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Full Length Research Article

# Variation Iteration Method for The Approximate Solution of Nonlinear Burgers Equation

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## ABSTRACT
In this study, we considered the numerical solution of the nonlinear Burgers equation using the Variational Iteration Method (VIM). The method seeks to examine the convergence of solutions of the Burgers equation at the expense of the parameters $x$ and t, of which the amount of errors depends. Numerical experimentation was carried out on the Burgers equation with the Variational Iteration Method (VIM). The resulting solution showed that the rate of convergence decreases with increase in the values of the parameters $x$ and t at each iterate level. However, as the number of iterations increases, there is a rapid rate of convergence of the approximate solution to the analytic solution. Results obtained with the Variational Iteration Method (VIM) on the Burgers equation were compared with the exact found in literature. All computational framework of the research were performed with the aid of Maple 18 software.
**keywords:** Variational Iteration Method, Burgers Equation, Partial Differential Equations, Approximate Solution, Mean Value Theorem, Schwartz Inequality.

## INTRODUCTION
The study of the Burgers' equation is an important aspect of compressible and incompressible fluid models and the critical analysis of these models have been triggered by many researchers in recent years for understanding the basic principles of a class of physical flows and for examining various computational procedures. The Burgers equation has relevant applications in many field of Mathematics which include, hydrodynamic, gas dynamic, time-space stochastic processes, rocket motor, acoustic, number theory, heat conduction, shock waves, etc. (Burger, 1948). Hence, obtaining the exact resolution of this equation for a precise analysis of models under consideration is of great significance.

However, available analytic methods are insufficient in handling these equations due to large computational and round-off errors which arise due to linearization and perturbation. Thus, approximate methods have become more relevant as developed by researchers over the years to effectively handle these problems. These include: the Variational Iteration Method (VIM) (He, 1998), the Reconstruction of Variation Iteration Method (RVIM) (Esfandyaripour, 2013), Homotopy Analysis Method (HAM) (Molabahrami and Khani, 2009), the Homotopy Perturbation Method (He, 2004 and 2005), the piecewise-adaptive decomposition method (Ramos, 2008) etc.

The Variational Iteration Method (VIM) was first proposed by the Chinese mathematician, J.H. He in He (1998) for solving both linear and nonlinear problems. The method involves the construction of a correction functional for the problem in question from which the Lagrange multiplier is derived optimally using the Variational Theory. This method has been extended to solve different classes of problems by various researchers such as, Fredholm and Volterra integro-differential equations (Mamadu and Njoseh, 2016; Wazwaz, 2011; Abdelkhani, 1993; Abbasbandy and Shivanian, 2009), partial differential equations (Duangpithak, 2012; He, 1998; He, 1999), delay differential equations (Liu *et al*., 2013), fractional diffusion equations (Gao *et al*., 2016), etc. Njoseh and Mamadu (2017) equally coupled the Variational Iteration Method and the Homotopy Perturbation Method in a modified sense for the numerical treatment of the Burgers equation.

In this paper, the Variation Iteration Method (VIM) is initiated to solve nonlinear partial differential equation of which the Burgers equation is a special kind. For this Burgers equation, the initial approximation is freely chosen so as to satisfy the analytic solution. The method gives the solution in a more compact rapidly convergent series. For numerical illustration, the method was applied to one-dimensional nonlinear Burgers equation of the form $\frac{\partial y}{\partial t} + y\frac{\partial y}{\partial x} = \frac{\partial^2 y}{dx^2}$ (1)

Subject to the initial condition:
$y(x,t) = \frac{1}{2} - \frac{1}{2}\tanh\left(\frac{x}{4}\right)$.

## METHODOLOGY

### Basic Ideas of VIM

Consider the general differential equation

$Ly(x,t) + Ny(x,t) = f(x,t)$ (2)

with prescribed auxiliary conditions, where $L$ is a linear operator of the highest derivative, $N$ a non linear term, $y(x,t)$ is unknown function and $f(x,t)$ is a source term.

Using VIM (He, 1999; Duangpithak, 2012; Esfandyaripour *et al.*, 2013), we can construct correction functional for equation (2) as:

$$y_{k+1}(x,t) = y_k(x,t) + \int_0^x \lambda(\xi)\left[Ly_k(x,\xi) + N\tilde{y}_k(x,\xi) - f(,x\xi)\right]d\xi,\ k \geq 0 \quad (3)$$

where $\lambda$ is a general Lagrange multiplier, $\tilde{y}_k(x,t) = 0$, i.e, $\tilde{y}_k(x,t)$ is a restricted variable, where it can be identified via variational theory (He, 1999, He and Wu, 2006). However, the Lagrange multiplier can still be obtained using the formula proposed by Abbasbandy and Shivanian (2009)

$\lambda(s) = \frac{(-1)^n(s-x)^n}{(n-1)!}$,

Where $n$ is the order of the derivative.

### Variation Iteration Method for the Burgers Equation

To apply the Variation Iteration Method (VIM), we redefine equation (2) as

$Ly(x,t) + Ny(x,t) = f(x,t)$ (4)

where

$Ly(x,t) = \frac{\partial^2 y}{\partial x^2}$, $Ny(x,t) = y\frac{\partial y}{\partial x}$, $f(x,t) = \frac{\partial y}{\partial t}$.

By the theory of Variational Iteration Method, we construct a correction functional for equation (4) as follows:

$$y_{k+1}(x,t) = y_k(x,t) + \int_0^t \lambda(s)\left[\frac{\partial^2 y_k(x,s)}{\partial x^2} - y_k(x,s)\frac{\partial y_k(x,s)}{\partial x} - \frac{\partial y_k(x,s)}{\partial t}\right]ds,\ k \geq 0 \quad (5)$$

and to obtain the Lagrange multiplier $\lambda(s)$, we take the variation of (5) with respect to $y_k(x,s)$, bearing in mind that the variation of $\tilde{y}_k(x,t)$ is zero, as:

$$\delta y_{k+1}(x,t) = \delta y_k(x,t) + \delta\int_0^t \lambda(s)\left[\frac{\partial^2 y_k(x,s)}{\partial x^2} - y_k(x,s)\frac{\partial y_k(x,s)}{\partial x} - \frac{\partial y_k(x,s)}{\partial t}\right]ds,\quad k \geq 0\ . \quad (6)$$

$$\delta y_{k+1}(x,t) = \delta y_k(x,t) + \delta\left[\lambda(s)\left(\frac{\partial y_k(x,s)}{\partial t} - y_k(x,s)\right)\right]_{s=t} - \int_0^t \lambda'(s)\left(\frac{\partial y_k(x,s)}{\partial t} - y_k(x,s)\right)ds,\quad k \geq 0 \quad (7)$$

Hence, equation (7) provides the stationary conditions

$\lambda(s) - 1 = 0$ (8)

$[\lambda'(s)]_{s=t} = 0, \quad [\lambda''(s)]_{s=t} = 0 \qquad (9)$
Equation (8) is the Lagrange multiplier and Equation (9) is the boundary condition.

We now rewrite Equation (5) as

$$y_{k+1}(x,t) = y_k(x,t) - \int_0^t \left[\frac{\partial^2 y_k(x,s)}{\partial x^2} - y_k(x,s)\frac{\partial y_k(x,s)}{\partial x} - \frac{\partial y_k(x,s)}{\partial t}\right] ds, \quad k \geq 0 \qquad (10)$$

Equation (10) is the Variation Iteration Method for the Burgers equation (1).

## Convergence Analysis of the Method

Let $H = (\alpha_0, \alpha_1) \times [0, T]$, H is a Hilbert space, and define

$y: H \to R, \ \int_H y^2(x) dx dw < +\infty.$

**Theorem 1.**
Let

$$\frac{\partial y}{\partial t} = V(y)$$

in equation (1) such that

$$V(y) = \frac{\partial^2 y}{\partial x^2} - y\frac{\partial y}{\partial x},$$

then the variational iteration method of equation (1) converges if;

$i.$ $(V(y) - V(Y), y - Y) \geq \alpha \|y - Y\|^2, \alpha > 0, \ y, Y \in H.$
$ii.$ For $w > 0$, there exist $I(w) > 0$ for all $\|y\| \leq w, \|Y\| \leq w, \quad y, Y \in H,$
then

$$(V(y) - V(Y), y - Y) \geq I(w)\|y - Y\|\|r\|, \qquad r \in H.$$

***Proof***
Let $y, Y \in H$, then for $\alpha > 0$, we have

$$(V(y) - V(Y), y - Y) = \left(\left(\frac{\partial^2 y}{\partial x^2} - y\frac{\partial y}{\partial x}\right) - \left(\frac{\partial^2 Y}{\partial x^2} - Y\frac{\partial Y}{\partial x}\right), y - Y\right).$$

Applying the Schwartz inequality, we have

$$\begin{aligned}(V(y) - V(Y), y - Y) &\leq \alpha_1 \left\|\frac{\partial^2 y}{\partial x^2} - y\frac{\partial y}{\partial x} - \frac{\partial^2 Y}{\partial x^2} + Y\frac{\partial Y}{\partial x}\right\| \|y - Y\| \\ &\leq \alpha_1 \left\|\frac{\partial^2}{\partial x^2}(y - Y) + (y - Y)\frac{\partial}{\partial x}(Y - y)\right\| \|y - Y\| \\ &\leq \alpha_1 \left\|\frac{\partial^2}{\partial x^2}e(x) + e(x)\frac{\partial}{\partial x}(-e(x))\right\| \|e(x)\|\end{aligned}$$

where

$$e(x) = y(x) - Y(x),$$

$y(x)$ is the computed solution, and $Y(x)$ is the exact solution.

By the mean value theorem, we have that

$$\big(V(y) - V(Y), e(x)\big) \geq \frac{1}{4}\alpha_1 w^2 \|e(x)\|^2.$$

Also, if $I(w) > 0$ for all $\|y\| \leq w, \|Y\| \leq w, \quad y, Y \in H$, then we have

$$\begin{aligned}\big(V(y) - V(Y), e(x)\big) &= \left(\frac{\partial^2}{\partial x^2}e(x) + e(x)\frac{\partial}{\partial x}\big(-e(x)\big), e(x)\right) \\ &\geq I(w)\|e(x)\|\|r\|, \ r \in H.\end{aligned}$$

This completes the proof.

**Theorem 2.**
The Burgers equation as defined in equation (1) converges rapidly to the exact for $t = \frac{\tau}{10^m}, \tau > 0$, and $n$ is the number of iterations.
(Readers are referred to Mamadu and Njoseh (2017) for the proof of Theorem 2.)

**Numerical Illustrations**
We consider the one dimensional nonlinear Burgers equation for illustration in order to show the effectiveness and reliability of the method with the aid of Maple 18 software for our computations. The results obtained are compared with the exact solution available in literature.
Given the Burgers equation

$$\frac{\partial y}{\partial t} + y\frac{\partial y}{\partial x} = \frac{\partial^2 y}{\partial x^2} \qquad (11)$$

subject to the initial condition:
$y(x,t) = \frac{1}{2} - \frac{1}{2}\tanh\left(\frac{x}{4}\right)$.

The analytic solution is
$y(x,t) = \frac{1}{2} - \frac{1}{2}\tanh\left(\frac{1}{4}(x - \frac{t}{2})\right)$.

By imposing the initial condition on the exact solution, we take the initial approximation as
$y_0(x,s) = \frac{1}{2} - \frac{1}{2}\tanh\left(\frac{x}{4}\right)$ .
Hence, for $k = 0$ in Equation (10), we have

$$y_1 = \frac{1}{2} - \frac{1}{2}\tanh\left(\frac{1}{4}x\right) + \frac{1}{4}\tanh\left(\frac{1}{4}x\right)\left(\frac{1}{4} - \frac{1}{4}\tanh\left(\frac{1}{4}x\right)^2\right)t - \left(\frac{1}{2} - \frac{1}{2}\tanh\left(\frac{1}{4}x\right)\right)\left(-\frac{1}{8} + \frac{1}{8}\frac{1}{4}\tanh\left(\frac{1}{4}x\right)^2\right)t$$

The **Tables** given below show the amount of absolute error for $y_1$ , $y_2$ and $y_4$ and the amount of error responses to the parameters $x$ and $t$. . Here, $x$ ranges from 0.1 to 1.0, and $t$ ranges from 0.2 to 3.0.

**DISCUSSION OF RESULTS**
Apparently from **Tables 1** to **3**, it is obvious that the amount of errors responses to the parameters $x$ and $t$ indicates that the convergence of the method as applied to the Burgers equation decreases with increase in the values of the parameters $x$ and $t$. It is also noted that the rate of convergence of the scheme improve adequately as the number of iterations increases. Thus, by theorem 2, $(n+1)$ iterations will see the approximate solution converging absolutely to the analytic solution.

**Table 1:** Computation of absolute errors for first approximation, $y_1$

| X | First approximation, $y_1$ | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | $t = 0.2$ | $t = 0.6$ | $t = 0.9$ | $t = 1.2$ | $t = 1.5$ | $t = 1.7$ | $t = 2.0$ | $t = 2.5$ | $t = 3.0$ |
| 0.1 | 5.2058e-06 | 1.1700e-08 | 7.8775e-05 | 2.7925e-04 | 6.5173e-04 | 1.0196e-03 | 1.7853e-03 | 3.7389e-03 | 6.6977e-03 |
| 0.2 | 1.2989e-05 | 7.0178e-05 | 7.8952e-05 | 3.7280e-07 | 2.1684e-04 | 4.6332e-04 | 1.0212e-03 | 2.5646e-03 | 5.0424e-03 |
| 0.3 | 2.0708e-05 | 1.3999e-04 | 2.3628e-04 | 2.7999e-04 | 2.1911e-04 | 9.5251e-05 | 2.5208e-04 | 1.3780e-03 | 3.3631e-03 |
| 0.4 | 2.8323e-05 | 2.0911e-04 | 3.9244e-04 | 5.5821e-04 | 6.5397e-04 | 6.5334e-04 | 5.1823e-04 | 1.8478e-04 | 1.6678e-03 |
| 0.5 | 3.5800e-05 | 2.7720e-04 | 5.4666e-04 | 8.3367e-04 | 1.0856e-03 | 1.2082e-03 | 1.2860e-03 | 1.0093e-03 | 3.5270e-05 |
| 0.6 | 4.3100e-05 | 3.4392e-04 | 6.9818e-04 | 1.1050e-03 | 1.5119e-03 | 1.7571e-03 | 2.0474e-03 | 2.1984e-03 | 1.7380e-03 |
| 0.7 | 5.0192e-05 | 4.0897e-04 | 8.4629e-04 | 1.3710e-03 | 1.9307e-03 | 2.2974e-03 | 2.7987e-03 | 3.3767e-03 | 3.4322e-03 |
| 0.8 | 5.7044e-05 | 4.7205e-04 | 9.9031e-04 | 1.6302e-03 | 2.3402e-03 | 2.8265e-03 | 3.5365e-03 | 4.5386e-03 | 5.1097e-03 |
| 0.9 | 6.3626e-05 | 5.3287e-04 | 1.1296e-03 | 1.8817e-03 | 2.7384e-03 | 3.3420e-03 | 4.2572e-03 | 5.6786e-03 | 6.7625e-03 |
| 1.0 | 6.9912e-05 | 5.9120e-04 | 1.2635e-03 | 2.1242e-03 | 3.1236e-03 | 3.8415e-03 | 4.9575e-03 | 6.7914e-03 | 8.3830e-03 |

**Table 2:** Computation of absolute errors for second approximation, $y_2$.

| X | Second Approximation, $y_2$ | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | $t = 0.2$ | $t = 0.6$ | $t = 0.9$ | $t = 1.2$ | $t = 1.5$ | $t = 1.7$ | $t = 2.0$ | $t = 2.5$ | $t = 3.0$ |
| 0.1 | 2.7302e-06 | 7.3753e-05 | 2.4869e-04 | 5.8835e-04 | 1.1457e-03 | 1.6635e-03 | 2.6959e-03 | 5.2125e-03 | 8.8929e-03 |
| 0.2 | 2.8429e-06 | 7.6970e-05 | 2.5999e-04 | 6.1618e-04 | 1.2021e-03 | 1.7475e-03 | 2.8374e-03 | 5.5036e-03 | 9.4201e-03 |
| 0.3 | 2.9406e-06 | 7.9778e-05 | 2.6991e-04 | 6.4074e-04 | 1.2521e-03 | 1.8222e-03 | 2.9639e-03 | 5.7658e-03 | 9.8984e-03 |
| 0.4 | 3.0221e-06 | 8.2155e-05 | 2.7837e-04 | 6.6182e-04 | 1.2953e-03 | 1.8871e-03 | 3.0743e-03 | 5.9969e-03 | 1.0323e-02 |
| 0.5 | 3.0876e-06 | 8.4084e-05 | 2.8530e-04 | 6.7927e-04 | 1.3313e-03 | 1.9416e-03 | 3.1678e-03 | 6.1949e-03 | 1.0692e-02 |
| 0.6 | 3.1364e-06 | 8.5551e-05 | 2.9066e-04 | 6.9296e-04 | 1.3600e-03 | 1.9853e-03 | 3.2436e-03 | 6.3584e-03 | 1.1001e-02 |
| 0.7 | 3.1679e-06 | 8.6551e-05 | 2.9442e-04 | 7.0281e-04 | 1.3812e-03 | 2.0179e-03 | 3.3013e-03 | 6.4863e-03 | 1.1248e-02 |
| 0.8 | 3.1827e-06 | 8.7081e-05 | 2.9657e-04 | 7.0880e-04 | 1.3947e-03 | 2.0393e-03 | 3.3407e-03 | 6.5779e-03 | 1.1432e-02 |
| 0.9 | 3.1802e-06 | 8.7144e-05 | 2.9713e-04 | 7.1095e-04 | 1.4006e-03 | 2.0496e-03 | 3.3616e-03 | 6.6329e-03 | 1.1552e-02 |
| 1.0 | 3.1617e-06 | 8.6752e-05 | 2.9611e-04 | 7.0930e-04 | 1.3989e-03 | 2.0488e-03 | 3.3643e-03 | 6.6517e-03 | 1.1609e-02 |

**Table 3:** Computation of absolute errors for fourth approximation, $y_4$.

| X | Second Approximation, $y_4$ | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | $t = 0.2$ | $t = 0.6$ | $t = 0.9$ | $t = 1.2$ | $t = 1.5$ | $t = 1.7$ | $t = 2.0$ | $t = 2.5$ | $t = 3.0$ |
| 0.1 | 2.6000e-09 | 6.5500e-07 | 5.1128e-06 | 2.2166e-05 | 6.9608e-05 | 1.3267e-04 | 3.0771e-04 | 9.8519e-04 | 2.5716e-03 |
| 0.2 | 2.7000e-09 | 6.2530e-07 | 4.8782e-06 | 2.1116e-05 | 6.6211e-05 | 1.2608e-04 | 2.9211e-04 | 9.3394e-04 | 2.4360e-03 |
| 0.3 | 2.4000e-09 | 5.8650e-07 | 4.5601e-06 | 1.9696e-05 | 6.1650e-05 | 1.1727e-04 | 2.7132e-04 | 8.6595e-04 | 2.2563e-03 |
| 0.4 | 2.3000e-09 | 5.3660e-07 | 4.1616e-06 | 1.7931e-05 | 5.6001e-05 | 1.0638e-04 | 2.4570e-04 | 7.8237e-04 | 2.0355e-03 |
| 0.5 | 2.0000e-09 | 4.7760e-07 | 3.6907e-06 | 1.5851e-05 | 4.9367e-05 | 9.3620e-05 | 2.1572e-04 | 6.8481e-04 | 1.7779e-03 |
| 0.6 | 2.0000e-09 | 4.1000e-07 | 3.1565e-06 | 1.3500e-05 | 4.1884e-05 | 7.9244e-05 | 1.8202e-04 | 5.7530e-04 | 1.4890e-03 |
| 0.7 | 1.4000e-09 | 3.3640e-07 | 2.5693e-06 | 1.0926e-05 | 3.3709e-05 | 6.3557e-05 | 1.4529e-04 | 4.5619e-04 | 1.1749e-03 |
| 0.8 | 8.0000e-10 | 2.5630e-07 | 1.9431e-06 | 8.1823e-06 | 2.5016e-05 | 4.6897e-05 | 1.0635e-04 | 3.3010e-04 | 8.4274e-04 |
| 0.9 | 7.0000e-10 | 1.7400e-07 | 1.2901e-06 | 5.3299e-06 | 1.5994e-05 | 2.9627e-05 | 6.6042e-05 | 1.9981e-04 | 4.9982e-04 |
| 1.0 | 6.0000e-10 | 8.9100e-08 | 6.2510e-07 | 2.4295e-06 | 6.8383e-06 | 1.2118e-05 | 2.5234e-05 | 6.8155e-05 | 1.5371e-04 |

## CONCLUSION

This paper has considered the Variational Iteration Method for solving the Burgers equations. The method is simple and straightforward with no hidden or weak assumptions. The results obtained in this research with VIM on Burgers equations agree with results in Esfandyaripour *et al*. (2013). All computational analysis was performed with the computer application software, Maple 18.

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