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Prediction and Simulation of Kainji Hydropower Reservoir Operation in Nigeria

A. A. Mohammed[1], B. F. Sule[2], A. W. Salami[3], A. G. Adeogun[4], A. M. Ayanshola[5], A. S. Oladeji[6]

[1,3,6]National Centre for Hydropower Research and Development University of Ilorin, Kwara State – Nigeria,
[2,3,5]Department of Water Resources and Environmental Engineering University of Ilorin, Kwara State – Nigeria,
[4]Department of Civil Engineering, Kwara State University, Malete, Kwara State – Nigeria.
[1]rasaq.muhammed@yahoo.com

Research Article

Abstract

Kainji hydropower reservoir streamflow was forecast from 2017 to 2050 using historical streamflow data and a Markov model. The model was evaluated with statistical parameters. Various percentages of water stored in the reservoir as Ecological Flow Release (EFR) were used to simulate future energy generation subject to operational constraints on storage and turbine releases. It was observed that the model forecast the sreamflow adequately with coefficient of determination (R^2), Mean Absolute Error (MAE), Relative Bias (RB) and correlation coefficient (r) of: 0.98, 0.10, 0.002 and 0.99 respectively. Total simulated energy generation reduced as EFR percentage increased. The energy generation simulated was greater than the observed generated energy for the scenarios. The outcomes of this study can be applied to the Kainji hydropower operational management.

Keywords
Forecast; Generated energy; Hydropower; Markov model; Reservoir; Streamflow

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1. Introduction

Streamflow measurement is very important in surface water hydrology (Mustafa and Yusuf, 2012). It is first and most essential requirement for planning and management of any water resource project (Patra, 2008; Vaghela and Vaghela, 2014). Streamflow data are very scarce in Nigeria and many Africa countries because many rivers are ungauged. Monthly streamflows are stochastic in nature, this necessitates modelling and predicting streamflow data for effective planning and management of water resources system (Sen, 1978). In modelling monthly reservoir inflow, the Thomas and Fierring model based on first order Markov model was used to synthesize flow series using historical streamflow data (Yurekli and Kurunc, 2015). Thomas and Fierring in 1962 used Markov model to simulate streamflow when monthly inflow, q_i are normally distributed with first order auto regressive model.

Alfa *et al.* (2018) assessed reliability of Thomas-Fiering method of streamflow prediction using gauged data of 1955-1973. The model was calibrated with multiple linear regressions. The results revealed that the model is highly reliable in predicting streamflow. Sharma *et al.* (2018) generated streamflow of *a* river in India using Thomas-Fiering model. Performance of the model was evaluated using statistical approaches. The results revealed that the model performed satisfactorily. Markov model is a basic concept of stochastic process used in modelling streamflows, rainfall, temperature or other phenomena whose values change with time (Loucks and Beek, 2005; Saminu *et al.*, 2014; Al-Mansori *et al.*, 2016).

The Kainji dam is located in New Bussa, Borgu Local Government Area of Niger State, Nigeria. Kainji hydropower reservoir is fed by Malando, Danzaki and Sokoto-Rima rivers. It lies at an altitude of 108 m above sea level, between Yelwa (latitude 10° 53'N: longitude 4° 45' E) and Kainji (latitude 9° 50'N: longitude 4° 35' E). Figure 1 shows the location of the dam on a map of Nigeria. It is underlained by basement complex rocks such as porphyritic granite, mica and quartzite (Ifabiyi, 2011). The reservoir that resulted from Kainji dam was built between 1964 and 1968 and commenced operation in 1968 for the purpose of generating electricity (Ifabiyi, 2011). The maximum water surface elevation is 141.9 m above sea level. Kainji Lake is the largest man-made lake in Nigeria with a surface area of 1270 km^2. The storage capacity is 15 x 10^9 m^3 with a total live storage of 12 x10^9 m^3. Kainji hydropower reservoir has an installed capacity of 760 MW. The maximum length, width are 136.8 km, 24.1 km, 60 m respectively, while the and mean depth is 11 m. Kainji reservoir is characterized by prolonged high temperature, low rainfall and low relative humidity; it exhibits evaporation values that are in excess of rainfall (Abam, 2001). The dam has eight plants with total installed capacity of 760 MW (four-80 MW, two-100 MW and two-120 MW). The spillway is equipped with radial gates having a total spilling capacity of 7,900 m^3/s (Jimoh, 2008).

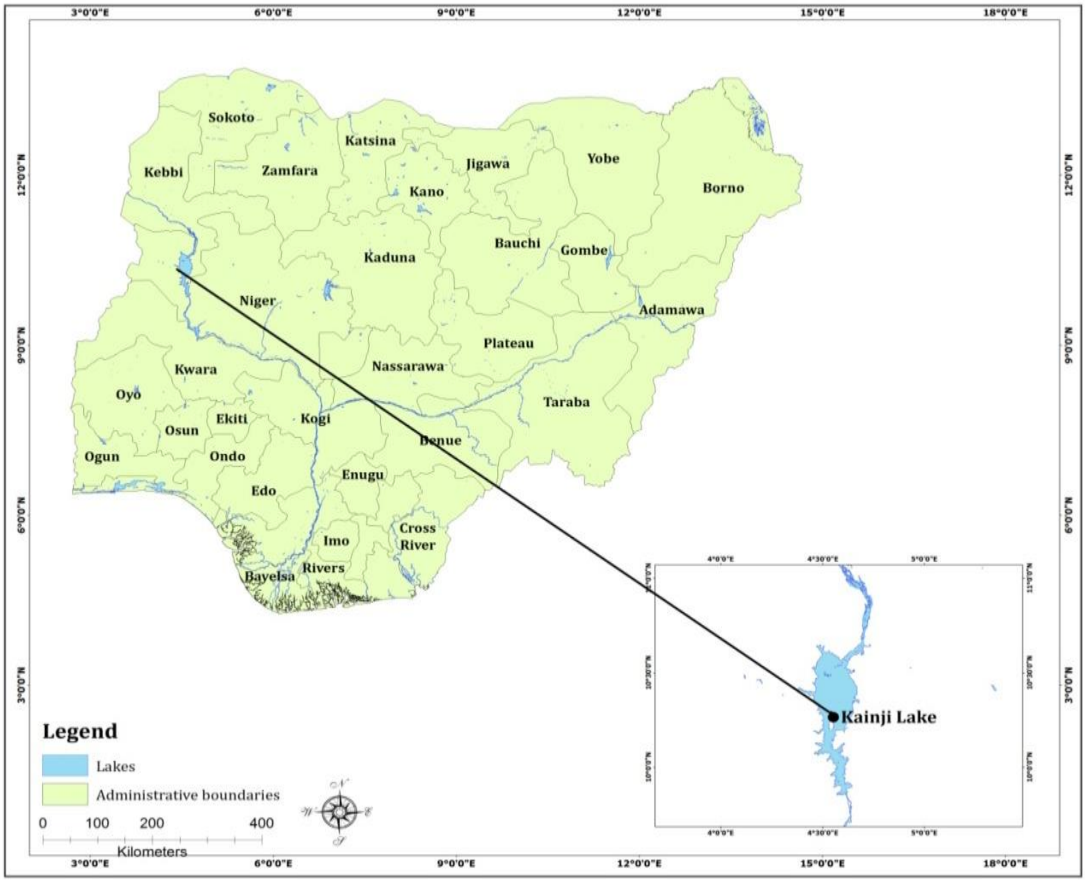

Figure 1: Map of Nigeria showing location of Kainji lake

This study aimed at predicting and simulating streamflows and energy generation at the Kainji hydropower station in Nigeria. Streamflows into the reservoir was predicted from the year 2017 to 2050 using historical data and Markov model based on Thomas Fierring approach. Monthly reservoir storage, turbine release and energy generation were simulated using the established mass balance and energy generation equations. Also, scenarios of ecological flow releases (EFR) of certain percentages of storage were introduced in order to determine its impacts on the energy generation.

2. Methodology

Kainji hydropower reservoir operation data on mean monthly reservoir storage, turbine discharge and energy generation were collected from the hydrology section of the Mainstream Energy Solution Limited (also known as Kainji hydropower station) for a period of fourty six years (1970-2016). Mean, standard deviation and lag one serial correlation were computed using the observed monthly historical streamflow data and Microsoft Excel. If there is N years of available data, the calculation of the terms in Thomas and Fierring model for each month, j = 1, 2, 3..., 12 according to (McMahon and Mein, 1978) are presented in (1) to (3).

$$\bar{q}_i = \sum_{i=1}^{N}\left(\frac{q_{j,i}}{N}\right) \quad \text{(i=j, 12+j, 24+j,...)} \tag{1}$$

$$S_j = \sqrt{\frac{\sum_{i=1}^{N}\left(q_{j,i} - \bar{q}_j\right)^2}{N-1}} \tag{2}$$

$$r_{j,\,j+1} = \frac{\sum_{i=1}^{N}\left(q_{j,\,i} - \bar{q}_i\right)\left(q_{j+1,i} - \bar{q}_{j+1}\right)}{\sqrt{\left\{\sum_{i=1}^{N}\left(q_{j,i} - \bar{q}_j\right)^2 \sum_{i=1}^{N}\left(q_{j+1,1} - \bar{q}_{j+1}\right)^2\right\}}} \tag{3}$$

Where N is the number of observed value, q_{j+1}, q_j are the generated flows during $(i+1)^{th}$ and i^{th} seasons from the beginning of the synthesized sequences and $\bar{q}_{j+1}$, $\bar{q}_j$ are the mean flow during $(j+1)^{th}$ and j^{th} seasons within a repetitive annual cycle of seasons, (for monthly period, $1 \leq j \leq 12$). S_{j+1}, S_j are standard deviations of flows during the $(j+1)^{th}$ and j^{th} seasons, while r_{j+1}, r_j correlation coefficients between flows in j^{th} and $(j+1)^{th}$ seasons.

Thomas and Fierring log-normal seasonal algorithm model presented in Equation 4 was adopted in this study (McMachon and Mein, 1978; Viessman *et al.,* 1989; Salami, 2007; Dashora *et al.*, 2015; Eldaw *et al.*, 2019) to predict the monthly streamflow into the Kainji hydropower reservoir from 2017 to 2050 because it has been established that it is an accurate model for predicting streamflow (Alfa *el al.*, 2018). The model is limited to log-normal distribution because monthly streamflow is assumed to be log-normally distributed (Sharma and Panu, 2012; Salami, 2007).

$$y_{i+1} = \mu_{i+1} + b_{j,\,j+1}\left(y_i - \mu_i\right) + k_{i+1}\,\sigma_{j+1}\left(\sqrt{\left(1 - r^2{}_{j,\,j+1}\right)}\right) \quad \text{i= 1 to } n \qquad (4)$$

where:

$y_i = \ln q_i$

μ_i = log transform of the mean annual historical inflows

y_i , y_{i+1} = generated log-normal inflow in the month i and i+1th respectively

$b_{j,\,j+1}$ = least squares regression coefficient for estimating (j+1)th flow from the jth flow, but $b_{j,\,j+1}$ can be computed with Equation 5.

$$b_{j,\,j+1} = r_{j,\,j+1}\left(\frac{S_{j+1}}{S_j}\right) \qquad (5)$$

Z_{i+1} = normal random number with zero mean and variance unit = Normsinv (rand ())

k_{i+1} = normally distributed random number with zero mean and unit variance

σ_{j+1} =standard deviation of annual inflows of the log transform

n = period of prediction in month

The other terms are as defined in (1) and (2).

Normally distributed random number k_i was considered to be a log-normal random number and was the only unknown variable in the model and for each time step it is estimated as a pseudo-random normal number. These numbers were samples from random variable between specified intervals that have equal probability. The generated random number and its appropriate use are very important in stochastic simulation. The k_i values were estimated using the Microsoft Excel functions NORMSINV (RAND ()) which is the inbuilt function for generating random number in Microsoft Excel. The values of k_i generated were multiplied by the random part of stochastic variable in Equation 1 to generate streamflows (Salami, 2007). The predicted streamflows were used to simulate reservoir storage, turbine release and energy generation subject to boundary condition observed from historical data.

Reservoir storage at next time step was simulated using storage mass balance equation with reference to inflow, turbine discharge, previous and current storages as presented in Equation 6, subject to boundary condition on storage presented in Equation 7. The water stored in the reservoir is bounded by dead storage (S_{min}) and reservoir capacity (S_{max}) as presented in Equation 8. The available turbine release at the next time step based on reservoir mass balance equation was simulated with the expression in Equation 9 subject to boundary conditions on turbine release presented in Equation 10. The water release through the turbine is bounded by minimum (R_{min}) and maximum (R_{max}) release as presented in Equation 11. In simulating reservoir operation, initial storage and turbine release were assumed to be full and optimum respectively being the maximum volume of water that can be drawn from the reservoir and the optimum draft from the reservoir that gives reasonable quantity of energy. The optimum turbine release was estimated as 2217.88 Mm^3 (Mohammed, 2018; Mohammed *et al.*, 2019). Relationship between the observed operational head, H_t (m) and reservoir storage, S_t (Mm^3) in Equation 12 was used to estimate the value of H_t with the boundary conditions presented in Equation 13. The operational head is bounded by minimum (H_{min}) and maximum (H_{max}) observed head as shown in Equation 14. Mean monthly energy was simulated using Equation 15 based on the relationship between the energy generation from a reservoir during period t which depends on the operational head (m) and reservoir release (m^3) (Wan *et al.*, 2020 Chen *et al.*, 2013).

$$S_{t+1} = S_t + I_t - R_t \qquad t = 1 \text{ to } n \qquad (6)$$

$$S_{t+1} = \begin{cases} \text{If} & S_{t+1} > S_{\max}, \quad S_{\max} \\ \text{If} & S_{t+1} < S_{\min}, \quad S_{\min} \\ \text{Otherwise,} & S_{t+1} \end{cases} \qquad t = 1 \text{ to } n \qquad (7)$$

$$S_{\min} \le S_t \le S_{\max} \qquad t = 1 \text{ to } n \qquad (8)$$

The limit on the reservoir capacity is given as

$$3000.00 \le S_t \le 12000.00$$

$$R_t = S_t + I_t - S_{t+1} \qquad t = 1 \text{ to } n \qquad (9)$$

$$R_t = \begin{cases} \text{If} & R_t > R_{\max}, \quad R_{\max} \\ \text{If} & R_t < R_{\min}, \quad R_{\min} \\ \text{Otherwise,} & R_t \end{cases} \qquad t = 1 \text{ to } n \qquad (10)$$

$$R_{\min} \le R_t \le R_{\max} \qquad t = 1 \text{ to } n \qquad (11)$$

The limit on the turbine release is given as

$$500.00 \le S_t \le 3900.00$$

where:

S_{t+1}= reservoir storage at time $t+1$ (Mm^3)
S_t = reservoir storage at full capacity (Mm^3)
I_t = simulated reservoir inflow (Mm^3)

R_t = optimized reservoir release (Mm^3)
t = time (month) and n = total number of months

$$H_t = 0.001S_t + 30.403 \qquad t = 1 \text{ to } n \qquad (12)$$

$$H_t = \begin{cases} \text{If} & H_t > H_{\max}, \quad H_{\max} \\ \text{If} & H_t < H_{\min}, \quad H_{\min} \\ \text{Otherwise,} & H_t \end{cases} \qquad t = 1 \text{ to } n \qquad (13)$$

$$H_{\min} \le H_t \le H_{\max} \qquad t = 1 \text{ to } n \qquad (14)$$

The limit on the operation head is given as

$$33 \le H_t \le 42$$

$$E_t = kR_tH_t \qquad t = 1 \text{ to } n \qquad (15)$$

where:
E_t = energy generation at time t (MWh)
k = constant
R_t = optimized reservoir release (Mm^3)
H_t = corresponding operational head (m)

To protect the fish habitats in the river channel, a time varying ecological flow requirement, EFR_t (Mm^3) must be maintained as shown in Equation 16.

$$R_t \geq EFR_t \qquad t = 1 \text{ to } n \qquad (16)$$

where:
EFR_t = ecological flow requirement at any time t, (Mm^3)

Various EFR_t scenarios with 5, 10, 15, 20 and 25% of the reservoir storage were used to simulate the various operation parameters of the Lake in order to test for the effect of increase in reservoir storage on the other operation parameters.

2.1 Model Performance Evaluation

The performance of the model was evaluated with the correlation coefficient (r), RMSE, MRE and RB as presented in Equations 17 to 20 respectively (Giri and Singh, 2014).

$$r = \frac{\sum\left(y_{pi} - \bar{y}_{pi}\right)\left(y_{oi} - \bar{y}_{oi}\right)}{\sqrt{\left(y_{pi} - \bar{y}_{pi}\right)^2\left(y_{oi} - \bar{y}_{oi}\right)^2}} \qquad (17)$$

$$RMSE = \left[\frac{1}{n}\sum_{i=1}^{n}\left(y_{pi} - y_{oi}\right)^2\right]^{\frac{1}{2}} \qquad (18)$$

$$MRE = \frac{1}{n}\sum_{i=1}^{n}\left|\frac{y_{pi} - y_{oi}}{y_{oi}}\right| \qquad (19)$$

$$RB = \frac{y_{oi} - y_{pi}}{y_{oi}} \qquad (20)$$

where:

y_{pi} = predicted parameter
y_{oi} = observed parameter
$\bar{y}_{pi}$ = mean predited parameter
y_{oi} = mean observed parameter
n = number of observations

3. Results and Discussions

Statistics of mean monthly predicted inflow parameters for Kainji hydropower reservoir is presented in Table 1. The basic descriptive statistics presented in Tables 1 revealed that the predicted mean monthly inflow into the Kainji reservoir between 2017 to 2050 indicted low inflow in April through June while high inflow was noticed in August through February this may be due to rainfall intensity, white and black floods experienced during the periods.

Table 1: Mean monthly predicted inflow parameters for Kainji hydropower reservoir (2017-2050)

Month	Mean $\bar{q}_j$(Mm^3)	Standard deviation (S_j) (Mm^3)	Coefficient of variation	Coefficient of skewness	Serial lag one correlation (r_j)	Regression coefficient (b_j)
Jan	3348.22	1201.43	0.36	-0.16	0.86	1.00
Feb	2526.44	1396.37	0.55	0.26	0.84	0.53
Mar	1141.76	883.69	0.77	1.34	0.96	0.37
Apr	385.24	337.68	0.88	1.63	0.80	0.41
May	218.08	173.76	0.80	1.41	0.37	0.62
Jun	343.90	294.80	0.86	1.78	0.60	0.88
Jul	940.37	435.06	0.46	1.14	0.80	2.01
Aug	2818.01	1096.39	0.39	1.05	0.82	1.28
Sep	4627.74	1704.24	0.37	0.50	0.62	0.85
Oct	3143.00	2313.36	0.74	-0.03	0.66	0.28
Nov	3435.17	964.44	0.28	-1.09	0.99	1.08
Dec	3652.16	1057.43	0.29	-1.09	0.91	1.04

Simulated mean annual reservoir storage, turbine release and energy generation with and without considering scenarios on effect of EFR between 2017 to 2050 are shown in Figures 2 to 4 respectively. The observed and predicted reservoir operation parameters with and without considering EFR scenarios are closely related which similar what was observed in (Eldaw *et al*,. 2019). The simulated mean annual reservoir storage with and without considering EFR presented in Figure 2 showed that there is variation in the simulated reservoir storage which is similar to what was observed in the mean annual turbine release and energy generation presented in Figures 3 and 4. This indicates that all things being equal, changes in simulated reservoir storage and turbine release will automatically results in changes in energy generation for all the scenarios.

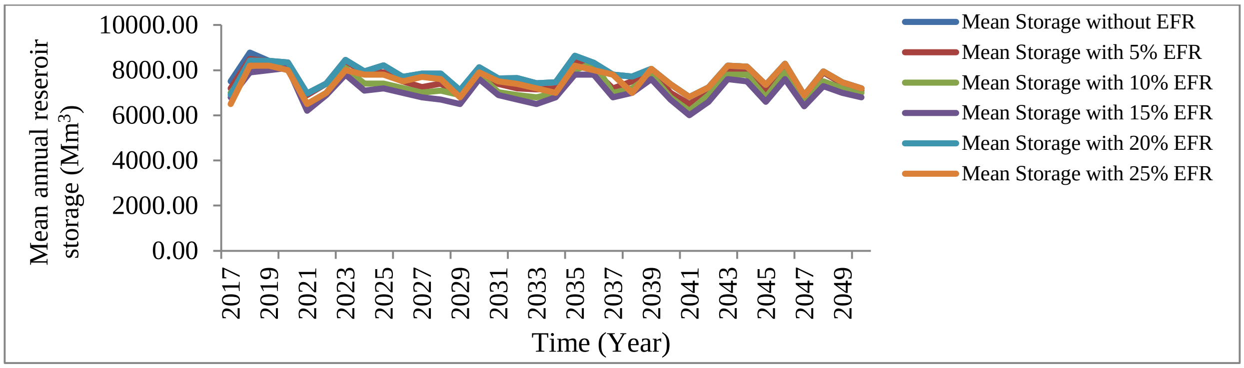

Figure 2: Simulated mean annual reservoir storage (2017-2050)

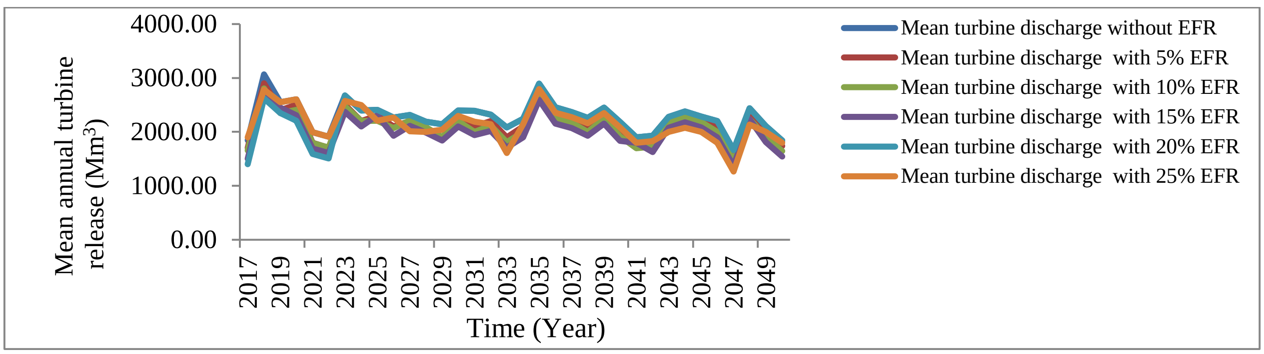

Figure 3: Simulated mean annual turbine discharge (2017-2050)

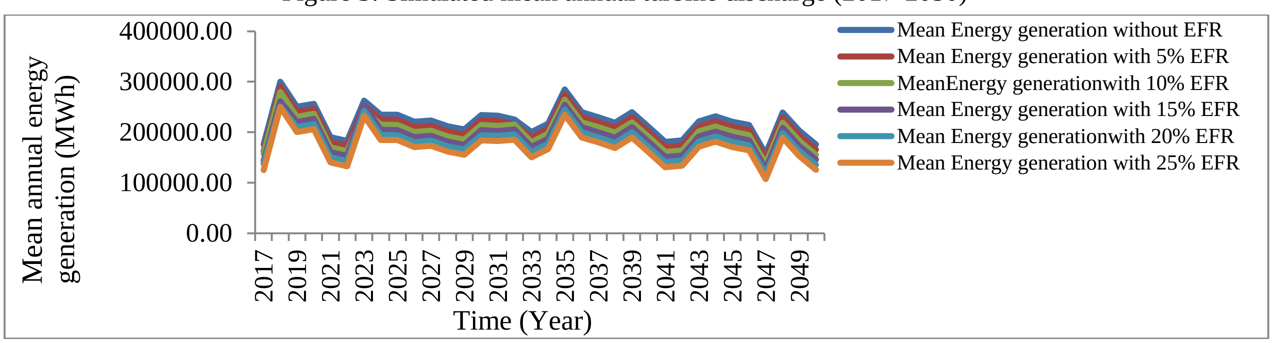

Figure 4: Simulated mean annual energy generation (2017-2050)

Table 2 indicated that the simulated total annual energy generation is higher than the total actual annual energy generation for all the scenarios. Also, it was observed that, as the EFR scenarios increase the simulated total annual energy outputs reduce. It was also observed that the total simulated annual energy generation decrease with increase in EFR percentages. It was noticed that the simulated energy generation is greater than the actual energy generation for all the scenarios.

Table 2: Difference between average total annual simulated and actual energy generation using EFR scenarios

EFR as % of storage	Actual energy output (GWh)	Simulated energy output (GWh)	% Diff. simulated to actual
0	2519.46	2648.94	4.89
5	2519.46	2647.7	4.84
10	2519.46	2645.77	4.77
15	2519.46	2641.5	4.62
20	2519.46	2639.47	4.55
25	2519.46	2637.99	4.49

Observed and predicted mean monthly inflow into the reservoir indicated that both follows similar trend pattern as shown in Figure 5. This indicated that the Markov model was able to predict the inflow adequately similar to what was observed in (Nayab and Faisal, 2018). Comparison between predicted and observed mean monthly inflow revealed that there is a very strong positive relationship between them.

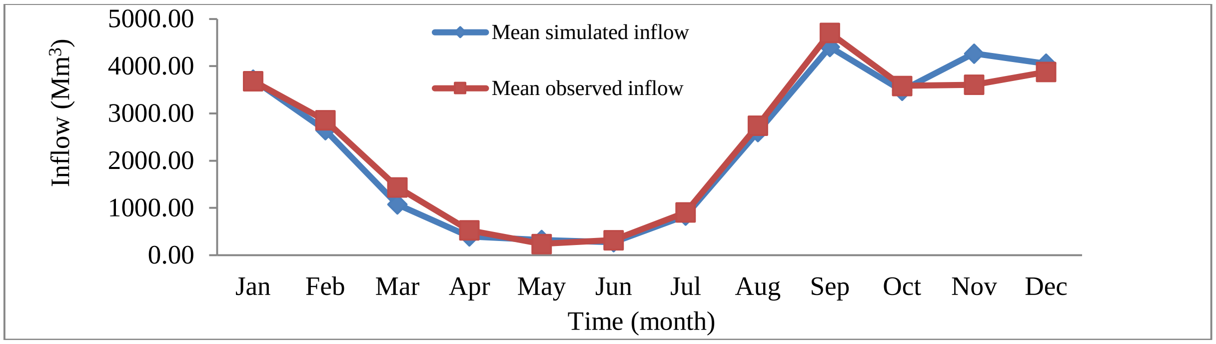

Figure 5: Comparison between predicted and observed mean monthly inflow (mm^3)

The correlation coefficient (r), mean absolute error (MAE), relative bias (RB) and coefficient of determination (R^2) were estimated using the observed and predicted mean monthly streamflow data with the equations 17 to 20 as: 0.99, 0.10, 0.002 and 0.98 respectively using the mean monthly observed and simulated inflow data presented in Table 4 and Equations 17 to 20. The results shown in Table 4 for the performance evaluation statistics of the model implies that the model predicted the reservoir inflow accurately.

Table 3: Performance evaluation of the Markov model

Month	Mean monthly observed inflow (1970-2016) (mm^3)	Mean simulated inflow (2017-2050) (mm^3)	Performance evaluation statistics
Jan	3687.34	3690.39	Correlation coefficient, r = 0.99
Feb	2855.49	2527.06	
Mar	1436.97	1117.09	
Apr	527.13	409.96	MAE = 0.10
May	239.24	367.84	
Jun	322.48	270.21	
Jul	906.91	893.02	Relative bias = 0.002
Aug	2741.95	2670.10	
Sep	4704.89	4472.42	
Oct	3582.43	3216.87	coefficient of determination, R^2 = 0.98
Nov	3609.45	4059.79	
Dec	3878.84	4067.90	

4. Conclusion

The study was carried out in order to predict and simulate hydropower reservoir operation parameters at the Kainji hydropower station in Nigeria. Streamflows into the reservoir was predicted from the year 2017 to 2050 using historical data and Markov model based on Thomas Fierring approach. Monthly reservoir storage, turbine release and energy generation were simulated using the established mass balance and energy generation equations. The model was validated by comparing the mean monthly observed inflow from 1970 to 2016 with the mean simulated inflow from 2017 to 2050) using statistical parameters like: Correlation coefficient (r), Mean Absolute Error (MAE), Relative Bias and Coefficient of Determination (R^2). Also, scenarios of ecological flow releases (EFR) as certain percentages of storage were introduced into the model order to determine its impacts on the energy generation. The results of the model show that the Markov model the predicted reservoir inflow is adequate. Total simulated annual energy generation decrease with increase in EFR percentages. It was also noticed that the simulated energy generation is higher than the actual energy generation for all the scenarios. It is recommended that all things being equal if the management of the Kainji hydropower station can adopt the approaches of the reservoir storage scenarios used in this study, it will improve the future energy generation at the station. Also, this study can be adopted for similar work

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