Available online at http://ajol.info/index.php/njbas/index ISSN 0794-5698
Nigerian Journal of Basic and Applied Science (2010), 18(2): 198-203

Queueing and Service Patterns in a University Teaching Hospital

F.O. Ogunfiditimi and E.S. Oguntade

Department of Maths, Stat & Computer Science
University of Abuja
[Author of correspondence: olusodayofrank@yahoo.com]

ABSTRACT: Analysis of queueing and service times is essential for designing effective congestion control at a service point. The objective of this is to be able to offer satisfactory service to waiting customers with minimum delay. In this study, using University of Abuja Teaching Hospital as a case study, we compared the queueing and service patterns in major key service departments of the hospital. Analysis of the data gathered showed that arrival and departure times follow Poison distribution with the M/M/I queueing model. Results from data analysis show that service delay which leads to long queue occur mostly at the consultancy department, followed by the pharmacy department. Very little or no delay at all occur at the records department

INTRODUCTION

In real life, waiting for service is a common phenomenon. We wait for service in bars and restaurants; we queue up for service in the banks, schools, supermarkets, filling stations, post offices and hospitals etc. As a system gets congested, service delay is inevitable in the system; as the service delay in the system is on the increase waiting time gets longer. A good understanding of the relationship between congestion and delay is essential for designing effective congestion control algorithms at a service points such as the ones obtainable in the university teaching hospital. Queuing theory provides all the tools needed for this analysis.

Queuing theory is part of the mathematical theory of the formation and behaviour of queues or waiting lines. The objective of queuing analysis is to offer a reasonably satisfactory service to waiting customers. It determines the measure of performance of waiting lines such as the average waiting time in queuing and the productivity of the service facility (Taha, 2007). These are used to design the service installations. For the mathematician, queuing theory is particularly interesting because it is concerned with relative simple stochastic process which are in general non-markovian and possibly stationary.

THEORETICAL FRAMEWORK

Many authors have studied and applied queue theory model to various real life situations. Prominent among them is: Erlang(1908), the principal pioneer of queuing theory. He studied problems of telephone congestion. That is, the waiting times of subscribers in a manually operated system, the average waiting time and the chance that a subscriber will obtain service immediately without waiting and he examined how much the waiting time will be affected if the number of operators is altered or conditions are changed in any other way. Phillips et al (1976), Taha (2007), Kendal (1953) introduced an A/B/C queuing notation that can be found in all standard modern works on queuing theory. Kleinrock, (1960) used queue theory in all modern packed switching networks. Another contributor is Little,(1961). Little's theorem states that; the average number of customers (N) can be determined from the following equation:

$N = \lambda T$ where lambda is the average customer arrival rate and T is the average service time for a customer.

The Arrival process is defined as the probability distribution of the pattern of arrivals of customers in time. This is called the input process. Service process is defined as the service distribution, the probability distribution of the random time to serve a customer (or group of customers in the case of batch service).

The Queue discipline represents the order in which customers are selected from a queue. It is an important factor in the analysis of queuing models. The most common discipline is First Come, First Serve (FCFS). Other
The queuing behaviour of customers plays a role in waiting line analysis. Human customers may jockey from one queue to another in the hope of reducing waiting time. They may also balk from joining a queue altogether because of anticipated long delay, or they may renege from a queue because they have been waiting too long.
The Erlang family of distributions is a subclass of the Gamma family (Phillips et al, 1976).The exponential distribution used in
A special case of the Erlang distribution is the exponential distribution. Here random inter arrival and service times are described quantitatively in queuing model by the distribution which is defined as:

$$f(x) = \begin{cases} \lambda e^{-\lambda x} & ; \quad x > 0 \\ 0 & ; \quad \text{otherwise} \end{cases}$$

where λ is the rate per unit at which events (arrivals or departures) are generated.
Randomness here means that the occurrence of an event e.g. arrival of a customer or completion of a service is not influence by the length of time that has elapsed since the occurrence of the last event.
A Poisson process models random events (such as a customer arrival) as emanating from a memory less process. That is, the length of the time interval from the current time to the occurrence of the next event does not depend upon the time of occurrence of the last event. In the Poisson probability distribution, the observer records the number of events that occur in a time interval of fixed length. In the negative exponential probability distribution, the observer records the length of the time interval between consecutive events. In both, the underlying physical process is memory less. The Poison process is a special case of the Markov process in which the state is discrete while the time is continuous. For instance consider a process of point event occurring on the real axis.

disciplines include Last Come, First Serve (LCFS) and Services in Random Order (SIRO). Customers may also be selected from the queue based on some order of priority (Taha, 2007).
queuing theory is a negative exponential distribution. It is a special case of Erlang distribution named after A.K Erlang, one of the founding fathers of queuing theory. The Probability Density Function of the Erlang distribution is given as :

$$f(x) = \begin{cases} \dfrac{b^{k+1} x^k e^{-bx}}{K!} & ; o \leq x \leq \infty \\ 0 & ; \text{ otherwise.} \end{cases}$$

Let $N(t,\delta t)$ denote the number of events occurring in time interval $(t,\delta t)$;the state is discrete while the time is continuous. Example of event that can be modelled by Poisson process is the arrival of patients for service (treatment) in University of Abuja Teaching Hospital. For this study we take $\delta t = 5$.We now postulate as follows;

(i) The numbers of events happening in disjoint interval of time are independent random variables. That is, for every integer $\delta t = 5, 10, 15$ etc.
$N(t_0,t_1), N(t_1,t_2), N(t_2,t_3)...N(t_{ti-1,}t_i).$ are independent.
(ii) For any time t and positive number δt, the probability distribution of $N(t,t+\delta t)$, (the number of events occurring between t and $t+\delta t$), depend only on the interval length δt and not on the time t.
(iii) The probability of at least one event happening in a time period of duration δt is:
$\Pr\{ N(t,t+\delta t) \geq 1 \} = \lambda \delta t + 0(\delta t)$ as $\delta t \to 0$
where λ is the rate of occurrence of events and $0(\delta t)$ denotes unspecified remainder term of smaller order than δt.
(iv) The probability of two or more events happening in time δt is $0(\delta t)$ or
$\Pr \{N(t, t+\delta t) \geq 2\} = 0(\delta t)$ as $\delta t \to 0$
It is good to note that assumption (iii) is a specific formulation of the notion that events are rare. Assumption (iv) is tantamount to excluding the probability of

the simultaneous occurrence of two or more events.

Using the above assumptions, the defining properties of the Poisson processes for some positive constant λ are given as follows:

(i) Pr{ N(t, t+δt) =0} = 1 - Pr{N(t, t+δt) ≥ 1} = 1- λδt + 0(δt)

(ii) Pr{N(t, t+δt) = 1} = λδt +0(δt)

(iii) Pr{N(t, t+δt) >1} = 0(δt)

And further that N(t, t+δt) is completely independent of occurrence in (0,t)

Assumptions (i) to (iv) above led to the probability density function of Poison distribution as:

$$p_i(t) = \frac{e^{-\lambda t}(t\lambda)^i}{i}, i = 0,1,2,3\ldots$$

where $p_i(t)$ = Pr(N(t) = i) = Pr(i events have occurred at time t) (Hogg,& Craig, 1970)

Some of the varieties of queueing models are:

(i) The M│M│1 Queue model

Arrivals follow a Poisson process; service times are exponential distributed; and there is a single server. X (t) denotes the number of customers in the system at time t.

(ii) The M│M│ ∞ Queue

There are exponential and exponentially distributed service times. Any numbers of customers are processed simultaneously and independently. Often self-service situations may be described by this model. In other words, for this model; there are unlimited number of servers and as such all customers in the system at any instant are simultaneously being served.

(iii) The M│GI│1 Queue model

In this process, there are Poisson arrivals but arbitrarily distributed service times.

The analysis proceeds with the help of an embedded Markov Chain.

DATA ANALYSIS

The University of Abuja Teaching Hospital (UATH) is located at Gwagwalada in the Federal Capital Territory (FCT). Its services include both primary and secondary health care. Aside the National Hospital, Abuja, UATH is the second specialist health care facility in the FCT. UATH has several healthcare service departments like other teaching hospitals in the country.

However our study covers three basic healthcare departments namely: Records, Consultancy and Pharmacy departments. The reason for these choices is because our preliminary observation before data collection showed that these departments have the highest traffic of persons requesting for service. Like every other hospital in Nigeria, anybody coming to UATH for health service will first go to the records department to register and get a file opened for him/her. From the records department, the patient will proceed to the consultancy unit where he/she will see a doctor for complaints and drug prescription. From here the patient will proceed to the pharmacy department after making necessary payments to collect his/her drugs.

The data for the study were collected from the three service departments directly. The number of arrivals and departures together with the service times at each service point were recorded. The interval of time used was five minutes. The arrival and departures were recorded for every five minutes interval from 8.00am to 2.00pm from Monday to Friday.

Analysis of the data shows that arrivals follow a Poissson process and service times at each point are exponentially distributed. Each service point has a single server.

Thus, queue structures in all the service points follow the M/M/1 queue model. The data obtained on arrivals and departures were Poison distributed. The mean arrival and departure rates **λ** for the various service points are tabulated in Tables 3.1- 3.9.

We show below how the entries in the first row of Table 3.1 were calculated.

Average Arrival Rate (λ) =Total number of arrivals/Total time taken

Average Departure Rate (λ) =Total number of departure/Total time taken

For Monday 8.00-10.00
Total number of arrivals=62 persons
Total time taken=120 minutes

Average Arrival Rate (λ) = $\frac{62}{120}$ persons per minute or = $\frac{62}{120}\ X\ 5$ persons per 5minutes

= □ 3 persons per 5minutes

For Monday 10.00-12.00
Total number of arrivals=65 persons
Total time taken=120 minutes

Average Arrival Rate (λ) = $\frac{65}{120}$ persons per minute or = $\frac{65}{120}\ X\ 5$ persons per 5minutes

= □ 3 persons per 5minutes

For Monday 12.00-2.00
Total number of arrivals00=43 persons
Total time taken=120 minutes

Average Arrival Rate (λ) = $\frac{43}{120}$ persons per minute or = $\frac{43}{120}\ X\ 5$ persons per 5minutes

= □ 2 persons per 5minutes

Table 3.1: Records Dept – Mean Arrival Rate (per 5mins period)

	8am- 10am	**10am- 12noon**	**12pm-2pm**
Monday	3	3	2
Tuesday	3	2	2
Wednesday	3	2	2
Thursday	3	2	1
Friday	3	2	1

Table 3.2: Records Dept – Mean Departure Rate

	8am- 10am	**10am- 12noon**	**12pm-2pm**
Monday	2	3	3
Tuesday	2	3	2
Wednesday	3	2	3
Thursday	3	1	2
Friday	2	2	2

Table 3.3: Records Dept – mean service time

	8am- 10am	**10am- 12noon**	**12pm-2pm**
Monday	2.00	1.92	2.21
Tuesday	2.10	1.87	1.94
Wednesday	1.84	1.71	1.91
Thursday	1.94	2.01	1.85
Friday	2.44	2.98	2.10

Tables 3.1 & 3.2 show the parameter **λ** i.e. mean rate of arrivals and departures of persons at the records department per five minutes interval period in the designated time belt. Table 3.3 shows the parameter **λ** i.e. the mean service time in minutes per individual that arrived at the department to register.

Comparing Tables 3.1 and 3.2, we can infer that the queue pile up in the mornings and then thin down later in the day. On the average, the arrival rate is slightly higher than the rate of departures. Whereas later in the day, the number of persons departing from the queue on the average is higher than arrivals on the queue. We can say that patients do not experience much delay on the queue in the records department because on the average the mean service time is approximately 2 minutes. This can be seen from Table 3.3. The entries in Table 3.3 show that all the values can be approximated to 2 except that of mid-morning of Friday.

Table 3.4: Consultancy Dept – Mean Arrival Rate

	8am- 10am	10am- 12noon	12pm-2pm
Monday	2	1	2
Tuesday	3	1	2
Wednesday	2	1	2
Thursday	2	1*	2
Friday	2	1	1

Table 3.5: Consultancy Dept – mean departure

	8am- 10am	10am- 12noon	12pm-2pm
Monday	1	2	1
Tuesday	2	2	1
Wednesday	1	2	2
Thursday	2	1	1
Friday	1*	3	1

Table 3.6: Consultancy Dept – mean service time

	8am- 10am	10am- 12noon	12pm-2pm
Monday	4.52	3.42	3.04
Tuesday	4.01	3.11	2.12
Wednesday	6.25	2.18	2.05
Thursday	5.41	3.21	2.54
Friday	7.50	4.85	2.83

Tables 3.4 & 3.5 show the parameter λ i.e. mean rate of arrivals and departures of persons at the consultancy department per five minutes interval period in the designated time belt. Table 3.6 shows the parameter λ i.e. the mean service time in minutes per individual that arrived at the department to see doctor. Comparing Tables 3.4 and 3.5, we can infer that the queue pile up in the mornings and continues like that throughout the day. On the average, the arrival rate is consistently higher than the rate of departures. Again looking at Table 3.6, we can say that patients do experience much delay on the queue in the consultancy department because on the average the mean service time is consistently more than four minutes in the morning period. The entries in Table 3.6 show that on the average service time is higher in the morning period thereby leading to queue pile up. However it can be observed that service time get faster later in the day. The reason for this may not be unconnected with the feeling of urgency to clear up the long queue.

Table 3.7: Pharmacy Dept - Mean Arrival Rate.

	8am- 10am	10am-12noon	12pm-2pm
Monday	1	3	2
Tuesday	1	2	1
Wednesday	2	2	1
Thursday	1	2	1
Friday	1	2	1

Table 3.8: Pharmacy Dept – Mean Departure Rate.

	8am- 10am	10am- 12noon	12pm-2pm
Monday	1	3	2
Tuesday	1	2	1
Wednesday	2	1	2
Thursday	1	1	2
Friday	1	2	1

Table 3.9: Pharmacy Dept – Mean service time.

	8am- 10am	10am- 12noon	12pm-2pm
Monday	2.20	2.60	2.40
Tuesday	2.45	3.46	1.99
Wednesday	2.51	3.96	2.25
Thursday	3.21	2.45	2.12
Friday	3.45	1.97	2.43

Tables 3.7 & 3.8 show the parameter λ i.e. mean rate of arrivals and departures of persons at the pharmacy department per five minutes interval period in the designated time belt. Table 3.9 shows the parameter λ i.e. the mean service time in minutes per individual that arrived at the department to collect the prescribed drugs.

Comparing Tables 3.7 and 3.8, we can infer that the queue do not pile up at all for any appreciable length of time. There is cleared up as soon as they tend to grow. On the average, the arrival rate is slightly higher than the rate of departures in the mid-morning to afternoon. We can say that patients do not experience any delay on the queue in the pharmacy department. Table 3.9 shows that on the average the service time is between two and four minutes.

Conclusion: Results from data analysis show that service delay which leads to long queue occur, mostly at the consultancy department, followed by the pharmacy department. Very little or no delay at all occur at the records department. It is hereby recommended that UATH should provide more service points that are more consulting units at the consultancy department. The current service levels at the Records and Pharmacy departments should be strengthened.

REFERENCES

Beevers K.R.(2006): CSCI-6971 Lecture Notes: Stochastic Processes, Department of Computer Science, Rensselaer Polytechnic Institute.www.relexsoftware.com

Hogg, R. V. and Craig, A.(1997) Introduction to Mathematical Statistics, 3rd ed. Macmillan Publishing Co. Inc. New York.

Phillips, D.T, Ravindran, A and Solberg, J. (1976): Operations Research Principles And Practice, John Wiley & Sons, New York.

Taha, H. A. (2007): Operations Research; an Introduction. (Eighth ed.) Pearson Education Inc. USA.

Tijms, H. C. (2003): A First Course in Stochastic Models, Wiley, Chichester.