(x) = \alpha^{-1}\ exp\left(\left(-\frac{x-\xi}{\alpha}\right) - \left[exp\left(-\frac{x-\xi}{\alpha}\right)\right]\right) \quad (5)$$

$$Q_t = \xi + \alpha y_t \quad (6)$$

Where $y_t = -LN\ \left[-LN\left(1-\left(\frac{1}{T}\right)\right)\right]$

T is the desired return period.

Generalized Pareto distribution (GPA)

The Generalized Pareto distribution (GPA) originally introduced by the Italian economist Vilfredo Pareto, is a simple distribution useful for describing extremes. It is exceptionally useful for modeling events like daily rainfall and all floods above a moderate threshold (Izinyon and Ehiorobo, 2015; Keast and Elliso, 2013). Similar to GEV, it uses three parameters, namely location (ξ), scale (α) and shape (κ).

The GPA is also widely used; it is the recommended distribution in India and one of the four recommended distributions in Australia (Selaman *et al.*, 2007). Recently Izinyon and Ehiorobo (2015) employed its use amongst two other notable distributions (GLO and GEV) in their study for Ogun State Nigeria, and noted it as the best fit. Its CDF, PDF and quantile function as defined in Izinyon and Ehiorobo (2015) are as expressed in Equations 7 - 9.

$$(CDF)\ F(x) = 1 - \left[1-\frac{\kappa(x-\xi)}{\alpha}\right]^{\frac{1}{\kappa}} \quad (7)$$

$$(PDF)\ f(x) = \alpha^{-1}\left[1-\kappa\frac{(x-\xi)}{\alpha}\right]^{\frac{1}{\kappa}-1} \quad (8)$$

$$Q_t = \xi + \left(\frac{\alpha}{\kappa}\right)\{1-(1-F)^{\kappa}\} \quad (9)$$

Where: $F = \frac{T-1}{T}$

Generalized Logistic distribution (GLO)

The generalized logistic (GLO) distribution was introduced to the mainstream of the hydrological literature in 1997 by Hosking and Wallis (Izinyon and Ehiorobo, 2015). As with the GEV distribution, it has three parameters, namely location (ξ), scale (α) and shape (κ). Since its admittance, a couple of researchers have employed its use and found it quite useful. For instance, Shabri *et al.* (2011) identified it as one of the most suitable distributions for representing the statistical properties of extreme rainfall in Selangor, *Malaysia. Also, it is the recommended frequency* distribution for the UK (Selaman *et al.*, 2007). Its CDF, PDF and quantile function as defined in Izinyon and Ehiorobo (2015) are as expressed in Equations 10 – 12.

$$(CDF)\ F(x) = \left[1 + \left(1 - \frac{\kappa}{\alpha}(x - \xi)\right)^{1/\kappa}\right]^{-1} \quad (10)$$

$$(PDF)\ f(x) = \alpha^{-1}\frac{\gamma^{(1-\kappa)}}{(1+\gamma)^2} \quad (11)$$

$$Q_t = \xi + \left(\frac{\alpha}{\kappa}\right)\left\{1 - \left(\frac{1-F}{F}\right)^{\kappa}\right\} \quad (12)$$

Where :

$$\gamma = \left[1 - \frac{\kappa(x-\xi)}{\alpha}\right]^{\frac{1}{\kappa}}\ for\ \kappa \neq 0$$
$$F = \frac{T-1}{T}$$

Three parameter Log normal (LN3) Galton distribution

LN3 distribution also known as Galton distribution is a combination of the normal distribution and the modified logarithmic transformation. LN3 has also seen many applications in hydrology and has been positively reviewed (Aucoin *et al.*, 2011; Galoie *et al.*, 2013).It is recommended in Indiana, USA, Continental, USA, Saskatchewan, Canada, Australia and South-western, USA (Selaman *et al.*, 2007). It also has three parameters, namely location (ξ), scale (α) and shape (κ). Its CDF, PDF and quantile function as defined in Ware and Lad (2003) are presented in Equations (13) to (15).

$$(CDF)\ F(x) = \Phi\left(\frac{LN(x-\xi) - \alpha}{\kappa}\right) \quad (13)$$

$$(PDF)\ f(x) = \frac{1}{(x-\xi)\sqrt{2\pi\kappa^2}}exp\left[-\frac{1}{2}\left(\frac{LN(x-\xi)-\alpha}{\kappa}\right)^2\right] \quad (14)$$

$$Q_t = \xi + exp\{\alpha + \kappa\Phi^{-1}(F)\} \quad (15)$$

Where: Φ is the standard Normal distribution function.

METHODS

Study Area

Makurdi, the Benue State capital is located between latitudes 7°37’ and 7°47’ North and longitude 8°27’ and 8°40’ East and is within the floodplain of the lower Benue river, central Nigeria. Temperatures are generally high throughout the year, with mean maximum and minimum values of 32°C and 26°C respectively. March and April are the hottest months. The mean monthly relative humidity varies from 43% in January to 81% in July-August period (Tyubee, 2009). The climatic condition of Makurdi as stated by Ologunorisa and Tersoo (2006) is influenced by two air masses namely the warm, moist south-westerly air mass, and the warm, dry north-easterly air mass. The south-westerly air mass is a rain-bearing wind that brings about rainfall from the months of May to October. While the dry north-easterly air mass blows over the region from November to April, thus bringing about seasonal drought.

The physiographical characteristics span between 73 m to 167 m above sea level (Ocheri and Okele, 2012). A huge part of the town is waterlogged and flooded during extreme rainfall due to its location in a flood plain. The town is drained basically by the Benue River (which traverses it into north and south banks) and its tributaries, such as Rivers Idye, Genebe, Urudu, Kpege and Kereke (Ocheri *et al.*, 2010).

Data Collection

Daily rainfall data for Makurdi with station number (0708.43) at latitudes 7°41’ N and longitude 8°37’E, for a period of 48-years, spanning from 1968 to 2015 was obtained from the Nigeria Meteorological Agency (NIMET) Oshodi, Lagos.

Data Analysis Techniques

The extreme rainfall frequency analysis was done for both annual maximum and partial duration series, using GEV, EV1, GPA, GLO and LN3 as described in theoretical background of the study. The linear moments (L) method was used for parameter estimation, and the AD, KS and χ2 were used for the goodness of fit analysis. For annual series, the annual maximas were extracted from the rainfall record sheets; for partial series, rainfall values greater than or equal to 50 mm were extracted. A five percent (5%) level of significance was used for the study, while Microsoft excel 2010 was used for the analysis of the results. The Mean rainfall for the study period (1955 to 2015, exempting 1965, 1966 and 1977) was 1208.94 mm with a standard deviation of 221.55 mm and skewness of -0.06. The maximum annual rainfall through the study period was 1729.9 mm, which occurred in 1993 and the minimum annual rainfall through the study period is 678.1 mm, which occurred in 1958.

Parameter Estimation

Once the probability distribution model has been selected the next step is to estimate the parameters of the distribution from the sample data, so that required quantiles can be calculated from the fitted model. The parameters for each of the five distributions were estimated using the L moments method. The L-Moments is a modification of the probability weighted moments (PWMs) (Izinyon and Ehiorobo, 2015). Based on its characteristics and advantages, the L-Moments method is thus particularly useful in providing accurate quantile estimates of hydrological data in developing countries where small sample size typically exists (Izinyon and Ehiorobo, 2015).

The L-Moments λr, of a random sample X is definable in terms of PWMs (Chen *et al.*, 2006). In particular, the first four L- moments ($\lambda 1$, $\lambda 2$, 3 $and \lambda 4$) as given by Das *et al.* (2013) are as expressed in Equations 16 to 19. Other useful ratios are L-CV (τ2), L-Skewness (τ3) and L-Kurtosis (τ4), which are defined by Equations (20) to (21). Table 1 summarizes the distributions’ parameters.

$$\lambda_1 = L1 = M_{100} \quad (16)$$
$$\lambda_2 = L2 = 2M_{110} - M_{100} \quad (17)$$
$$\lambda_3 = L3 = 6M_{120} - 6M_{110} + M_{100} \quad (18)$$
$$\lambda_4 = L4 = 20M_{130} - 30M_{120} + 12\,M_{110} - M_{100} \quad (19)$$
$$\tau_2 = L2/L3 \quad (20)$$
$$\tau_3 = L3/L2 \quad (21)$$

$$\tau_4 = L4/L2 \quad (22)$$

Where, the PWMs as given by Das *et al.* (2013) are defined by Equations 23 to 26.

$$M_{100} = Sample\ mean = \frac{1}{N}\sum_{i=1}^{N} Q_i \quad (23)$$

$$M_{110} = \frac{1}{N}\sum_{i=1}^{N} \frac{(i-1)}{(N-1)} Q_i \quad (24)$$

$$M_{120} = \frac{1}{N}\sum_{i=1}^{N} \frac{(i-1)(i-2)}{(N-1)(N-2)} Q_i \quad (25)$$

$$M_{130} = \frac{1}{N}\sum_{i=1}^{N} \frac{(i-1)(i-2)(i-3)}{(N-1)(N-2)(N-3)} Q_i \quad (26)$$

Table 1: Distributions' parameters

Distribution	Parameter	Formula
GEV	κ	7.8590 [illegible] .9554 [illegible]
	c	$\frac{2}{3+\tau_3}$ [illegible] $\frac{LN\,2}{LN\,3}$
	α	[illegible] $\lambda_2 \kappa$ / $(1-2^{-\kappa})\Gamma(1+\kappa)$
	ξ	$(1-2^{-\kappa})\Gamma(1+\kappa)$ Γ $\lambda_1 - \alpha\{1-\Gamma(1+\kappa)\}/\kappa$ is the gamma function
EV1	α	[illegible] $\frac{\lambda_2}{log\,2}$
	ξ	λ [illegible] ([illegible] γ is Euler's constant (0.5772)
GPA	κ	$\frac{(1-[illegible]\tau_3)}{(1+\tau_3)}$
	α	[illegible] 1)([illegible] 2)]
	ξ	[illegible] 2)
GLO	κ	$-\tau_3$
	α	[illegible] $\Gamma(1+\kappa)\Gamma(1-\kappa)$
	ξ	[illegible]
LN3	κ	0.999281 [illegible] .006118 [illegible] 0.000127 [illegible]
	z	[illegible])Φ [illegible] $\left(\frac{1+\tau_3}{2}\right)$
	α	[illegible] $LN\left(\frac{\lambda_2}{erf(\kappa/2)}\right) - \frac{\kappa^2}{2}$
	ξ	[illegible] $\lambda_1 - exp\left(\alpha + \frac{\kappa^2}{2}\right)$

Computation of CDFs and PDFs

The estimated parameters were used to compute the CDFs and PDFs for each of GEV, EV1, GPA, GLO and LN3 using their individual CDF and PDF equations described earlier.

Goodness of fit test

The goodness of fit of a statistical model describes how well it fits a set of observations. Measures of goodness of fit typically summarize the discrepancy between the observed values and the expected under the model in question (Galoie *et al.*, 2013). The goodness of fit test was done using AD, KS and χ2tests.

Anderson-Darling test (AD)

The AD test compares an observed CDF to an expected CDF (Shukla and Kumar, 2010). The AD test statistic (A2) is defined by Equation 27 (Das *et al.*, 2013).

$$A^2 = -n - \frac{1}{n}\sum_{i=1}^{n}(2i-1)\left[LNF(x_i) + LN(1 - F(x_{n-i+1}))\right] \qquad (27)$$

Where:

LN is the natural logarithmic value,
F(xi) is the CDF value
i is the ordered data point
n is the sample size.

Kolmogorov-Smirnov (KS) test (D)

KS is based on the largest vertical distance from the theoretical and empirical CDFs (Galoie *et al.*, 2013). It is non-parametric and used to test for equality of probability distributions (Tramblay *et al.*, 2013). The test statistic (D) is defined in Equation 28 (Das *et al.*, 2013)

$$D = \mathrm{Max}\left(F(x_i) - \frac{i-1}{n}, \frac{1}{n} - F(x_i)\right) \qquad (28)$$

Where:

LN is the natural logarithmic value,
F(xi) is the CDF value
I is the ordered data point
n is the sample size.

Chi-Squared test (χ2)

For this study the number of bins (k) as adopted from Das *et al.* (2013) was determined by Equation (29):

$$k = 1 + \log_2 N \qquad (29)$$

The test statistic (χ2) was calculated by Equation 30 (Das *et al.*, 2013).

$$\chi^2 = \sum_{i=1}^{k}\frac{(O_i - E_i)^2}{E_i} \qquad (30)$$

Where:

Oi is the observed frequency;
Ei is the expected frequency,
E_i=NPi;
Pi is the difference between upper and lower cumulative probability for the interval i; and,
N is the sample size.

Table 2: Parameter estimation results

Distributions	Parameters	Annual series	Partial Series
GEV	κ	-0.1563	-0.279
	α	19.0087	10.832
	ξ	76.7313	59.862
EV1	α	22.4576	15.129
	ξ	78.1864	61.457
GPA	κ	0.1399	-0.062
	α	37.9694	19.051
	ξ	57.8387	49.871
GLO	κ	-0.2739	-0.362
	α	13.7149	8.372
	ξ	84.3898	64.342
LN3	κ	0.5706	0.764
	α	3.7426	2.947
	ξ	41.4793	44.680

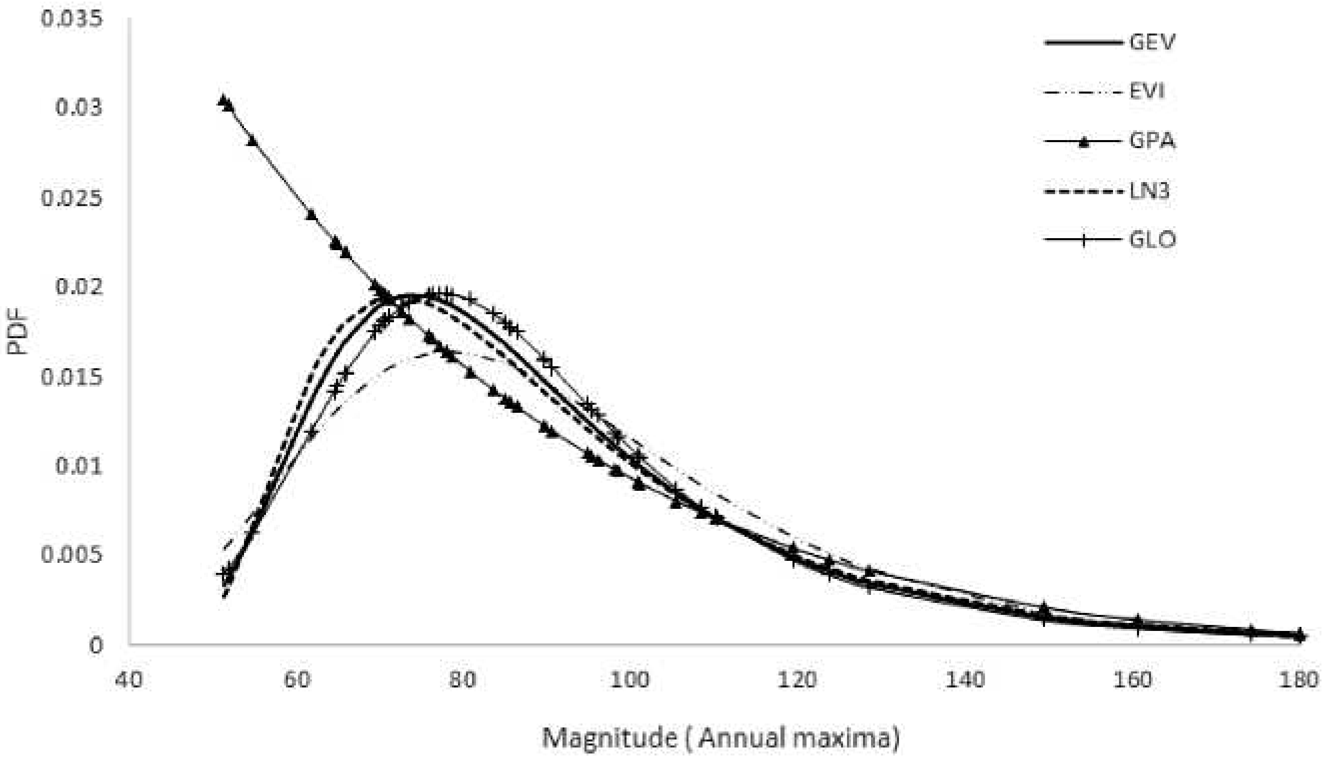

Figure 1: Annual series PDF plot of the selected distributions

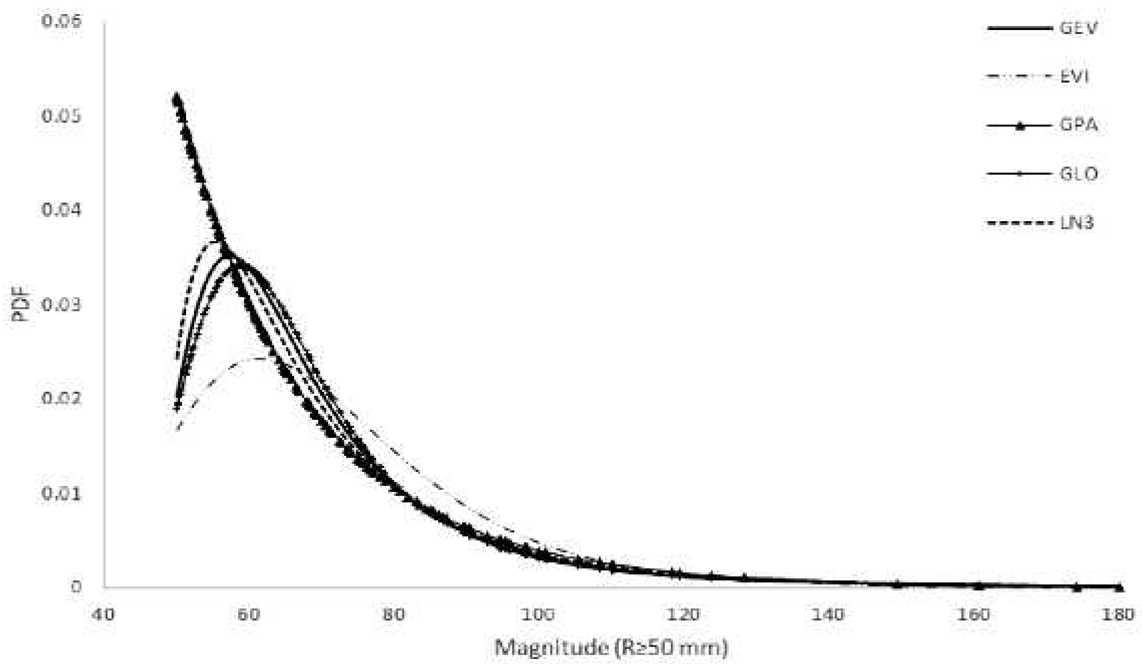

Figure 2: Partial series PDF plot of the selected distributions

Table 3: Goodness of fit ranking

PDF	Annual series					Partial series				
	AD	KS	χ^2	Total Score	Rank	AD	KS	χ^2	Total Score	Rank
GEV	5	4	4	13	1	3	3	2	8	3
EV1	3	3	0	6	3	0	0	0	0	5
GPA	1	0	0	1	4	5	5	4	14	1
GLO	4	5	0	9	2	2	2	3	7	4
LN3	2	2	5	9	2	4	4	5	13	2

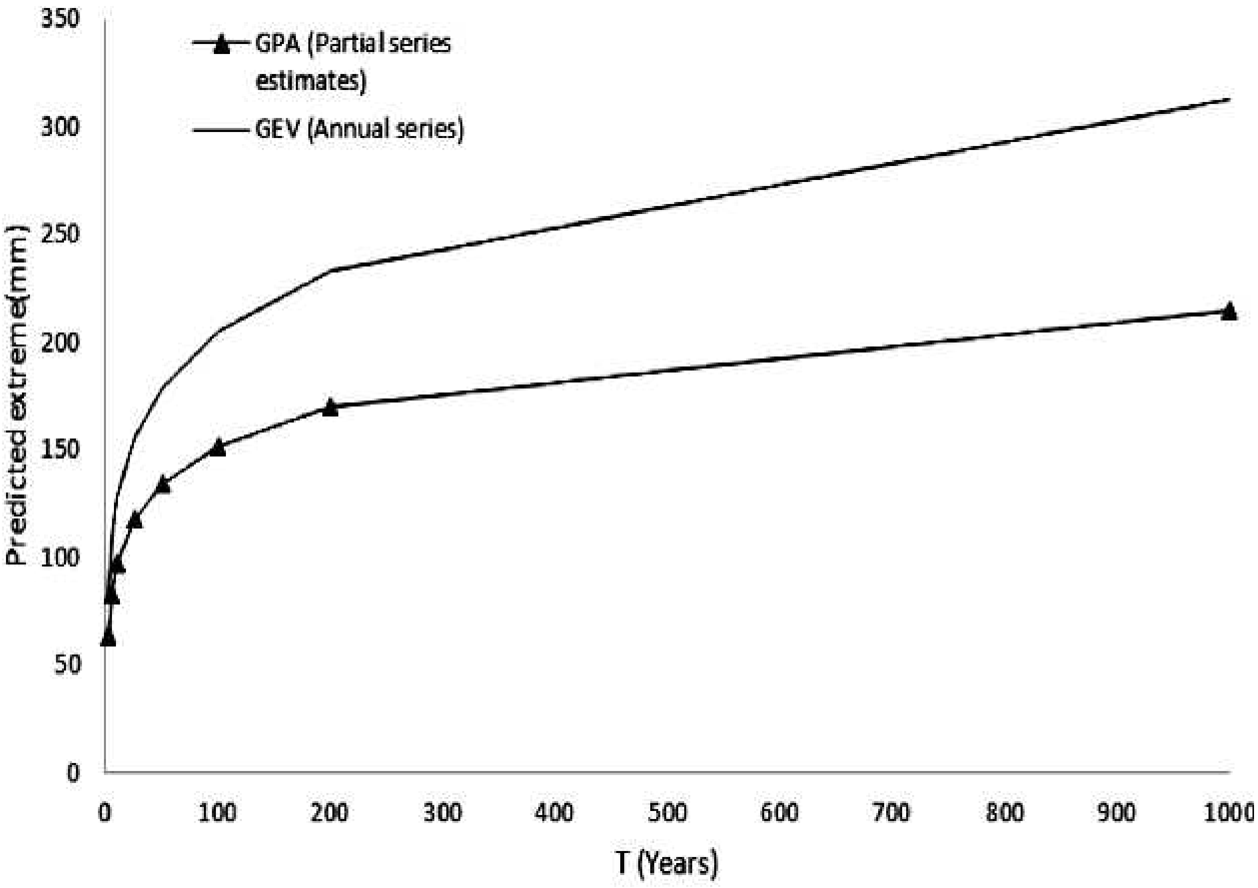

Figure 3: Quantile estimates for GEV/annual series and GPA/partial series

CONCLUSION

The outcome of this study shows that GEV distribution is the best fit distribution for annual series and the best fit distribution for partial series is the GPA. Predictions from both partial and annual series indicate a progressive increase in extreme rainfall magnitudes in the coming years.

The GEV/annual series could be adopted as the most preferred series for Makurdi extreme rainfall analysis based studies and designs; because it predicts higher extreme quantiles than the partial series (i.e. it presents the worst case scenario). However, water budget analysis is recommended for the study area in order to identify periods of excess and deficit precipitations

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