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Full Length Research Article

Modelling Subsidy as a Cooperative Advertising Channel Coordination Mechanism

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ABSTRACT

This work considers the use of subsidy as channel coordination strategy in vertical cooperative advertising in which the manufacturer is the Stackelberg game leader and the retailer is the follower. While the retailer is directly involved in advertising, the manufacturer is indirectly involved through the provision of subsidy to aid the retailer in advertising the product. The work models the demand function using a multiplicative advertising-price-demand function, and obtains the players' prices, the retail advertising effort, the manufacturer's subsidy rate and the payoffs. The work observes that with increasing subsidy, the manufacturer's price margin increases while that of the retailer reduces and eventual becomes zero with total subsidy. However, the manufacturer should not totally subsidise retail advertising since it would be counterproductive for him, while at the same time would lead to very large retail payoff. Thus with appropriate subsidy strategy, the prices and the payoffs, and eventually the entire channel can be coordinated.

Keywords: Channel coordination, Vertical cooperative advertising, Stackelberg game, Advertising price-demand function, Subsidy rate.

INTRODUCTION

Cooperative advertising is an advertising arrangement in which the manufacturer pays for a fraction of the advertising cost which the retailer incurs in the process of advertising the manufacturer's product. In the cooperative advertising literature Berger (1972) is considered to be the first to develop a mathematical model on cooperative advertising. He considered cooperative advertising support as price discount from the supplier (manufacturer) to the retailer. Based on his work, Dant and Berger (1996) considered cooperative advertising in a franchising system. Bergen and John (1997) considered the effect of horizontal level competition among retailers and among manufacturers, and advertising spillover on subsidy (participation) rate and concluded on guidelines for the provision of subsidy. Huang *et al*. (2002) worked on a manufacturer-retailer situation where the manufacturer is the channel leader, and when the channel members are involved in partnership. They observed that the payoff is larger in a partnership. Another consideration of partnership relationship in cooperative advertising was done by Xie and Wei (2009) in their work on channel coordination. They studied a noncooperative Stackelberg game and Nash cooperative game. They observed that a cooperative relationship results in better channel coordination. In a shift from the traditional bilateral monopolistic model Wang et al. (2011) considered cooperative advertising involving a monopolistic manufacturer and competing retailers in a duopoly using four game structures. They observed that to achieve good channel coordination the players should engage in cost-sharing contract. Another consideration of a situation involving a manufacturer and two retailers was considered by Ghadimi et al. (2013). Using the concepts of cooperative games and group equilibria they obtained an appropriate allocation strategy for sharing channel profit. Further, another consideration of cooperative advertising using one manufacturer and two retailers was considered by Aust and Buscher (2014). They observed that demand is sensitive to advertising and price. They also observed that end-users benefit from retail competition, and that integration is not usually suitable for retail-duopoly.

It is a known fact that uncoordinated channel leads to double marginalization leading to inefficiencies (Gerstner and Hess, 1995). Hence, channel coordination is very important.

Kazemi and SaeedMohammadi (2016) employed cooperative advertising together with price to coordinate a two-level channel. Tibrewala et al. (2018) devised a strategy for coordinating a two-stage supply channel for a product that has a known expiration date (limited life span). They noted that proper coordination leads to increase in channel payoff and also reduces the resultant effect of demand uncertainty on payoff. Cooperative advertising is an effective tool in channel coordination as observed by Lu and Zhang (2019). They observed that channel members can find themselves in a prisoner's dilemma, of which cooperative advertising provides an escape route.

Cooperative advertising models are characterised by the support given by the manufacturer to the retailer. This is usually referred to as subsidy. Considering a two-period bilateral monopoly, Martin-Herran and Sigue (2017) observed that the retailer should engage in advertising the manufacturer's product in both periods, but should only be provided with subsidy in the second period. Ezimadu and Nwozo (2018) compared a situation where the manufacturer subsidises retail advertising while still engaging in manufacturer's national advertising, with the situation in He et al. (2009) where the manufacturer subsidises retail advertising without directly engaging in advertising. They showed that the manufacturer's direct involvement in advertising should be encouraged. In an extension of the traditional cooperative advertising model from a manufacturer-retailer to a manufacturer-distributor-retailer setting Ezimadu (2019) considered a channel structure involving the transfer of subsidy from the manufacturer to the retailer through the distributor. He observed that the channel members should ensure that the manufacturer's provided subsidy reaches the retailer, and in the event that the manufacturer does not provide subsidy, the distributor should step in to subsidise retail advertising.

Works in the cooperative advertising literature usually consider the effect of price, demand, subsidy, quality and the likes on the payoffs, but have not been able to capture the possibility of switching between subsidy and wholesale price margin which by extension influences the retail price. This is the centre of this work. Thus, we consider for the first time a situation where price margin and subsidy can be interchanged purposely for channel coordination. We attempt to provide advice for managerial implementation in cooperative advertising in a bilateral monopoly. The work addresses the condition under which the manufacturer should totally subsidise retail advertising, and determine whether the wholesale price can be strategically used to determine the optimal subsidy rate. Further, it will determine whether subsidy can be interchanged with wholesale price margin.

THE MODEL

In this work we assume that the retailer sells only the manufacturer's product brand within the product class. To increase product demand he (the retailer) engages in advertising, while the manufacturer on the other hand supports him by subsidising his advertising expenditure.

List of Notations

We will use of the following notations:

$P_M > 0$ Wholesale price (The price the manufacturer sells the product to the retailer)

$P_R > 0$ Retail Price (The price the retailer sells the product to the consumers)

$a_R > 0$ Retail advertising effort

$S \in [0,1]$ Subsidy rate given to the retailer by the manufacturer

$\alpha \in (0, 1]$ Advertising effectiveness parameter

$\theta > 0$ Rate of decrease of demand with respect to the retail price

R_{Pay} Retailer's payoff

M_{Pay} Manufacturer's payoff

The Advertising-Price-Demand Function

It is well known that advertising influences product demand (Xie and Wei (2009)). Also retail price affects demand (He et al 2009). Thus we use the multiplicative effect of advertising and price to model the end-user demand:

$D(a_R, P_R) = f(a_R)g(P_R)$,

where $f(a_R)$ is the impact of advertising on demand; and $g(P_R)$ is the impact of price on demand. This advertising-price-demand function is well known in the advertising literature (Kuehn, 1962; Xie and Wei, 2009).

Now, considering the known saturation effect of advertising on sale we adopt the function

$f(a_R) = \alpha a_R^{\frac{1}{2}}$,

where the advertising effectiveness parameter α indicates the effect of advertising on sale. Clearly this is a concave function of the advertising effort a_R. This ascertains the possibility of diminishing return on advertising which is a common phenomenon in the advertising literature.
Just as it is in the cases of many other demand-price models, we assume that demand is a linearly decreasing function of price. Thus we express the impact of price on demand as follows:

$g(P_R) = 1 - \theta P_R$,
where θ is a positive constant. To simplify our discussion, we normalize the maximum value of $g(P_R)$ to 1.

The Players' Payoffs
We observe that
$R_{pay} = Retailer's\ Price\ Margin \times Demand - Retailer's\ Expenditure.$
Also we have that
$M_{Pay} = Manufacturer's\ Price\ Margin \times Demand - Manufacturer's\ Expenditure$
Thus we have that the manufacturer and retailer's payoffs are given by:

$$M_{pay} = P_M(1 - \theta P_R)\alpha a_R^{\frac{1}{2}} - S a_R \quad (1)$$
and
$$R_{Pay} = (P_R - P_M)(1 - \theta P_R)\alpha a_R^{\frac{1}{2}} - (1 - S)a_R \quad (2)$$
respectively.

RESULTS

The Optimal Problem, Strategies and Payoffs
We model this work as a Stackelberg game in which the manufacturer being the game leader first announces his wholesale price P_M through which he intends to transfer the product to the retailer; and the participation (subsidy) rate S which he intends to give the retailer as his advertising support. Subsequently the retailer takes a decision on his advertising effort a_R and retail price P_R which he is willing to sell the product to the end-users. We obtain the Stackelberg equilibrium through backward induction by first solving the retailer's optimal problem

$$\text{Max } R_{Pay} = (P_R - P_M)(1 - \theta P_R)\alpha a_R^{\frac{1}{2}} - (1 - S)a_R,$$
$$s.t \quad P_R > 0, \ a_R > 0. \quad (3)$$
Now,
$$\frac{\partial R_{Pay}}{\partial a_R} = (P_R - P_M)(1 - \theta P_R)\alpha a_R^{-\frac{1}{2}} - (1 - S). \quad (4)$$

Equating (4) to 0 we have:

$$a_R = \frac{\alpha^2(P_R - P_M)^2(1 - \theta P_R)^2}{2^2(1 - S)^2}. \quad (5)$$

Also from (3) we have that

$$\frac{\partial R_{Pay}}{\partial a_R} = (1 - \theta P_R)(1) + (P_R - P_M)(-\theta). \quad (6)$$

Equating (6) to 0 we have

$$P_R = \frac{1 + \theta P_M}{2}. \quad (7)$$

Now, from (2) and (5) we have that

$$R_{Pay} = \frac{\alpha^2(P_R - P_M)^2(1 - \theta P_R)^2}{2^2(1 - S)}. \quad (8)$$

From the above discussion we have the following result:
Proposition 1 Given the retailer's optimal problem (3), then the retailer's advertising effort, price and payoff are given by (5), (7) and (8) respectively.

From (1) we have that the manufacturer's optimal problem is:

$$\text{Max} M_{Pay} = P_M(1-\theta P_R)\alpha a_R^{\frac{1}{2}} - S a_R, \qquad (9)$$
$$s.t \quad P_M > 0, \;\; S \in [0,1].$$

Using (5) and (7) in (9) we have

$$\text{Max} M_{Pay} = \alpha P_M \left(1-\theta\frac{1+\theta P_M}{2\theta}\right)\frac{\alpha\left(\frac{1+\theta P_M}{2\theta}-P_M\right)\left(1-\theta\frac{1+\theta P_M}{2\theta}\right)}{2(1-S)}$$
$$-S\frac{\alpha^2\left(\frac{1+\theta P_M}{2\theta}-P_M\right)^2\left(1-\theta\frac{1+\theta P_M}{2\theta}\right)^2}{2^2(1-S)^2}$$

$$s.t \quad P_M > 0, \;\; S \in [0,1]$$

That is

$$\text{Max} M_{Pay} = \frac{\alpha^2 P_M}{2\theta(1-S)}\left(\frac{1-\theta P_M}{2}\right)^3 - \frac{\alpha^2 S}{2^2\theta^2(1-S)^2}\left(\frac{1-\theta P_M}{2}\right)^4, \qquad (10)$$
$$s.t \quad P_M > 0, \;\; S \in [0,1].$$

Now,

$$\frac{\partial M_{Pay}}{\partial P_M} = \frac{\alpha^2}{2^4\theta(1-S)}\left(-3\theta P_M(1-\theta P_M) + (1-\theta P_M)^3\right)$$
$$+\frac{4\alpha^2\theta S}{2^6\theta^2(1-S)^2}(1-\theta P_M)^3 = 0 \qquad (11)$$

Equating (11) to 0 we have

$$P_M = \frac{1}{4\theta - 3\theta S}. \qquad (12)$$

Thus from (7) and (12) we have that

$$P_R = \frac{1+\theta\left(\frac{1}{4\theta-3\theta S}\right)}{2\theta} = \frac{5-3S}{2\theta(4-3S)}. \quad (13)$$

Also from (10) we have that

$$\frac{\partial M_{Pay}}{\partial S} = \frac{\alpha^2 P_M}{2\theta}\left(\frac{1-\theta P_M}{2}\right)^3\frac{1}{(1-S)^2}$$
$$-\frac{\alpha^2}{2^2\theta^2}\left(\frac{1-\theta P_M}{2}\right)^4\left[\frac{(1-S)^2 - 2(1-S)S(-1)}{(1-S)^4}\right]. \qquad (14)$$

Equating (14) to 0 we have:

$$P_M = \frac{1}{2\theta}\left(\frac{1-\theta P_M}{2}\right)\left(\frac{1+S}{1-S}\right),$$
$$\Rightarrow \quad S = \frac{5\theta P_M - 1}{3\theta P_M + 1}. \qquad (15)$$

Now, from (10) and (15) we have that

$$M_{Pay} = \frac{\alpha^2 P_M(3\theta P_M+1)}{2\theta(-2\theta P_M+2)}\left(\frac{1-\theta P_M}{2}\right)^3 - \frac{\alpha^2(3\theta P_M+1)(5\theta P_M-1)}{2^2\theta^2(-2\theta P_M+2)^2}\left(\frac{1-\theta P_M}{2}\right)^4. \quad (16)$$

Thus, from the above discussion we have the following result:

Proposition 2: Given the manufacturer's optimal problem (9), then the manufacturer's wholesale priceP_M, the subsidy rate S, and the payoff M_{Pay} are given by (12), (15) and (16) respectively.

Unsubsidised and Subsidised Equilibrium

Suppose the manufacturer does not subsidise the retail advertising effort, so that $S = 0$, then we have an unsubsidised equilibrium. Thus (12) becomes

$P_M = \frac{1}{4\theta}$. (17)

Using (17) in (7) we have

$P_R = \frac{1+\theta\frac{1}{4\theta}}{2\theta} = \frac{5}{8\theta}$. (18)

Using (17) and (18) in (5) we have

$$a_R = \frac{\alpha^2\left(1-\theta\frac{5}{8\theta}\right)^2\left(\frac{5}{8\theta}-\frac{1}{4\theta}\right)^2}{2^2} = \left(\frac{3}{8}\right)^4\left(\frac{\alpha}{2\theta}\right)^2. \quad (19)$$

Substituting (17), (18) and (19) into (2) we have that

$$R_{Pay} = \left(1-\theta\frac{5}{8\theta}\right)\left(\frac{5}{8\theta}-\frac{1}{4\theta}\right)\alpha\left(\left(\frac{3}{8}\right)^4\left(\frac{\alpha}{2\theta}\right)^2\right)^{\frac{1}{2}} - \left(\frac{3}{8}\right)^4\left(\frac{\alpha}{2\theta}\right)^2 = \frac{3^4}{4^7}\left(\frac{\alpha}{\theta}\right)^2.$$

Substituting (17), (18) and (19) into (1) we have that

$$M_{Pay} = \left(1-\theta\frac{5}{8\theta}\right)\left(\frac{1}{4\theta}\right)\alpha\left(\left(\frac{3}{8}\right)^4\left(\frac{\alpha}{2\theta}\right)^2\right)^{\frac{1}{2}} = \frac{3^3}{8^4}\left(\frac{\alpha}{\theta}\right)^2.$$

With the provision of subsidy we have that (12) and (15) leads to

$$P_M = \frac{1}{4\theta - 3\theta\left(\frac{5\theta P_M - 1}{3\theta P_M + 1}\right)}$$

$$\Longrightarrow \quad 3\theta^2 P_M^2 - 4\theta P_M + 1 = 0$$

$$\Longrightarrow \quad P_M = \frac{-(-4\theta) \pm \sqrt{(-4\theta)^2 - 4(3\theta^2)(1)}}{2(3\theta^2)} = \frac{4\theta \pm 2\theta}{6\theta^2}$$

$$= \frac{1}{\theta} \text{ or } \quad \frac{1}{3\theta}.$$

Observe that for $P_M = \frac{1}{\theta}$ we have that $P_R = \frac{1}{\theta}$ which (from (1) and (2)) implies that

$$R_{Pay} = M_{Pay} = 0,$$

implying that $P_M = \frac{1}{\theta}$ is inappropriate.

Now, using $P_M = \frac{1}{3\theta}$ in (7) we have

$$P_R = \frac{1+\theta\frac{1}{3\theta}}{2\theta} = \frac{2}{3\theta}. \quad (20)$$

Thus

$$P_M = \frac{1}{3\theta}. \quad (21)$$

Using (15), (20) and (21) in (5) we have

$$a_R = \frac{\alpha^2\left(1-\theta\frac{2}{3\theta}\right)^2\left(\frac{2}{3\theta}-\frac{1}{3\theta}\right)^2}{2^2\left(1-\frac{5\theta\frac{1}{3\theta}-1}{3\theta\frac{1}{3\theta}+1}\right)^2} = \left(\frac{\alpha}{12\theta}\right)^2.$$

From (2), (15), (20) and (21) we have that

$$R_{Pay} = \left(1-\theta\frac{2}{3\theta}\right)\left(\frac{2}{3\theta}-\frac{1}{3\theta}\right)\alpha\left(\frac{\alpha^2\left(1-\theta\frac{2}{3\theta}\right)^2\left(\frac{2}{3\theta}-\frac{1}{3\theta}\right)^2}{2^2\left(1-\frac{5\theta\frac{1}{3\theta}-1}{3\theta\frac{1}{3\theta}+1}\right)^2}\right)^{\frac{1}{2}} - \frac{\alpha^2\left(1-\theta\frac{2}{3\theta}\right)^2\left(\frac{2}{3\theta}-\frac{1}{3\theta}\right)^2}{2^2\left(1-\frac{5\theta\frac{1}{3\theta}-1}{3\theta\frac{1}{3\theta}+1}\right)^2}$$

$$= \left(\frac{1}{6}\right)^3\left(\frac{\alpha}{\theta}\right)^2.$$

From (1), (15), (20) and (22) we have that

$$M_{Pay} = \left(1-\theta\frac{2}{3\theta}\right)\left(\frac{2}{3\theta}\right)\alpha\left(\left(\left(\frac{\alpha}{12\theta}\right)^2\right)^{\frac{1}{2}} - \left(\frac{5\theta\frac{1}{3\theta}-1}{3\theta\frac{1}{3\theta}+1}\right)\left(\frac{\alpha}{12\theta}\right)^2\right)$$

$$= \left(\frac{1}{12}\right)^2\left(\frac{\alpha}{\theta}\right)^2.$$

DISCUSSION

We recall that α measures the effectiveness of advertising, so that $\alpha \in (0,1]$. Thus we let $\alpha = 0.3$. Further, θ measures the rate of decrease of the demand. As such we let $\theta = 0.005$.

Wholesale Price-Subsidy Coordination

Observe from Figure 1 that when there is no subsidy the retail price P_R is larger than the manufacturer's wholesaleprice P_M. This can be seen as the result of the retailer's effort to ensure that he stays in business when there is no support for his advertising effort. Thus the larger retail price margin is a form of compensation for his advertising spending. As the subsidy increases towards 1 (total subsidy) we observe that both prices become equal, which from (12) and (13) affirms that $P_M = P_R$ implies that

$$\frac{1}{4\theta-3\theta S} = \frac{5-3S}{2\theta(4-3S)}$$

$$\Longrightarrow \quad 2\theta(4-3S) = \theta(4-3S)(5-3S)$$

$$\Longrightarrow \quad 2 = 5-3S$$

$$\Longrightarrow \quad S = 1$$

This means that the manufacturer can only be willing to totally subsidise retail advertising if and only if both prices are equal. That is if the retailer's price margin is 0 (zero). But from (2) this will lead to $R_{pay} < 0$. Thus, it is practically impossible since the retailer is in business to make profit. This is clear from Figure 1 which shows that as subsidy increases the manufacturer's margin increases while the retailer's margin reduces and eventually becomes zero with total subsidy.

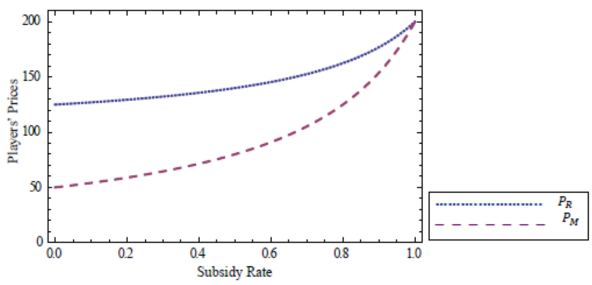

Figure 1: The effect of subsidy on the players' prices

However it is pertinent to note that the manufacturer must not totally subsidise the retail advertising effort. This is quite clear from (5) which suggests that total subsidy would lead to much advertising commitment such that the retailer's effort would become unbounded. This is unrealistic! Further (8) shows that this total subsidy and subsequently possible larger advertising effort will lead to very large retailer's payoff. In short it would lead to unbounded payoff. Again this is unrealistic.

On the other hand (10) shows that total subsidy will lead to a loss by the manufacturer. Observe from (10) that as $S \longrightarrow 1$ the term $(1-S)^2$ approaches 0 (zero) faster than $(1-S)$. Consequently the term $\frac{\alpha^2 S}{2^2\theta^2(1-S)^2}\left(\frac{1-\theta P_M}{2}\right)^4$ becomes unbounded much faster than $\frac{\alpha^2 P_M}{2\theta(1-S)}\left(\frac{1-\theta P_M}{2}\right)^3$. Thus M_{pay} approaches a negative value as the subsidy gets very large. Thus the manufacturer should provide only the optimal subsidy rate.

Obviously, with every additional increase in subsidy both players' prices increase. However, the manufacturer's rate of increase is higher than that of the retailer. This implies that for every additional unit of support to the retailer, the manufacturer should compensate himself by increasing his wholesale price.
From the foregoing we observe that subsidy can serve as price switching mechanism or strategy since every increase in subsidy leads to increase in wholesale price margin and vice versa, and every increase in subsidy leads to reduction in retail price margin. Thus the manufacturer has the option of using wholesale price margin or subsidy to influence the retail price. Thus as the channel leader, the manufacturer can use subsidy to coordinate the channel. This ensures proper price control of the manufacturer's product sold by the retailer to the end-users.

Subsidy-Price-Payoff Coordination Mechanism

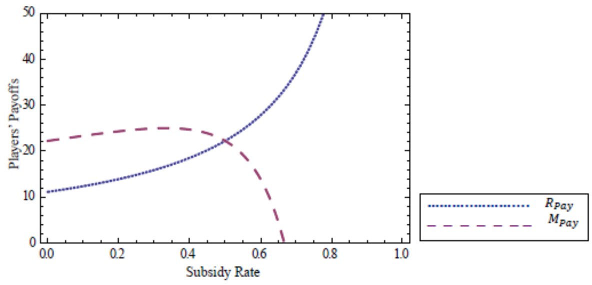

Figure 2: The effect of subsidy on Payoff.

We observe from Figure 2 that while the retailer's payoff R_{Pay} increases with subsidy, the reverse is the case with the manufacturer's payoff M_{Pay}. Thus at a certain subsidy level both payoffs become equal. Thus from (8) and (10) we have that equality of both payoffs would imply that

$$\frac{\alpha^2 P_M}{2\theta(1-S)}\left(\frac{1-\theta P_M}{2}\right)^3-\frac{\alpha^2 S}{2^2\theta^2(1-S)^2}\left(\frac{1-\theta P_M}{2}\right)^4=\frac{\alpha^2(P_R-P_M)^2(1-\theta P_R)^2}{2^2(1-S)}$$

$$\Rightarrow \quad \frac{P_M}{2\theta}\left(\frac{1-\theta P_M}{2}\right)^3-\frac{\alpha^2 S}{2^2\theta^2(1-S)}\left(\frac{1-\theta P_M}{2}\right)^4=\frac{\left(1-\theta\frac{1+\theta P_M}{2\theta}\right)^2\left(\frac{1+\theta P_M}{2\theta}-P_M\right)^2}{4}$$

$$\Rightarrow \quad S=\frac{4(1-(\theta+1)P_M)}{\theta(4-(3+\theta)P_M)}. \qquad (22)$$

From (15) and (22) we have that

$$\frac{5\theta P_M-1}{3\theta P_M+1}=\frac{4(1-(\theta+1)P_M)}{\theta(4-(3+\theta)P_M)}$$

which implies that

$$P_M=-\frac{\sqrt{(-24\theta^2+2\theta)^2+32\theta^3+12\theta^2}+24\theta^2-2\theta}{16\theta^3+6\theta^2} \quad (23)$$

or

$$P_M = -\frac{-\sqrt{(-24\theta^2 + 2\theta)^2 + 32\theta^3 + 12\theta^2} + 24\theta^2 - 2\theta}{16\theta^3 + 6\theta^2}. \qquad (24)$$

We note that $\sqrt{(-24\theta^2 + 2\theta)^2 + 32\theta^3 + 12\theta^2} + 24\theta^2 > 2\theta$, thus (24) is more appropriate.

Now, we recall from (12) that the manufacturer's wholesale price P_M increases with the subsidy rate S and vice versa. Thus increasing the wholesale price above (24) will lead to high subsidy which will eventually lead to the situation in Figure 2 where the provision of subsidy above a certain level will lead to the manufacturer's payoff being lower than the retailer's payoff. Thus appropriate wholesale price decision can be effectively used to decide appropriate subsidy rate, and subsequently coordinate the channel.

Implication of Findings

In this work we made the following contributions to the cooperative advertising literature:

(*i*) We have added to the static game theoretic cooperative advertising literature, and obtained closed-form solutions of the models.

(*ii*) Previous works in the literature always centre on using particular parameter(s) for channel coordination, but this work has shown that wholesale price margin and subsidy rate can be interchanged for channel coordination. In other words, any of them can be used instead of the other. An extension can consider incorporating other parameters/variables into the work. Further considerations can centre on switching or interchanging other parameters and/or variables in the cooperative advertising and channel coordination literature.

CONCLUSION

This work shows that it is possible for the manufacturer to use advertising subsidy or appropriate wholesale price to influence the retail price, and subsequently coordinate the supply channel. However, it is not advisable to totally subsidise retail advertising. This should only be done if the retail margin is zero.

REFERENCES

Aust, G. and Buscher, U. (2014).Vertical cooperative advertising in a retailer duopoly.*Computer and Industrial Engineering*, **72**(1): 247-254.

Berger, P. D. (1972). Vertical cooperative advertising ventures.*Journal of Marketing Research*, **9**(3): 309-312.

Bergen, M. and John, G. (1997). Understanding cooperative advertising participation rates in conventional channels. *Journal of Marketing Research*, **34**(3): 357-369.

Dant, R. P. and Berger, P. D. (1996).Modelling cooperative advertising decisions in franchising.*The Journal of Operational Research Society*, **47**(9): 1120-1136.

Ezimadu, P. E. (2019). A mathematical model of the effect of subsidy transfer in cooperative advertising using differential game theory.*Journal of Industrial Engineering International*, **15**(1): 351-366.

Ezimadu, P. E. and Nwozo, C. R. (2018).Modeling dynamic cooperative advertising in a decentralized channel, *Yugoslav Journal of Operations Research*, **28**(4): 539-566.

Gerstner, E. and Hess, J. D. (1995). Pull promotion and channel coordination. *Marketing Science,* **14**(1): 43–60.

Ghadimi, S., Szidarovszky, F., Farahani, R. Z. and Khiabani, A. Y. (2013).Coordination of advertising in supply chain management with cooperating manufacturer and retailers.*IMA Journal of Management Mathematics*, **24**(1):1-19.

He, X., Prasad, A. and Sethi, S. P. (2009). Cooperative advertising and pricing in a dynamic stochastic supply chain:

Feedback Stackelberg strategies. *Production and Operations Management,* **18**(1): 78-94.

Huang, Z., Li S. X. and Mahajan, V. (2002). An analysis of manufacturer-retailer supply chain coordination in cooperative advertising. *Decision Science*, **33**(3): 469-494.

Kazemi, A. and SaeedMohammadi, Z. (2016). A model for cooperative advertising and pricing decisions in manufacturer-retailer supply chain with discount: A game theory approach. *International Journal of Supply and Operations Management*, **2**(4): 1035-1063.

Kim, S. Y. and Staelin, R. (1999).Manufacturer allowances and retailer pass-through rates in a competitive environment.*Marketing Science*, **18**(1): 59-76.

Kuehn, A. A. (1962). How advertising performance depends upon other marketing factors, *Journal of Advertising Research*, **2**(1): 2-10.

Lu, F. and Zhang, J. (2019) Cooperative advertising: A way escaping from prisoner's dilemma in a supply chain with sticky price, *Omega*, **86**: 87-106.

Martin-Herran, G. and Sigue, S. P. (2017). An integrated framework of cooperative advertising: Should manufacturers continuously support retail advertising? *Journal of Business Research*, **70**(1): 67-73.

Tibrewala, R., Tibrewala, R. and Meena, P. L. (2018). Buy-back policy for supply chain coordination: a simple rule. *International Journal of Operational Research*, **31**(4): 545-572.

Wang, S., Zhou, Y., Min, J. and Zhong, Y. (2011). Coordination of cooperative advertising models in a one-manufacturer two-retailer supply chain system. *Computer and Industrial Engineering*, **61**(4):1053-1071.

Xie, J. and Wei, J. (2009). Coordination advertising and pricing in a manufacturer-retailer channel. *European Journal of Operational Research*, **197**(2), 785-791.

Zhang, J., Li, J., Lu, L. and Dai, R. (2017). Supply chain performance for deteriorating items with cooperative advertising. *Journal of Systems Science and Systems Engineering*, **26**(1): 23-49.