

## Comparison of Transportation Algorithms Using Data from Katsina State Transport Authority, Katsina State, Nigeria

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**ABSTRACT:** Transportation problem has been one of the most important applications of Linear programming. Transportation problems have become vastly applied in industrial organizations with multiple manufacturing units, warehouses and distribution centers. In this study, the methods of finding initial basic feasible solution of balanced transportation problem are studied and compared to find among them the best in terms of efficiency. The initial basic feasible solution tableaus of all the methods are constructed using data collected from Katsina State Transport Authority. The costing of the allocated cells associated with the initial basic feasible solutions of the five methods are computed and compared with that of optimal solution which was found to be ₦1,098,000:00. It was observed that Vogel's approximation method, Least-Cost method and Column minimum method yielded better starting solutions. The North-West Corner method and Row minimum methods though simple to compute yielded starting solutions far from the optimal solution. In addition, Vogel's approximation method is more difficult and requires more iteration. The best transportation network for Katsina State Transport Authority was obtained.

**Keywords:** Comparison, Transportation algorithms, Initial Basic Feasible Solution, Optimal solution, Linear programming solver

### INTRODUCTION

Transportation is an important domain of human activity as it supports and makes possible most other social and economic activities and exchanges. It is also a complex domain with several players and levels of decision where investments are capital-intensive and usually require long implementation delays (Crainic, 1998). According to Enyi (2007), "The primary objective of a business entity is to maximize owners' equity". A passenger transport service company maximizes profit from its operations by taking advantage of all situations by applying operational research techniques to aid decision making and to perform better than those that do not apply them. According to Fedra (2004), Transportation problems are among the most pressing strategic development problems in many cities, and can be addressed with a consistent and comprehensive approach and planning methodology that helps to design strategies for sustainable cities.

Some of the initial work on transportation problems was done by Hitchcock (1941) and Koopmans (1947) as quoted by Lee (1973). Imam *et al.* (2009) showed that the results of the five C++ programs are equal to the result of the Linear Programming (LP) solutions. Enyi (2007) affirmed that the number of vehicles available is not always equal to the population present. Kumar, *et al.* (2011) concluded that Vogel Approximation Method (VAM) one short-cut approach

to solve transportation problem with less iterations even though the issue of degeneracy is not avoided.

This research work is important as it studied Nigerian transport sector backed by inability to apply scientific based approach to vehicle capacity assignment and passenger volume projection which deters profit maximization for most transport companies both public and private.

### MATERIAL AND METHOD

Data for this research work was collected from Katsina State Transport Authority and used for comparison of transportation algorithms, with the view to finding the most efficient transportation algorithm(s) and the best transportation network for the benefit of the transport authority. Imam *et al.* (2009) techniques of obtaining initial basic feasible solutions using five different methods were employed. Linear Programming Solver (LiPS v1.11.0) computer software was used to find the optimal solution of the constructed Katsina State Transport Authority problem tableau.

### Transportation Problem

The transportation problem is one of the subclasses of linear programming problem where the objective is to transport various quantities of a single homogeneous product that are initially stored at various origins, to different destinations in such a way

that the total transportation cost is minimum. In the general transportation problem, the objective is to minimize the total transportation (and possibly) costs. Mathematically a transportation problem is nothing but a special linear programming problem in which the objective function is to minimize the cost of transportation subjected to the demand and supply constraints (Imam *et al.*, 2009).

The problem of finding the initial basic feasible solution of the Transportation Problem has long been studied and is well known to researchers in the field of operations research. Finding an optimal solution for transportation problems requires two stages. In the first stage, the initial basic feasible solution through the available methods such as: North West Corner, Least Cost, Row Minima, Column Minima and Vogel's Approximation. The next and last stage is the test for optimality using the Stepping stone or Modified Distribution method.

**Mathematical Formulation of Transportation Problem**

Transportation Problem is one of the original applications of linear programming models. Before solving the problem, there is need to translate the problem in to mathematical form. The transportation problem is an optimization problem with a linear objective function and linear constraints. If we ignore the restriction that the variables take on integer values, then it would fall into our standard framework.

Let us assume in general that there are m - sources  $S_1, S_2, \dots, S_m$  with capacities  $a_1, a_2, \dots, a_m$  and n - destinations (sinks) with requirements  $b_1, b_2, \dots, b_n$  respectively. The transportation cost from  $i^{th}$  - source to the  $j^{th}$  - sink is  $c_{ij}$  and the amount shipped is  $x_{ij}$ . If the total capacity of all sources is equal to the total requirement of all destinations, with  $i = 1, 2, \dots, m$  and  $j = 1, 2, \dots, n$ , Imam *et al.* (2009). To sum up, the following mathematical formulation of the transportation problem is presented below:

$$\text{Minimize: } Z = \sum_{i=1}^m \sum_{j=1}^n c_{ij} x_{ij} \quad (1)$$

$$\text{Subject to: } \sum_{j=1}^n x_{ij} = a_{ij} \quad i = 1, 2, \dots, m \quad (2)$$

$$\sum_{i=1}^m x_{ij} = b_{ij} \quad j = 1, 2, \dots, n \quad (3)$$

$$\text{and } x_{ij} \geq 0 \text{ for all } i \text{ and } j. \quad (4)$$

The above formulation looks like a linear programming problem (LPP). This special LPP will be called a Transportation Problem (TP).

**Solution of the Transportation Problem**

Setting up the transportation tableau is to summarize conveniently and concisely all relevant data and keep track of algorithm computations (Imam *et al.*, (2009). In this context, it serves the same role as the simplex tableau does for linear programming problems. Each cell in the tableau represents the amount transported from one source to one destination. The amount placed in each cell is therefore the value of a decision variable for the cell.

**Finding Initial Feasible Solution to TP**

An initial feasible solution can be found by several alternative methods. A feasible solution to a transportation problem is basic if and only if the corresponding cells in the transportation table do not contain a loop. That is a feasible solution to a transportation problem with m-origins and n-destination is said to be basic if the number of positive allocations are equal to (m+n-1), Imam *et al.* (2009).

Initial basic feasible solution for the balanced Katsina State Transport Authority transportation problem formulated could be obtained using the following methods: Northwest Corner, Minimum cost, Vogel's approximation, Row Minimum, and Column Minimum Methods. The methods differ in the "quality" of the starting basic solution and they produce better starting solution that yields smaller objective value. A brief description on how to use each method is presented below:

**The North-West Corner Rule**

This rule is implemented using the following steps:

- i) The first assignment is made in the cell occupying the upper left-hand (North West) corner of the transportation table. The maximum feasible amount is allocated there, i.e;  $x_{11} = \min(a_1, b_1)$ .
- ii) If  $b_1 > a_1$ , the capacity of origin  $O_1$  is exhausted but the requirement at  $D_1$  is not satisfied. So move down to the second row, and make the second allocation:  
 $x_{21} = \min(a_2, b_1 - x_{11})$  in the cell (2,1).  
 If  $a_1 > b_1$ , allocate  $x_{12} = \min(a_1 - x_{11}, b_2)$  in the cell (1, 2).

Continue this until all the requirements and supplies are satisfied.  $x_{ij}$  is the  $(i, j)^{th}$  cell allocation. "a"

and "b" represent individual columns (demand) and rows (supply) respectively.

**Least-Cost Method**

The minimum-cost method finds a better starting solution by concentrating on the cheapest routes. The method starts by assigning as much as possible to the cell with the smallest unit cost. Next, the satisfied row or column is crossed out and the amounts of supply and demand are adjusted accordingly (Imam *et al.*, 2009). If both a row and a column are satisfied simultaneously, only one is crossed out, the same procedure is applied as in the northwest –corner method. Next, one looks for the uncrossed-out cell with the smallest unit cost and repeats the process until exactly one row or column is left uncrossed out.

**Vogel's Approximation Method (VAM)**

*Vogel's Approximation Method* is an improved version of the minimum-cost method that generally produces better starting solutions (Imam *et al.*, 2009). To apply the VAM, we first compute for each row and column the penalty faced if we should ship over the *second best* route instead of the *least-cost* route. This can be done using the following steps:

- i) For each row of the transportation table, identify the smallest and the next to-smallest costs. Determine the difference between them for each row. Display them alongside the transportation table by enclosing them in parenthesis against the respective rows. Similarly compute the differences for each column.
- ii) Identify the row or column with the largest difference among all the rows and columns. If a tie occurs, use any arbitrary tie breaking choice. Let the greatest difference correspond to *i*<sup>th</sup> row and the minimum cost be  $C_{ij}$ . Allocate a maximum feasible amount:  $x_{ij} = \min(a_i, b_j)$  in the  $(i, j)$ <sup>th</sup> cell, and cross off the *i*<sup>th</sup> row or *j*<sup>th</sup> column.
- iii) Re compute the column and row differences for the reduced transportation table and go to step 2. Repeat the procedure until all the rim requirements are satisfied.

**Row Minimum Method**

Row minimum method start with first row and choose the lowest cost cell of first row so that either the capacity of the first supply is exhausted or the demand at *j*th distribution center is satisfied or both. According to Imam *et al.* (2009) three cases arise:

- i) If the capacity of the first supply is completely exhausted, cross off the first row and proceed to the second row.
- ii) If the demand at *j*th distribution center is satisfied, cross off the *j*th column and reconsider the first row with the remaining capacity.
- iii) If the capacities of the first supply as well as the demand at *j*th distribution center are completely satisfied, make a zero allocation in the second lowest cost cell of the first row. cross off the row as well as the *j*th column and move down to the second row.

Continue the process for the resulting reduced transportation table until all the rim conditions (supply and demand condition) are satisfied.

**Column Minimum Method**

Column minimum method starts with first column and chooses the lowest cost cell of first column so that either the demand of the first distribution center is satisfied or the capacity of the *i*th supply is exhausted or both (Imam *et al.*, 2009). Three cases arise:

- (i) If the demand of the first distribution center is satisfied, cross of the first column and move right to the second column.
- (ii) If the capacity of *i*th supply is satisfied, cross off *i*th row and reconsider the first column with the remaining demand.
- (iii) If the demands of the first distribution center as well as the capacity of the *i*th supply are completely satisfied, make a zero allocation in the second lowest cost cell of the first column. Cross of the column as well as the *i*th row and move right to the second column.

Continue the process for the resulting reduced transportation table until all the rim conditions are satisfied.

**Construction of KTSTA Transportation Problem Tableaus**

Consider a transportation firm Katsina Transportation Authority (KTSTA) that allocates buses to different routes on a normal day and the income generated on each route per day as well as the total passengers transported daily is presented in Table 1.

The problem in Table 1 can be written mathematically as:

$$\text{Maximize: } Z = \sum_{i=1, j=1}^n c_{ij}x_{ij} \tag{5}$$

$$\left. \begin{aligned} Z = & c_{1A}x_{1A} + c_{1B}x_{1B} + c_{1C}x_{1C} + c_{1D}x_{1D} + c_{2A}x_{2A} + c_{2B}x_{2B} + c_{2C}x_{2C} + c_{2D}x_{2D} \\ & + c_{3A}x_{3A} + c_{3B}x_{3B} + c_{3C}x_{3C} + c_{3D}x_{3D} + c_{4A}x_{4A} + c_{4B}x_{4B} + c_{4C}x_{4C} + c_{4D}x_{4D} \end{aligned} \right\} \tag{6}$$

$$\text{Subject to: } \left. \begin{aligned} x_{1A} + x_{1B} + x_{1C} + x_{1D} &= 972 \\ x_{2A} + x_{2B} + x_{2C} + x_{2D} &= 204 \\ x_{3A} + x_{3B} + x_{3C} + x_{3D} &= 236 \\ x_{4A} + x_{4B} + x_{4C} + x_{4D} &= 58 \end{aligned} \right\} \quad (7)$$

$$\left. \begin{aligned} x_{1A} + x_{2A} + x_{3A} + x_{4A} &= 822 \\ x_{1B} + x_{2B} + x_{3B} + x_{4B} &= 174 \\ x_{1C} + x_{2C} + x_{3C} + x_{4C} &= 438 \\ x_{1D} + x_{2D} + x_{3D} + x_{4D} &= 36 \end{aligned} \right\} \quad (8)$$

**Table 1:** Daily Transportation (to-fro) From Katsina to Kano, Kaduna, Abuja and Sokoto.

Route	Vehicle make	Number of vehicles allotted		Total passengers carried	Trip per vehicle	Total income (₦)
		TO	FRO			
Katsina to Kano	T/Hiace	05	05	720	04	324,000.00
	MB 1721	01	01	102	01	30,600.00
Katsina to Kaduna	T/Hiace	02	02	72	01	57,600.00
	MB 1721	01	01	102	01	66,300.00
Katsina to Abuja	T/Hiace	04	04	144	01	201,600.00
	IVM	01	01	58	01	69,600.00
	MB 0500	02	02	236	01	283,200.00
Katsina to Sokoto	T/Hiace	01	01	36	01	50,400.00
<b>Total</b>		<b>17</b>	<b>17</b>	<b>1,470</b>		<b>1,083,300.00</b>

Table 2 represent the actual cost of transportation and total money generated from the four routes plied by the buses to the various destinations and back.

Initial cost from Table 2 is evaluated as follows using equation (6) to obtain **₦ 1,083,300.00**.

**Table 2:** Initial Tableau for KTSTA Transportation Problem:

Route VM	A KANO	B KADUNA	C ABUJA	D SOKOTO	TOTAL DEMAND
<b>T/HIACE</b> (1)	450 720	800 72	1400 144	1400 36	972
<b>MB 1721</b> (2)	300 102	650 102	1200	1200	204
<b>MB 0500</b> (3)	300	650	1200 236	1200	236
<b>IVM</b> (4)	300	650	1200 58	1200	58
<b>TOTAL SUPPLY</b>	822	174	438	36	1,470

Key: VM means Vehicle Make

**Obtaining the Initial Basic Feasible Solutions (IBFS)**

**North-West Corner Method**

The initial basic feasible solution is obtained using North-West Corner methods following the algorithm presented starting from the north-west corner cell until all the demand and supply are met as shown in

Table 3. The objective function value equation (6) is evaluated as ₦ 1,074,300.00.

**Least-Cost Method**

The Least-cost method is applied starting allocation to the cell with minimum cost following the algorithms mentioned earlier and is shown in Table 4. The objective function value is ₦ 1,083,300.00.

**Table 3:** The IBFS using North-West Corner method

Route VM	A KANO	B KADUNA	C ABUJA	D SOKOTO	TOTAL DEMAND
T/HIACE (1)	450 822	800 150	1400	1400	972
MB 1721 (2)	300	650 24	1200 180	1200	204
MB 0500 (3)	300	650	1200 236	1200	236
IVM (4)	300	650	1200 22	1200 36	58
<b>TOTAL SUPPLY</b>	<b>822</b>	<b>174</b>	<b>438</b>	<b>36</b>	<b>1,470</b>

Key: VM means Vehicle Make

**Table 4:** The IBFS using Least-Cost Method

Route VM	A KANO	B KADUNA	C ABUJA	D SOKOTO	TOTAL DEMAND
T/HIACE (1)	450 324	800 174	1400 438	1400 36	972
MB 1721 (2)	300 204	650	1200	1200	204
MB 0500 (3)	300 236	650	1200	1200	236
IVM (4)	300 58	650	1200	1200	58
<b>TOTAL SUPPLY</b>	<b>822</b>	<b>174</b>	<b>438</b>	<b>36</b>	<b>1,470</b>

Key: VM means Vehicle Make

**Vogel's Approximation Method**

The VAM was applied to the illustrative example and after seven iterations the final tableau was obtained following the algorithm mentioned earlier as shown in Table 5. The objective function value associated to this method is obtained as ₦ 1,083,300.00.

**Row Minimum Method**

The algorithm for the Row minimum method is applied using the data obtained from KTSTA as shown in Table 6. The associated objective function value is the same as the one obtained using the North-West Corner method that is ₦ 1,074,300.00.

**Table 5:** The IBFS using VAM

Route	A KANO	B KADUNA	C ABUJA	D SOKOTO	TOTAL DEMAND
<b>T/HIACE</b> (1)	450 324	800 174	1400 438	1400 36	972
<b>MB 1721</b> (2)	300 204	650	1200	1200	204
<b>MB 0500</b> (3)	300 236	650	1200	1200	236
<b>IVM</b> (4)	300 58	650	1200	1200	58
<b>TOTAL SUPPLY</b>	822	174	438	36	1,470

Key: VM means Vehicle Make

**Table 6:** The IBFS using the Row Minimum method

Route VM	A KANO	B KADUNA	C ABUJA	D SOKOTO	TOTAL DEMAND
<b>T/HIACE</b> (1)	450 822	800 150	1400	1400	972
<b>MB 1721</b> (2)	300	650 24	1200 180	1200	204
<b>MB 0500</b> (3)	300	650	1200 236	1200	236
<b>IVM</b> (4)	300	650	1200 22	1200 36	58
<b>TOTAL SUPPLY</b>	822	174	438	36	1,470

Key: VM means Vehicle Make

**Column Minimum Method**

The Column minimum is also applied to obtain the starting solution of the example provided. Table 7 depicts the solution after applying the algorithm and

the value of the objective function is the same with the ones obtained using VAM and Least-cost methods.

**Table 7:** The IBFS using the Column Minimum method

Route VM	A KANO	B KADUNA	C ABUJA	D SOKOTO	TOTAL DEMAND
T/HIACE (1)	324	174	438	36	972
MB 1721 (2)	204				204
MB 0500 (3)	236				236
IVM (4)	58				58
TOTAL SUPPLY	822	174	438	36	1,470

Key: VM means Vehicle Make

**RESULTS AND DISCUSSION**

Using the Linear Program Solver (LiPS v1.11.0) software the optimal solution is obtained as **₦1,098,000.00**. The objective function values obtained using the five methods are presented in Table 8 which also provides the difference between them and that obtained using the LiPS computer software:

**Table 8:** Differences between IBFS and the Optimal Solution

Methods	IBFS obtained (₦)	Difference	% Diff.
North-West Corner	1,074,300	23,700	2.16
Least-Cost	1,098,000	0	0.00
Vogel's Approximation	1,098,000	0	0.00
Row Minimum	1,074,000	23,700	2.16
Column Minimum	1,098,000	0	0.00

Difference = Optimal solution (OS) – Initial Basic Feasible Solution (IBFS)

$$\% \text{ Difference} = \frac{\text{OS} - \text{IBFS}}{\text{OS}} \times 100\%$$

From Table 8, it could be deduced that VAM, Least-Cost and Column Minimum methods yields the best initial basic feasible solution very close to the optimal solution but in the case of VAM the computations take long time. The North-West Corner and Row

Minimum methods are quick methods because solutions take short time but yield large error, it is very far from optimal solution.

Least-Cost and Vogel's approximation methods tackle the problem of finding a good initial solution by taking into account the costs associated with each route alternative. This is something that the North-west corner method could not accomplish.

**CONCLUSION**

An advantage of transportation problem algorithms is that the solution process involves only main variables, artificial variables are not required as in the simplex method. Large transportation problems could relatively be solved using the transportation problem algorithms. It was shown that solving balanced transportation problems was easier using the North-west Corner, Least-cost, Row/Column minimum methods using the transportation problem formulated for Katsina State Transport Authority. VAM was found to be very difficult yet it was more nearer to an optimal solution. It was therefore concluded that VAM was not quite as simple as the other approaches, but it facilitated a better initial solution. The optimal solution for the constructed Katsina State Transport Authority problem tableau using the Linear Program Solver (LiPS v1.11.0) software was found to be **₦1,098,000.00**.

**RECOMMENDATION**

In nature, the passenger population (demand) is always greater than the vehicle supply, but this can only be verified if the actual data management and record is available for researches to be carried out on

them. Further research needs to be carried out in the area of bus services and maintenance looking at the number of times the buses could break down.

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