



Empirical Investigative Analysis on the Effect of Triangular and Gaussian Membership Function on Fuzzy-Based Controlled Vehicle Platooning

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Research Article

Abstract

Fuzzy controlled vehicle platoon system provides a simplified and robust approach to achieving platoon string stability and uniform inter-vehicular gap keeping in autonomous vehicle platoon. However, the fuzzification and defuzzification method adopted affects the characteristics of the platoon to a large extent, while also determining velocity stability timing, although researcher select a fuzzification/defuzzification method based on comfort, familiarity or simplicity. This work undertakes to investigate the effect of fuzzification and defuzzification method on vehicle platoon, to provide evidence on the selection criteria and how it affects the controlled system. The results obtained show that the best performing combination was reported as triangular/centroid with 4.44 secs velocity stability for vehicle V3, and 1.91 secs distance stability for follower vehicle, when compared to Gaussian/MoM with 79.88 secs velocity stability V3 and 85.89 secs for follower vehicle being the worst performing combination.

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1. Introduction

Autonomous vehicles are also referred to as Connected and Automated Vehicles (CAVs), or driverless vehicles (Elliott *et al.*, 2019). Autonomous car assumed to be able to communicate with other vehicles and road infrastructure, relying on onboard sensors for information gathering and decision making. The communication technology adopted for autonomous vehicles for sharing information is in the form of V2X, a paradigm that covers several communications such as Vehicle-to-Infrastructure (V2I), Vehicle-to-Vehicle (V2V) and Vehicle-to-Pedestrian (V2P) (Shrivastava, 2019). V2X is enabled by two communication technologies based on the Vehicular Ad Hoc Network (VANET) which are Dedicated Short Range Communication (DSRC) based on IEEE 802.11p (Shukla *et al.*, 2020) and Long-Term Evolution (LTE) cellular, sometimes referred to as Cellular-V2X (Abou-Zeid *et al.*, 2019).

Vehicle platooning system is the arrangement of multiple vehicles in motion, such that they aim to maintain the same velocity and keep equal distance between adjacent vehicles (Horowitz *et al.*, 2000). One of the earliest platooning system is PATH program in California (Nowakowski *et al.*, 2015), earlier platoon system relied only on Radar-based sensor for data acquisition.

The benefit of vehicle platooning includes fuel consumption (Axelsson *et al.*, 2016), improved traffic efficiency, increased road throughput, road and vehicle safety (Abdulnabi, 2017). Some of the performance measures of platoon are obtained from amount of fuel saved, road throughput with platoon compared to without platoon, string stability, the distance between vehicles and velocity of individual vehicles.

Conventional vehicular platoon control strategies require good knowledge of the entire system for an efficient model to be developed, where controllers such as proportional-integral-derivative (PID) are used (Fiengo *et al.*, 2019), robust and adaptive control capabilities are usually lacking due to the non-linearity and time-varying nature of the entire system (Xavier and Pan 2009). The use of fuzzy logic control systems (FLC) provides a robust control methodology, using a simplified fuzzy rule based on the understanding of vehicular operation and limitation, while also considering dynamic non-linear nature of entire system.

FLC system is an intelligent process control system that adapts to human un-precise concept and knowledge directly to control a process. The generalized block diagram of fuzzy control system is shown in Figure 1.

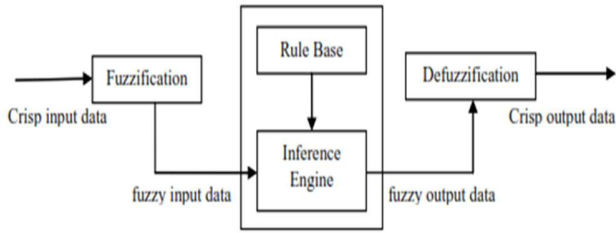


Figure 1: Generalized Fuzzy Logic Control System

Fuzzy Logic was first introduced by Lotfi A. Zadeh, of the University of California at Berkeley in a 1965 paper (Pappis and Mamdani, 1977). FLC system has been used earlier on in vehicle related research areas.

Fuzzy Logic control system can provide a simplified approach for platoon implementation in unmanned ground vehicles to take advantages of platooning and intelligent control methodology, however, the selection of the best fit fuzzification/defuzzification technique will increase overall robustness and assist in simplifying implementation based on evidence.

Fuzzification process converts crisp value (real world) classical data into fuzzy data referred to as membership function. There are several types of membership function applied in fuzzy control system, the most common is the Triangular, Gaussian, Trapezoidal, generalized bell and sigmoidal membership function, which is depicted in Figure 2(a-e).

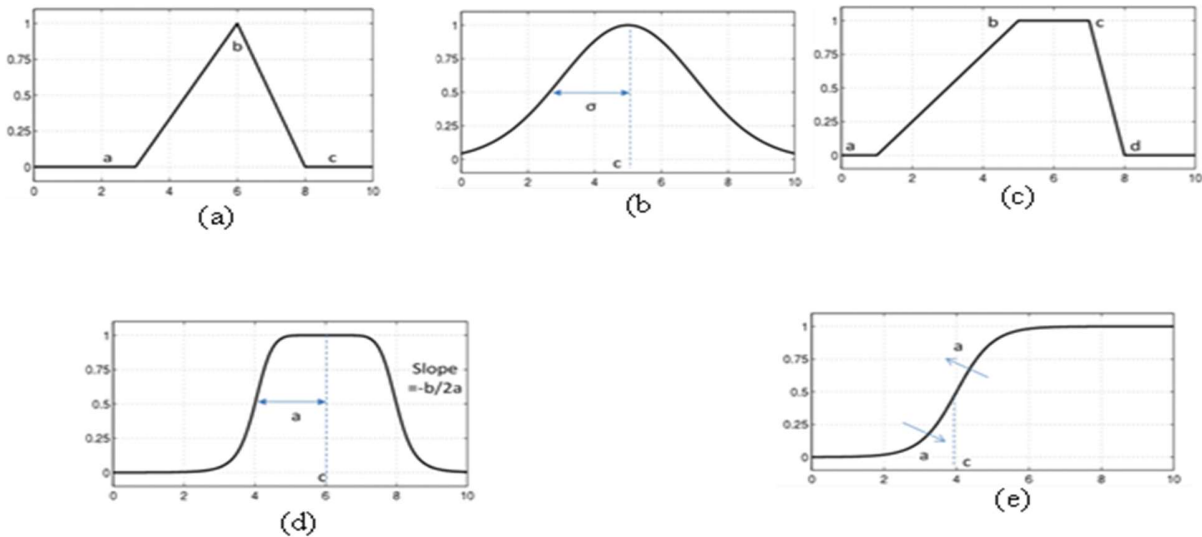


Figure 2: Fuzzy Membership Function Types (a) Triangular (b) Gaussian (c) Trapezoidal (d) Generalized Bell Sigmoidal (Nishida and Sugeno, 1985)

Defuzzification process is where an inferred fuzzy knowledge from the inference engine is converted into crisp value for driving the actuator. This is performed according to the output membership function (Ma *et al.*, 2018). Some of the defuzzification methods in use are; Centroid of Gravity (CoG), Weighted Average, Bisector of Area (BOA) and Maxima methods which includes; First of Maxima (FOM), Last of Maxima (LOM) and Mean of Maxima (MOM). Literatures report that the COG is the most frequently used defuzzification method, due mainly to physical appeal (He and Peng, 2020). This paper therefore attempts to provide empirical evidence of the effect of fuzzification/defuzzification combination on a platoon system control by investigating a triangular, Gaussian, centroid, mean of maxima and bisector combinations.

Some of the works carried out in the field of vehicle platooning that relate to fuzzy control system details the

adoption of particular fuzzification and defuzzification methodology often without justification or comparison to other selectable combinations. Some of these research works include; Ma *et al.*, (2018). In their work, Hierarchical Fuzzy Logic Based variable structure control applied to Platoon of Vehicles was proposed. A two-layer fuzzy controller is developed for robustness in uncertain operations triangular membership function was adopted. In He and Peng, (2020), the authors proposed a Gaussian learning-based fuzzy predictive cruise control platooning system, applied for improvement in safety of connected vehicles. Also, the work presented by Li *et al.*, (2018) developed a Fuzzy Logic Control System for Vehicle Platooning dependent on V2V communication. The authors used a combination of PID and fuzzy logic control technique to maintain variable time-gap within vehicles in platoon, and implemented a mean-of-maxima defuzzification method without stated justification.

The rest of the paper is structured as follows. Section one introduces the fuzzy controlled vehicle platoon system, while section two discusses the modelling of the vehicle platoon system. Fuzzy logic membership functions are given in section three, while the simulation scenario is given in section four. Results and discussions are given in section five, and finally section six gives the conclusion.

2. System Model

In obtaining a Platoon of 3 vehicles all homogenous and having the same model representing the BMW Series 5 Sedan, some parameters of importance are the desired velocity the vehicle in platoon are expected to travel, the desired distance between all the cars in the platoon, the initial velocity of the cars in the platoon and also the acceleration/deceleration capabilities of the vehicles.

Assumptions were made to enable the implementation of the platoon. These assumptions are as follows:

- i. The cars were assumed to be capable of accelerating up to $50ms^{-2}$ and deceleration of $-40ms^{-2}$. A distance keeping range of between $0-20m$ is implemented, the distance between vehicles is adjustable as required before simulation is carried out. An illustration of the platoon layout is presented in Figure 3.

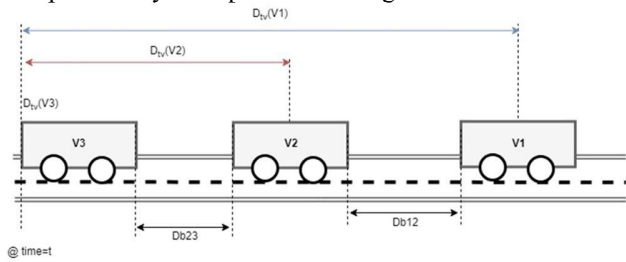


Figure 3: Three Vehicle Platoon Arrangement

Vehicle parameters are presented in Table 1, showing the manufacturer's specification

Table 2.1: Technical specification for BMW 520d model [24]

Parameter	Value
Wheelbase	2975mm
Dimension	4936/1868/1467mm
Drag Coeff.	0.42
Rolling Friction Coeff.	0.012
Air Resistance (ρ)	0.22X2.35
Wheel radius	2785mm
Max torque	400Nm
Speed	235km/h

As illustrated from Figure 3, the distance between vehicles in the platoon is computed and used to determine the platoon stability. The distance travelled by vehicle number 3 (V3) at time t is zero, distance travelled by vehicle number 2 (V2) is $D_v(V2)$, while that travelled by vehicle (V1) is $D_v(V1)$. Likewise, the distance between each of the cars in the platoon is computed, with distance between vehicle (V1) and (V2) is $Db(12)$ while the distance between vehicle (V2) and (V3) is $Db(23)$, the dynamic distance model of each vehicle in the platoon is obtained by equation (1).

$$D(t + \delta t) = D(t) + v(t)\delta t + \frac{1}{2}a(\delta t)^2 \quad (1)$$

The velocity of each vehicle in the platoon is determined by equation (2), where initial velocity can be determined at the start of the simulation.

$$V_f = V_i + a(\delta t) \quad (2)$$

The acceleration or retardation of the vehicle is assumed between $50ms^{-2}$ to $-40ms^{-2}$.

3. Fuzzy Logic Membership Functions

The membership functions used for the research are:

3.1 Triangular Membership Function

Triangular membership functions are functions such that each value can be dynamically adjusted. The ranges of the functions are defined as:

$$\mu_A(v) = \begin{cases} 1 - \frac{|v - P_v|}{w} & , \text{for } |v - P_v| \leq w \\ 0 & \text{otherwise} \end{cases} \quad (3)$$

The equation describing the membership is given as [25]:

$$\text{trimf}(x; a, b, c) = \begin{cases} 0 & x \leq a \\ \frac{x - a}{b - a} & a \leq x \leq b \\ \frac{c - x}{c - b} & b \leq x \leq c \\ 0 & c \leq x \end{cases} \quad (4)$$

Where trimf denotes triangular membership function, x is the universal discourse, a , the lower limit, c the upper limit, and a value b .

$$\mu_A(v) = \begin{cases} 1 - \frac{|v - P_v|}{w} & , \text{for } |v - P_v| \leq w \\ 0 & \text{otherwise} \end{cases} \quad (5)$$

Where w is the width of the triangular curve, when v is equal to P_v . This ensures that the membership function is 1, when the vehicle velocity reaches the desired set platoon velocity.

3.2 Gaussian Membership Function

The Gaussian membership function make use of the linguistic variables and annotation. The membership functions are defined as:

$$\mu_A(x) = e^{-\frac{(x-m)^2}{zk^2}} \quad (5)$$

The Gaussian membership function is described by a central value c , and a standard deviation $\sigma > 0$. Gaussian is characterized by the smaller the value of σ , the narrower the bell shape. In this paper, the Mamdani inference engine provided in MATLAB® is used.

3.3 Defuzzification: Centroid of Gravity

For this paper, three defuzzification methods are employed; centroid, bisector and mean of maxima method. Equation (6) describes centroid of gravity.

$$x^* = \frac{\sum_{i=1}^n x_i \mu(x_i)}{\sum_{i=1}^n \mu(x_i)} \quad (6)$$

Equation (6) was applied to discrete membership function, where the defuzzifier value denoted by x^* , x_i indicates the sample element, $\mu(x_i)$ represents the membership function and n is the number of elements in the sample. However, for continuous membership function, x^* is defined as

$$x^* = \frac{\int x \mu_A(x) dx}{\int \mu_A(x) dx} \quad (7)$$

3.4 Bisector

The method of defuzzification is defined as in equation (8):

$$\int_{\alpha}^{x^*} \mu_A(x) dx = \int_{x^*}^{\beta} \mu_A(x) dx \quad (8)$$

Where, $\alpha = \min\{x|x \in X\}$ and $\beta = \max\{x|x \in X\}$

3.5 Mean of Maxima

For the mean of maxima method, the defuzzifier value is defined as [26]:

$$x^* = \frac{\sum_{x_i \in Mx_i} x_i}{|M|} \quad (9)$$

Where, $M = \{x|\mu_A(x_i)\}$, is equal to the height of the fuzzy set $\{A\}$ and $|M|$ is the cardinality of the set M .

4. Simulation Scenario

The performance evaluation of the 3-vehicle platoon under fuzzy control effort is carried out by simulation in phases, where the desired platoon velocity and intervehicle distance is set while the fuzzification and defuzzification methods are changed to observe the stability time, and performance through the distance, acceleration and velocity graphs. This is shown in Table 1

Table 1: Simulation Scenarios for Method Comparison

Platoon Settings (Scenarios)		Controller Settings	
Initial settings	Desired Settings	Fuzzification	Defuzzification
Db _{i12}	Db ₁₂	Triangular	Centroid
Db _{i23}	Db ₂₃	Triangular	Mean of Maxima
V _{iV}	P _V	Triangular	Bisector
V _{iA}	V _A	Gaussian	Centroid
		Gaussian	Bisector

Table 1 discusses the simulation scenarios which includes the initial settings of the vehicle platoon, the desires setting, fuzzification and defuzzification. While the linguistic variables defined for the platoon control system is given in Table 2.

Table 2: Linguistic Variable Definition

Linguist variable	Notation
Distance between vehicles (D _{n,n-1}) – where n= 1, 2,3., and D _b Desired inter-vehicle distance.	
Too Far (when the distance is far > D _b)	TF
Far (when the distance is > D _b)	FR
Okay (when the distance is about = D _b)	OK
Too Close (when the distance is far < D _b)	TC
Close (when the distance is < D _b)	CL
Velocity of vehicles (V _n) – where n= 1, 2,3., and P _V is set platoon velocity.	
Too Slow (when the velocity is far < P _V)	TS
Slow (when the velocity is < P _V)	SL
Okay (when the velocity is about = P _V)	OK
Fast (when the velocity is > P _V)	FS
Too Fast (when the velocity is far > P _V)	TF
Acceleration of vehicles (A _n) – where n= 1, 2,3.,	
Accelerate High	AH
Accelerate	AC
Okay	OK
Decelerate	DC
Decelerate High	DH

5.0 Results and Discussions

The following results were obtained from Matlab 2022a environment. The results obtained are as follows:

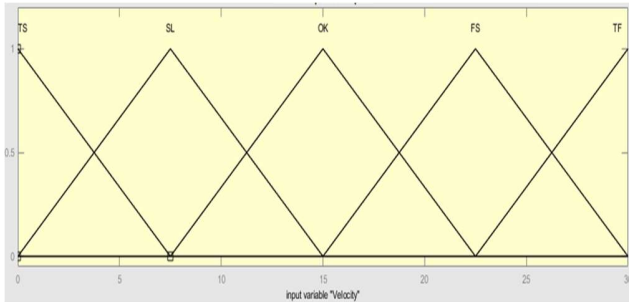


Figure 4: Triangular Membership function at 15m/s desire velocity

From Figure 4, the desired velocity defined by the defined membership function serves as one of the fuzzy control inference system using a triangular fuzzification system entirely for both inputs (velocity and distance) and also for the output method (acceleration). The triangular membership function for the acceleration is shown in Figure 5.

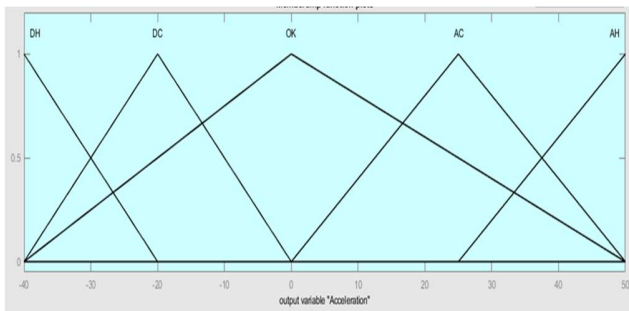


Figure 5: Acceleration (Output) Membership Function

The triangular membership function for the acceleration control as output -40 m/s^2 to 50 m/s^2 . As the velocity and distance of the vehicles in the platoon changes for follower vehicles, the deceleration also changes, depending on the rule base, from Decelerate High assigned -40 m/s^2 to -20 m/s^2 , while the acceleration is considered Okay at 0 m/s^2 .

Figure 6 shows the Gaussian membership function representing distance parameters.

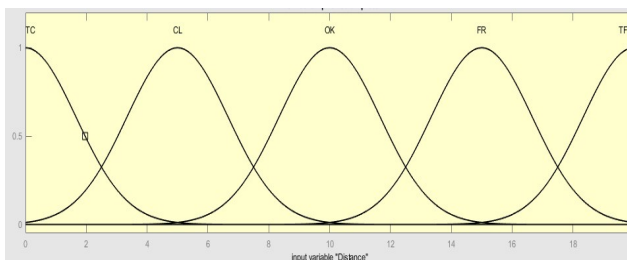


Figure 6: Gaussian Membership Function for Distance

From Figure 6, it depicts the case of the lead vehicle in which only velocity and acceleration membership functions are required, as it does not compare distance between any other vehicles in the platoon. While Figure 7 shows the Gaussian membership function variable representing acceleration parameter.

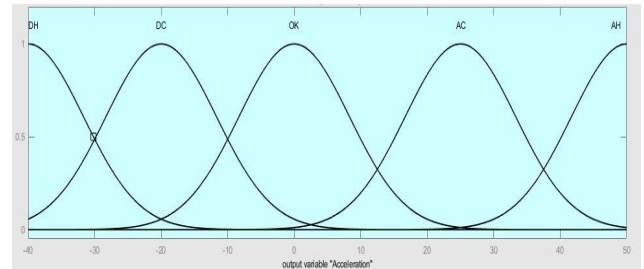


Figure 7: Output Membership function for Acceleration

From Figure 7, the acceleration membership function is the output of the fuzzy logic control system, which is required by both the lead vehicle and the follower vehicles.

The follower vehicles have a total of 25 rules, in the form of ‘If-and-If-Then’ relating the Distance and Velocity membership function to the output acceleration function. Also, the lead vehicle is governed by only 5 rules due to the absence of distance consideration.

Figure 8 shows the fuzzy rule surface viewer which displays the relationship between the velocity, distance and acceleration.

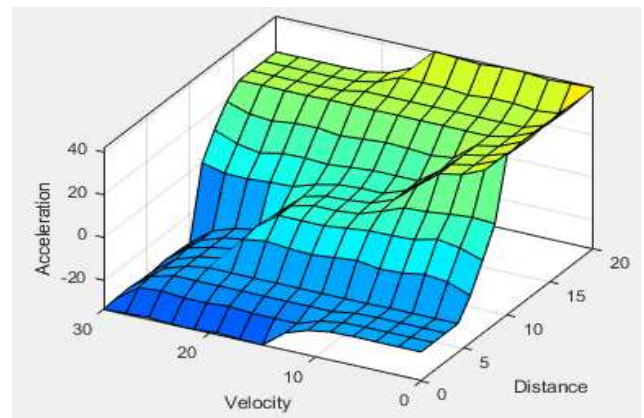


Figure 8: Fuzzy Rule Surface Viewer

Considering the acceleration plot of the vehicles at the output, the performance of the platoon vehicles are compared under different combination of fuzzification and defuzzification methods as stated in Table 1. Figure 9 shows the acceleration performance plot of the platooning vehicles under triangular and centroid methods.

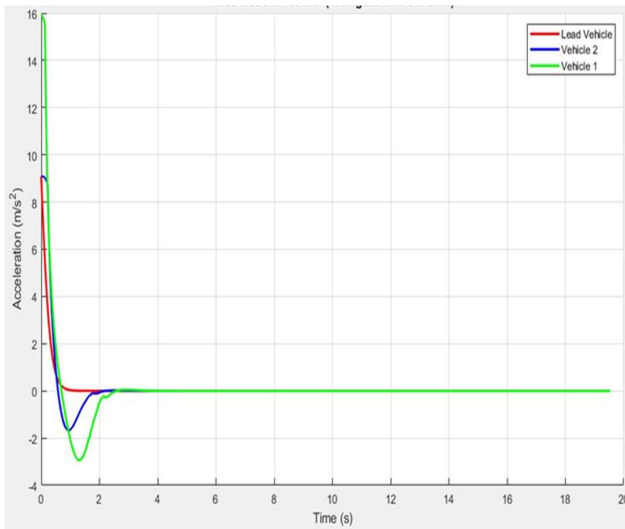


Figure 9: Platoon performance under triangular and centroid methods

From Figure 9, it showed that the lead vehicle accelerates to about 9 m/s^2 , and no change in acceleration from about 2 seconds into the journey, while vehicle V2 starts at about 9 m/s^2 , but decelerates to about 1.8 m/s^2 before reaching the required velocity. Likewise, the third vehicle has maximum acceleration of 16 m/s^2 , while the maximum deceleration of about 3 m/s^2 .

Figure 10 shows the performance of the platooning system under triangular/bisector method.

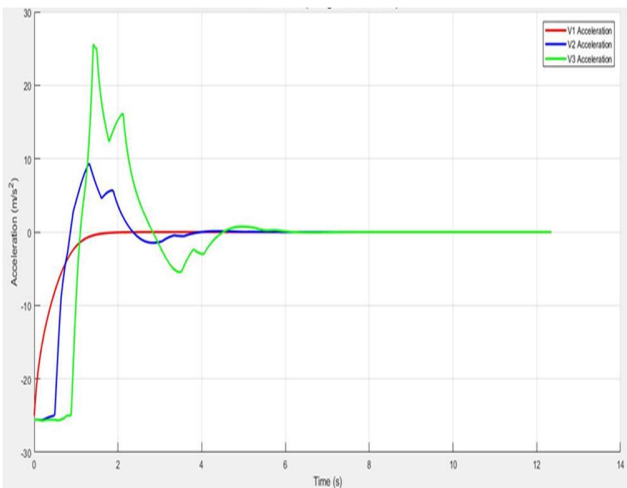


Figure 10: Platoon Vehicle Acceleration plot with Triangular and Bisector Method

From Figure 10, it showed that the vehicles start in deceleration state of about 25 m/s^2 , this is due to the initial velocity of the vehicles as set during the platoon setup phase. Although, the final value reaches zero without any positive acceleration in the case of the lead vehicle at about 1.5 seconds, the second and third platoon vehicles experienced

positive acceleration to reach desired velocity in a time of about 5 and 6 seconds respectively.

Figure 11 shows the performance of the platooning system under Gaussian and centroid methods.

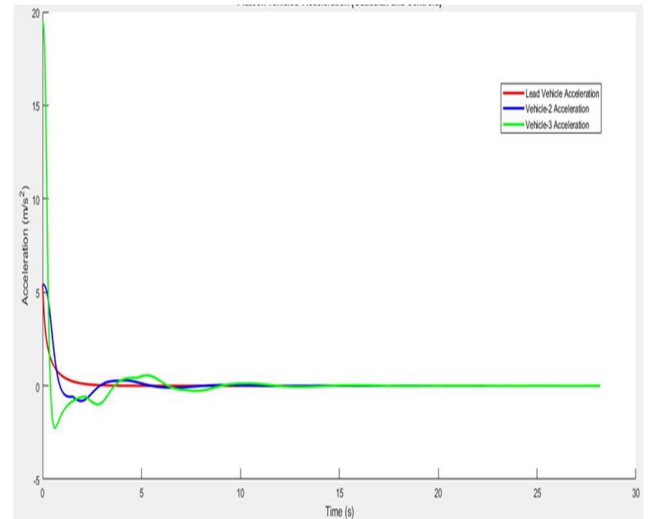


Figure 11: Platoon acceleration under Gaussian and Centroid methods

From Figure 11, it showed that the vehicle acceleration took longer time to reach zero under the Gaussian and centroid methods. The lead vehicle reached 0 m/s^2 after 3 seconds, V2 reached 0 m/s^2 after 6 seconds, while the third vehicle V3 reached 0 m/s^2 after about 8 seconds. This showed that the acceleration response of the vehicles under Gaussian and centroid is slower when compared to triangular fuzzification method. This affects the overall performance of the various method combination in terms of velocity and distance stability times.

A comparative analysis was carried out between the various methods discussed above with respect to velocity and distance stability respectively. This is given in Table 3.

From Table 3 above, it can be observed that the combination of fuzzification and defuzzification methods have significant effect on the performance of the fuzzy control system. Different combination of triangular fuzzification and defuzzification were compared along with different Gaussian methods. The combination with the best stability time measure is triangular/centroid at 1.7 secs lead vehicle stability, 4.44 secs V3 vehicle stability and 1.91 secs distance stability between V2 and V3. The most underperforming combination is Gaussian/mean of maxima with 43.64 secs velocity stability for lead vehicle and 85.89 secs distance stability between V2 and V3 respectively.

Table 3: Fuzzification/Defuzzification Comparison

Platoon Settings	Method	Velocity Stability (s)	Distance Stability (s)
$P_{iv} = 10$ m/s	Triangular Centroid	Lead (V1) = 1.74 secs	
		V2 = 3.75 secs	V1 and V2 = 1.51 secs
		V3 = 4.44 secs	V2 and V3 = 1.90 secs
$P_v = 12$ m/s	Gaussian Centroid	Lead (V1) = 7.02 secs	
		V2 = 15.2 secs	V1 and V2 = 15.02 secs
		V3 = 24.5 secs	V2 and V3 = 17.77 secs
$Db_{i23} = 7$ m	Gaussian Bisector	Lead (V1) 27.12 secs	
		V2 = 39.2 secs	V1 and V2 = 12.11 secs
		V3 = 33.35 secs	V2 and V3 = 37.07 secs
$Db_p = 6$ m	Triangular Bisector	Lead (V1) = 3.5 secs	
		V2 = 6.91 secs	V1 and V2 = 3.07 secs
		V3 = 9.15 secs	V2 and V3 = 5.27 secs
	Triangular and MoM	Lead (V1) = 22.22 secs	
		V2 = 32.21 secs	V1 and V2 = 22.71 secs
		V3 = 33.80 secs	V2 and V3 = 41.05 secs
	Gaussian and MoM	Lead (V1) = 43.64 secs	
		V2 = 79.91 secs	V1 and V2 = 83.10 secs
		V3 79.88 secs	V2 and V3 = 85.89 secs

6. Conclusions

paper considers the investigation of various fuzzification/defuzzification methodology comparison to establish empirical evidence on performance to selectable fuzzy control methods when applied as a control technique in system. The fuzzy control approach was applied to a 3-vehicle autonomous platoon, a combination of triangular/centroid, triangular/bisector, triangular/mean of

maxima was compared Gaussian/centroid, Gaussian/bisector and Gaussian/mean of maxima under the same platoon parameter settings. The best performing combination was reported as triangular/centroid with 4.44 secs velocity stability for vehicle V3, and 1.91 secs distance stability for follower vehicle, when compared to Gaussian/MoM with 79.88 secs velocity stability V3 and 85.89 secs for follower vehicle being the worst performing combinations.

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