Available online at http://ajol.info/index.php/njbas/index
Nigerian Journal of Basic and Applied Science (2010), 18(2): 204-208
ISSN 0794-5698

Numerical Solution of Ordinary Differential Equations (ODES) from Reformulated block 3-step AdamsBashforth method

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ABSTRACT: In this paper, efficient AdamsBashforth Runge-Kutta (ABRK) method are constructed.This is achieved by reformulating the block AdamsBashforth methods as a class of Runge-Kutta methods for 3-step.The reformulated methods are of orders 4 and their absolute stability regions construc ted are shown to be A-Stable.The newly constructed methods are tested on non stiff initial value problems and the solution reveals that the the methods are efficient.

INTRODUCTION

The use of numerical techniques has become an integral part of modern engineering and scientific studies .The advent of computers has immensely facilitate the speed, power and flexibility of numerical computation. In view of this, most of the Nigerians universities now offer courses in numerical methods in their engineering and scientific curriculum. The increasing importance of numerical methods in applied sciences have led to the developing a new numerical method for the solution of applied problem when ordinary analytical method fail. Several attempts were made by several researchers in developing the reliable numerical method to solve ordinary differential equations. Hojjati *et al* (2004) developed a multistep method for solving systems of ordinary differential equations (ODEs).Ahmad *et al* (2005) an explicit Runge-Kutta-like method is developed and shown to be efficient not only for stiff but also for ordinary differential equation.Hojjati *et al* (2006) presented a new class of second derivative multistep methods with improved stability region. Also Wells *eta al* (1982) implements Multirate linear multistep methods for the solution of systems of ordinary differential equations. Onumanyi *et a*l (1994, 1999,2005) resolve to continous linear k-step methods which provide sufficient number of simultaneous discrete methods used as self starting single integrator.Those methods have been considered Onumanyi et al(1999) and further investigation on block by Chollom(2007), Sirisena et al(2004) and Fatokun (2007).Several classes of Runge-Kutta methods have been constructed various authors and is still being pursued today due to its advantages.

In this paper,the single method approach is pursued leading to the construction of a new Runge-Kutta method approach is pursued leading to the construction of a new Runge-Kutta method based on AdamsBashforth 3-step linear multistep method of order four.This is achieved by reformulating the block AdamsBashforth as a Runge-Kutta method.The rest of the paper is divided as follows:The derivation of the new method is done in section two;section three contains the convergence analysis of the new method.Numerical experiment to test the efficiency of the new method is done in section four and result compared to standard existing methods and the concluding remarks in section five.

2.0 Derivation of the method

Let v_n be an approximate solution to v(t_n) let

$$\{f_{n+j}\ v_{n+j}\}\ i.e\ \sum_{j=0}^{s}\alpha_j v_{n+j} = k\{\sum_{j=0}^{s}\beta_j f_{n+j}\}$$

be a computational method.The S-step multistep collocation method with m collocation points and t interpolation points is constructed as follow,we find a poly v(x) of degree p=t+m-1,m>0 of the form

$$v(x) = \sum_{j=0}^{t-1} \alpha_j(x) v(x_{n+j}) + k\{\sum_{j=0}^{m-1} \beta_j(x) f(x_j, v(x_j))\} \qquad (2.1)$$

Such that it satisfies the conditions

$$\left.\begin{array}{ll} v(x_{n+j}) = v_{n+j} \\ v(x_j) = f(x_j, v(x)) \end{array}\right\} \qquad (2.2)$$

Where $\alpha_j(x)$ & $\beta_j(x)$ are assume poly of the form

$$\left.\begin{array}{l} \alpha_j(x) = \sum_{i=0}^{t+m-1} \alpha_{i,i+1} x^i, \qquad h\beta_j(x) = h\sum_{j=0}^{t+m-1} \alpha_{j,j+1} x^j, \\ i \in \{0,1,2,\ldots\ldots\ldots\ldots t-1\} \quad , \quad j \in \{0,1,2,\ldots\ldots\ldots\ldots m-1\} \end{array}\right\} \qquad (2.3)$$

(2.1) produces the matrix of DC=I (2.4)

Where I is an identity matix and C is the matrix coefficient to be determine.Where

$$D = \begin{pmatrix} 1 & \cdots & x_n^{t+m-1} \\ \vdots & \ddots & \vdots \\ 1 & \cdots & (t+m-1)x_{m-1}^{t+m-2} \end{pmatrix}$$

$$C = D^{-1}$$

It follows from (2.4) that D = C-I where C-I gives the values of the continous coefficients αj(x) and βj(x). The continous coefficients are substituted into (2.3) to form the continous interpolant.Evaluating the continuous interpolant at both grid and off grid points to produce the invidual discrete members of the block which are used as a single integrator.

2.1 Derivation of the three step AdamsBashforth method

Expressing the method into the general form of the AdamsBashforth method gives

$$v(x) = \alpha_2(x) v_{n+2} + k\{\beta_0(x) f_n + \beta_1(x) f_{n+1} + \beta_2(x) f_{n+2}\} \qquad 2.1.0$$

The interpolation and collocation points put in tabular form produces the D matrix in (2.1.0)

$$D = \begin{bmatrix} 1 & x_n + 2k & (x_n + 2k)^2 & (x_n + 2k)^3 \\ 0 & 1 & 2x_n & 3x_n^2 \\ 0 & 1 & 2x_n + 2k & 3(x_n + k)^2 \\ 0 & 1 & 2x_n + 4k & 3(x_n + 2k)^2 \end{bmatrix} \qquad 2.1.1$$

The inverse of the matrix is obtained using the Maple code to give the elements known as the continouos coefficients represented by C with the coefficients

$\alpha_2, \beta_0, \beta_1, \beta_2$ *for each* $x_j = x_{n+j}$ *where* $j = 0,1,2,3$ 2.1.2 to be determine and as listed in (2.1.2) below .The continous coefficients are substituted into (2.1.0) which yield the continous interpolant of the AdamsBashforth method .The continous interpolant (2.1.2) evalated at the points $x_j = x_{n+j}$ *where* $j = 0,1,2,3$ to produces the descrete schemes below

$$\left.\begin{aligned} v_n &:= v_{n+2} - \frac{1}{3}k\left(f_n + 4f_{n+1} + f_{n+2}\right) \\ v_{n+1} &:= v_{n+2} - \frac{1}{12}k\left(-f_n + 8f_{n+1} + 5f_{n+2}\right) \\ v_{n+3} &:= v_{n+2} + \frac{1}{12}k\left(5f_n - 16f_{n+1} + 23f_{n+2}\right) \end{aligned}\right\} \qquad 2.1.3$$

2.2 The new Runge-Kutta Method via AdamsBashforth method

The derive three step block AdamsBashforth method is reformulated as Runge-Kutta by first expressing it in the form:

$$\begin{aligned} v(x) = v_n + k\{\alpha_0 f(x_n, v_n) + \alpha_1 f(x_n + k_1 k, v_n + b_1 k) + \\ \alpha_2 f(x_n + k_2 k, v_n + b_2 k) + ...\alpha_p f(x_n + \mu k_p k, v_n + b_p k) \end{aligned} \qquad 2.2.1$$

And by letting

$$f_n = k_1,\ f_{n+1} = k_2,\ f_{n+2} = k_3,\ f_{n+\mu} = k_\mu .... f_{n+p} = k_{p+1} \qquad 2.2.2$$

Where

$$\left.\begin{aligned} k_1 &= f(x_n, v_n) \\ k_2 &= f(x_n + k, v_{n+1}) \\ k_3 &= f(x_n + 2k, v_{n+2}) \\ k_{p+1} &= f(x_n + (p+1)k, v_n + \{p+1\}) .... k\alpha_p k_p \end{aligned}\right\} \qquad 2.2.3$$

The block method (2.1.3) written in the form of (2.2.3) and rearrnging yields the new Runge-Kutta method through three step block AdamsBashforth method in (2.1.3)

$$\left.\begin{aligned} K_1 &= f(x_n, v_n) \\ K_2 &= f(x_n + k, v_n + \frac{k}{12}(5k_1 + 8k_2 - k_3)) \\ K_3 &= f(x_n + 2k, v_n + \frac{k}{3}(k_1 + 4k_2 + k_3)) \\ K_4 &= f(x_n + 3k, v_n + \frac{3k}{4}(k_1 + 3k_3)) \\ v_{n+1} &= v_n + \frac{k}{12}(5K_1 + 8K_2 - K_3) \end{aligned}\right\} \qquad 2.2.4$$

3.0 Stability analysis

3.1 Order of the (ABRK) method

Applying the method of chollom et al (2007) for the order of the block (ABRK) method gives

$$C_0 = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}, C_1 = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}, C_2 = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}, C_3 = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}.$$

Since $C_0 = C_1 = C_2 = C_3 = 0,\ and\ C_4 \neq 0$,then the (ABRK) block method(2.2.4) for k=3 is of order p = 3 and its error constant is $C_4 = \begin{pmatrix} \frac{1}{24} \\ 0 \\ 0 \end{pmatrix}$

3.2 Absolute stability regions of the block(ABRK) methods:

To plot the regions of absolute stability regions of the block (ABRK) method for k=3,equation (2.2.4) is expressed in the form of choolom,et al(2007).to yield the the region of absolute stability as shown below in fig 2

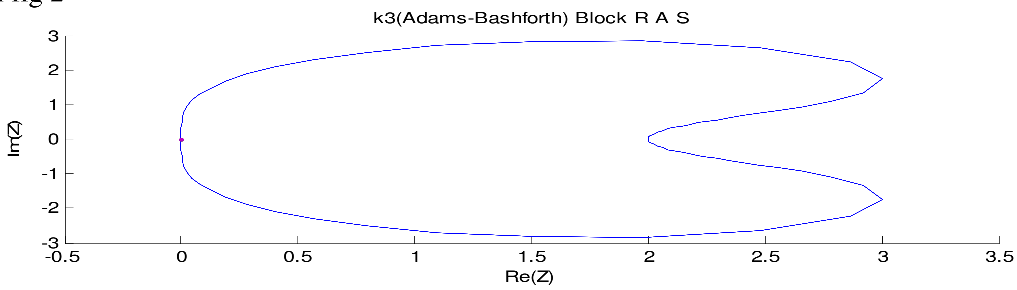

Fig.1 The region of absolute stability for k=3 of (ABRK) method.

Application

In this section we present numerical results for initial value problem to illustrate the proposed method.

Example 1

Consider an initial value problem for first order differential equation

$y' = 100(1-y), y(0) = 0 \quad 0 \leq t \leq 0.1, h = 0.01$ The exact solution is $y(t) = 1 - e^{-100t}$

Table1: Comparison of exact solution with ABRK-method solution

x	(ABRK) Method	Exact solution	ABS-ERR
0	0	0	0
0.01	0.965613374	0.632120559	0.333492816
0.02	0.993504329	0.864664717	0.128839612
0.03	0.998626728	0.950212932	0.048413796
0.04	0.999532727	0.981684361	0.017848366
0.05	0.999657913	0.993262053	0.00639586
0.06	0.999638503	0.997521248	0.002117255
0.07	0.999592316	0.999088118	0.000504198
0.08	0.999541170	0.999664537	0.000123368
0.09	0.999489106	0.999876590	0.000387484
0.10	0.999436871	0.999954600	0.000517729

Example 2

Consider an initial value problem for first order differential equation

$v' = \exp(v), v(0) = 1, 0 \le x \le 0.1, h = 0.01$ The exact solution is

$$v(x) = -\ln\left(\frac{xe-1}{e}\right)$$

Table2: Comparison of exact solution with (ABRK) method solution

x	(ABRK)Method	Exact Solution	ABS-ERR
0.00	0.000000000	0.00000000	0.000000000
0.01	1.027182586	1.027558914	0.000376328
0.02	1.055115595	1.055899399	0.000783804
0.03	1.083841293	1.085065591	0.001224298
0.04	1.113405601	1.115108968	0.001703367
0.05	1.143858529	1.146082806	0.002224277
0.06	1.175254673	1.178046374	0.002791701
0.07	1.207653792	1.214568458	0.006914665
0.08	1.241121477	1.245212763	0.004091286
0.09	1.275729933	1.280567898	0.004837965
0.1	1.311558893	1.317217501	0.005658608

Conclusion: In this paper an order explicit (ABRK) method has been constructed by reformulating the three step block AdamsBashforth method .The new method is of order four and the stability region ploteted shows that it is A-stable.The new method is sujected to test equations and results displayed on table (1) , table(2) and figure (1) comparing the results obtained and the region of absolute stability respectively, using the new method and the exact solution reveal that the new method is efficient.

The results genereted in this study could be compared with the other methods generated by other reseachers cited in this research.

Future extensions to this paper will include the construction of higher explicit ABRK methods and also discuss implementation strategies.

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