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Construction of Association Scheme Using Some (123)-avoiding Class of Aunu Patterns

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ABSTRACT: This paper presents some backgrounds research on association scheme using a class of (123)-avoiding pattern of Aunu numbers as an application area. It also attempts to highlight some further applications in other set structures. The finding in this research has shown that there is some interrelationship between the succession scheme used in generating Aunu numbers and the concept of association scheme. This research also shows us that the Aunu patterns can be used in design theory.
Keywords: Aunu Numbers, Aunu Patterns, sets, Associate classes, Association schemes.

INTRODUCTION

Association schemes are structures that appear in many different forms in the field of combinatorics, and statistics. The theory is one of the branches of algebraic combinatorics. Bose and Nair (1939) reported that, association schemes where first used in the context of experimental design in statistics, it was later found that the basis algebra of an association scheme, and the isomorphism to the intersection algebra were constructed by Bose and Mesner (1959), and for this reason the basis algebra is often called the bose-mesner algebra of the association scheme. Godsil (1993) reported in his research article that the origins of association schemes lie in statistics, in the work of Bose and his co-workers. Similarly Bailey (2004) reported that, the subject has close connections with permutation groups. In the same vein Ibrahim (2004) reported a group theoretical approach for the interpretation of the various patterns of succession involving five-obligatory prayers by identifying some cyclic structures in the compensation schemes there by forming a cyclic group from permutation of various succession models. Infact, permutations with forbidden patterns have also been studied extensively in relation to network topology Drager and Fettweis (2002) to computational mathematics and computer science, and very recently, to permutation statistics. The combinatorics of pairing based on a precedence relation was earlier reported by Ibrahim (2005) using a 5-element sample. This paper identified a connection between the method of success ion as used in generating cyclic structures in Ibrahim (2005) and associated scheme.

METHODOLOGY

The basic procedure for generating the special (123)-avoiding permutation pattern is well explained in (Ibrahim, 2007) and concerns as a pairing scheme involving pairs of numbers associated by some precedence relation. The governing conditions for generating those numbers were identified as follows:

(i) Elements are paired in order of precedence such that for λ_i, λ_j in Aunu patterns we have ; $i\,\Theta\, j,\; i, j \in C_n$

Where C_n is a cycle of length n while λ_i, λ_j are in the i^{th} *and* j^{th} positions in the permutation pattern generated by the precedence parameter Θ

(ii) The precedence parameter acts on the elements to produce pairs as in element and first successor, element and second successor, up to the element and n^{th} successor.

(iii) Under the given conditions, it is required that the j^{th} partner shifts in position incrementally corresponding to the n^{th} succession so that

$$j = i+1,\; i+2,\; \ldots i+m \quad where\ m \le n \quad (1)$$

Example 2.1. construction of associate classes using Aunu patterns of size n=5.

Now for n = 5, we can obtain all the permutations of these elements and partition them into some associate classes.

Consider the five element sample (Ibrahim, 2005), with their respective succession where
Ý = {1, 2, 3, 4, 5}

We assume 1 to be the first element in the order of precedence while 2, 3, 4, are respectively the second, third, and forth in that order. The permutation will then be as follows;
Element and first successor 1 Ū 2 Ū 3 Ū 4 Ū 5 = (1, 2, 3, 4, 5)
Element and second successor 1 Ū 3 Ū 5 Ū 2 Ū 4 = (1, 3, 5, 2, 4)
Element and third successor 1 Ū 4 Ū 2 Ū 5 Ū 3 = (1, 4, 2, 5, 3)
Element and fourth successor 1 Ū 5 Ū 4 Ū 3 Ū 2 = (1, 5, 4, 3, 2)
These types of permutations are called Aunu patterns.

APPLICATIONS OF ASSOCIATION SCHEMES IN AUNU PATTERNS

In this section, we provide some theoretic constructs of associationôs schemes as follows:
We construct the table for Aunu patterns of those sets, and partition all the sets obtained from outline of Aunu patterns into some associate classes as in the Tables 1-4.

Table 1: Partition of elements and first successor in a five element sample

	1	2	3	4	5
1	1,1	1,2	1, 3	1,4	1,5
2	2,1	2,2	2, 3	2,4	2,5
3	3,1	3,2	3, 3	3,4	3,5
4	4,1	4,2	4, 3	4,4	4,5
5	5,1	5,2	5, 3	5,4	5,5

Table 2 partition of elements and second successor

	1	3	5	2	4
1	1,1	1,3	1,5	1,2	1,4
3	3,1	3,3	3,5	3,2	3,4
5	5,1	5,3	5,5	5,2	5,4
2	2,1	2,3	2,5	2,2	2,4
4	4,1	4,3	4,5	4,2	4,4

Table 3 partition of elements and its third successor

	1	4	2	5	3
1	1,1	1,4	1,2	1,5	1,3
4	4,1	4,4	4,2	4,5	4,3
2	2,1	2,4	2,2	2,5	2,3
5	5,1	5,4	5,2	5,5	5,3
3	3,1	3,4	3,2	3,5	3,3

Table 4 partition of element and fourth successor a 5-element sample

	1	5	4	3	2
1	1,1	1,5	1,4	1,3	1,2
5	5,1	5,5	5,4	5.3	5,2
4	4,1	4,5	4,4	4,3	4,2
3	3,1	3,5	3,4	3,3	3,2
2	2,1	2,5	2,4	2,3	2,2

Using the entries in table1 above, the following are drivable:
$(i)\, R_0 = \{(1,1)(2,2)(3,3)(4,4)(5,5)\}$ Where R_0 is the diagonal subset $\Omega\times\Omega$

$(ii)\, R_1$ = {(1, 2), (2, 1), (1, 3), (3, 1), (1, 4), (4, 1)(1, 5), (5, 1), (2, 3), (3, 2), (2, 4), (4, 2), (2, 5), (5, 2), (3, 4), (4, 3)(3, 5), (5, 3), (4, 5), (5, 4)}

Note: that R_0 represents the class in which every pair (i, j) has equal values. That is i=j in the association scheme. While R_1 stands for the contrary. (i.e where $i\neq j$)

Table 2 gives the partitions $(\Omega\times\Omega)$ *where* $\Omega=\{1,3,5,2,4\}$

Let R_0(Ŭ) = {ɓ ∈ Ý : (Ŭ, ɓ) ∈ R_0 where Ŭ= ɓ}. Then R_0 = {(1,1) (3,3) (5,5) (2,2) (4,4)} (2)

Now let R_1(Ŭ) = {ɓ ∈ Ý : (Ŭ, ɓ) ∈ R_1 where Ŭ Í ɓ are both even} (3)

R_2(Ŭ) = {ɓ ∈ Ý : (Ŭ, ɓ) ∈ R_2 where Ŭ Í ɓ are both odd} (4)

R_3(Ǔ) = {ɓ ∈ Ý : (Ǔ, ɓ) ∈ R_3 where Ǔ Í ɓ are both prime} (5)

R_4(Ǔ) = {ɓ ∈ Ý : (Ǔ, ɓ) ∈ R_4 where Ǔ Í ɓ, Ǔ is a factor of ɓ } (6)

R_5(Ǔ) = {ɓ ∈ Ý : (Ǔ, ɓ) ∈ R_5 where Ǔ Í ɓ, Ǔ is divisible by ɓ} (7)

Now for i = {1, 2, é , 5}, we can obtain the following values for R_i as follows;

From equation (3) we have R_1 = {(2, 4)(4, 2)} (8)

From equation (4) we have R_2 = {(3, 5)(5, 3)} (9)

From equation (5) we have R_3 = {(2, 3)(3, 2)(2, 5)(5, 2)(3, 5)(5, 3)} (10)

From equation (6) we have R_4 = {(1, 4)(4, 1)(2, 4)(4, 2)} (11)

From equation (7) we have R_5 ={(1, 2)(2, 1)(1, 3)(3, 1)(1, 4)(4, 1)(1, 5)(5, 1)(2, 4)(4, 2)} (12)

Further, ɓ ∈ R_k(Ǔ) it follows that (Ǔ, ɓ) ∈ R_k for some k, in which case $|R_i$(Ǔ) ž R_j(ɓ)$| = P_{ij}^{k}$

Using equation (8) we have $R_1(2)$ = {(2, 4)(4, 2)}, (13)

From Table 3 we can obtain the following values of R_i , where R_0 is the diagonal values of the table in which i = j and R_i is the diagonal of each row in which i is not equal to j this can be obtain as follows:
R_0 = {(1, 1), (4, 4), (2, 2), (5, 5), (3, 3)}

R_1 = {(4, 1), (1, 4), (2, 4), (4, 2)}

R_4 = {(2, 1),(1, 2), (5, 4), (4, 5), (3, 2), (2, 3)}

R_2 = {(5, 1), (1, 5), (3, 4) ,(4, 3)}

R_5 = {(3, 1), (1, 3),}

R_3 = {}

Other partitions of Aunu patterns also exist through permuting the points (numbers) in the pattern.

From Table 4 the following entries are driveable
Since R_0 is the diagonal subset of $(\Omega\times\Omega)$
Then, R_0 = {(1,1), (5,5), (4,4), (3,3), (2,2)}

Suppose $R_1=\{(\alpha,\beta)\in\Omega\times\Omega:\alpha\neq\beta\}=\Omega\times\Omega\setminus R_0$

Then R_1= {(1,5), (1,4,) (1,3), (1,2) ,(5,1), (5,4), (5,3), (5,2), (4,1), (4,5), (4,3), (4,2) (3,1,) (3,5), (3,4), (3,2) ,(2,1), (2,5), (2,4), (2,3)}

This can be determine as an association scheme with only one associate class R_1 . An association scheme with only one associate class is called trivial association scheme.

CONCLUSION
This paper has proposed some interrelationship between the set theoretic construct of a 5-element sample reported earlier and the association scheme. It also identified the existence of some valuable application areas in terms of partition, the paper also establishes that some of the Aunu pattern form an association scheme. It is hoped that further communications should centre on graph theoretic aspects and should attempt to capture the enumeration segment of association scheme.

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