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# Modified Robust Regression-Type Estimators with Multi-Auxiliary Variables Using Non-Conventional Measures of Dispersion

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## ABSTRACT

Auxiliary variables correlated with the study variables have been identified to be useful in improving the efficiency of ratio, product and regression estimators both at planning and estimation stages. The existing regression-based estimators are functions of auxiliary variables which are sensitive to outliers. In this paper, a modified class of estimators is proposed using robust non-conventional measures of dispersion which are robust against outliers or extreme values. The properties (Biases and Mean Squared Errors (MSEs)) of the modified class of estimator were derived up to the first order of approximation using Taylor series approach. The empirical studies were conducted using stimulation to investigate the efficiency of the proposed estimators over the efficiency of the existing estimators. The results revealed that the proposed estimators have minimum MSEs and higher Percentage Relative Efficiencies (PREs) among all the competing estimators. These results implied that the proposed estimators are more efficient and can produce better estimate of the population mean compared to other existing estimators considered in the study. Therefore, it can be concluded that proposed estimators have better predictive power for estimating population mean when the study (interest) variables are characterized with outliers or extreme values.

**Keywords:** Robust estimators, Non-conventional measures, Efficiency, Outliers

**Symbols and Notations**

*N- Population Size*
*n- Sample Size*
*f = n/N- Sampling fraction*
*Y- Study Variable (Primary variable)*
*X- Auxiliary Variable (Secondary variable)*

$R = \frac{\bar{Y}}{\bar{X}}$

$\bar{Y} = \frac{1}{N}\sum_{i=1}^{N} Y_i$ *- Population Mean of Study Variable Y*

$\bar{X} = \frac{1}{N}\sum_{i=1}^{N} X_i$ *- Population Mean of Auxiliary Variable X*

$S_y = \sqrt{\sum_{i=1}^{N}\left(y_i - \bar{Y}\right)^2 / \left(N-1\right)}$ *- Population Standard Deviation of Study Variable Y*

$S_x = \sqrt{\sum_{i=1}^{N}\left(x_i - \bar{X}\right)^2 / \left(N-1\right)}$ *- Population Standard Deviation of Auxiliary Variable X*

$\rho_{yx} = S_{yx} / \left(S_y S_x\right)$ *- Population Correlation Coefficient between Study and Auxiliary Variables*

$S_{yx} = \sum_{i=1}^{N}\left(y_i - \bar{Y}\right)\left(x_i - \bar{X}\right) / \left(N-1\right)$ *- Population Covariance between Study and Auxiliary Variables*

$C_y = S_y / \bar{Y}$ *- Population Coefficient of Variation of Study Variable Y*

$C_x = S_x / \bar{X}$ *- Population Coefficient of Variation of Auxiliary Variable X*

$u_r = \frac{1}{N-1}\sum_{i=1}^{N}\left(X_i - \bar{X}\right)^r$

$\beta_{1(x)} = \frac{u_3}{u_2^{3/2}}$ *- Population Coefficient of Skewness of Auxiliary Variable X*

$\beta_{2(x)} = \frac{u_4}{u_2^2}$ *- Population Coefficient of Kurtosis of Auxiliary Variable X*

$HL = Median[(X_j + X_k)/2, 1 \le j \le k \le N]$ - *Hodges-Lehmann Estimator*

$MR = \frac{X_{(1)} - X_{(N)}}{2}$ - *Population mid-range of Auxiliary Variable*

$G = \frac{4}{N-1}\sum_{i=1}^{N}\left(\frac{2i-N-1}{2N}\right)X_{(i)}$ -*Gini's Mean Difference for Auxiliary Variable*

$D = \frac{2\sqrt{\pi}}{N-1}\sum_{i=1}^{N}\left(i - \frac{N+1}{2N}\right)X_{(i)}$ - *Downton's Method for Auxiliary Variable*

$QD = \frac{Q_3 - Q_1}{2}$ - *Population Quartile Deviation of Auxiliary Variable*

$DM = \frac{D_1 + D_2 + ... + D_9}{9}$ - *Decile Mean for Auxiliary Variable*

$TM =$ - *Trim Mean for Auxiliary Variable*

$S_{pw} = \frac{\sqrt{\pi}}{N^2}\sum_{i=1}^{N}(2i - N - 1)X_{(i)}$ - = *Probability Weighted Moments for Auxiliary Variable*

$$R_h = \frac{\bar{Y}}{\bar{X}_h}, \quad h = 1, 2, ...r$$

$$S_{x_h}^2 = \frac{1}{N_h - 1}\sum_{i=1}^{N_h}\left(X_h - \bar{X}_h\right)^2, \qquad h = 1, 2, ...r$$

$$S_{yx_h} = \frac{1}{N_h - 1}\sum_{i=1}^{N_h}\left(X_{hi} - \bar{X}_h\right)\left(Y_{hi} - \bar{Y}_h\right), \qquad h = 1, 2, ...r$$

$$\varphi_h = \frac{A_h \bar{X}_h}{A_h \bar{X}_h + B_h}, \qquad h = 1, 2, ...r$$

$$\bar{X}_h = \frac{1}{N_h}\sum_{i=1}^{N_h} X_{hi}, \qquad h = 1, 2, ...r$$

$A_j$ & $B_j$ *are any of coefficients of variation, skewness, kurtosis and standard deviation of auxiliary variable* $X_j$.

**INTRODUCTION**

Many modifications of the ratio and product estimators have been done to improve their efficiency by using a number of known parameters of the auxiliary variable such as the coefficient of variation $C_x$, the coefficient of kurtosis $\beta_2(x)$, standard deviation $\delta_x$, the coefficient of skewness $\beta_1(x)$, the correlation coefficient between the study variable and an auxiliary variable $\rho_{yx}$. Sisodia and Dwivedi (1981) have suggested a modified ratio estimator using the coefficient of variation $C_x$ of an auxiliary variable X for estimating the population mean $\bar{Y}$. Upadhyaya and Singh (1999) suggested another modified ratio estimator using linear combinations of the coefficient of variation $C_x$ and coefficient of the kurtosis $\beta_2(x)$. Singh and Tailor (2003) proposed another estimator using the correlation coefficient $\rho_{yx}$ between X and Y. By using the population variance $S_x^2$ of an auxiliary variable X, Singh (2003) proposed another modified ratio estimator which uses a linear combination of the coefficient of kurtosis $\beta_2(x)$ and standard deviation $\delta_x$, and the coefficient of skewness $\beta_1(x)$ for estimating the population mean of the study variable $\bar{Y}$. Motivated by Singh (2003), Yan and Tian (2010) used a linear combination of the coefficient of kurtosis $\beta_2(x)$, coefficient of skewness $\beta_1(x)$, coefficient of variation $C_x$ of the auxiliary variable X. More recently, Subramani and Kumarapandiyan (2013) suggested a new modified ratio estimator using known population median $M_d$ of an auxiliary variable. Subramani and Kumarapandiyan (2012, 2013) have also suggested modified ratio estimators using the known median and the coefficient of kurtosis, median and coefficient of skewness, median and the coefficient of variation, and median and the coefficient of correlation. Other authors that had worked in this direction include Hartley and Ross (1954), Quenouille (1956), Singh (1965, 1967), Naik and Gupta (1991), Kadilar and Cingi (2003), Singh and Espejo (2003), Shabbir and Yaab (2003), Abu-Dayeh *et al*. (2003), Kadilar and Cingi (2005), Jhajj et al.

(2006), Khoshnevisan *et al*. (2007), Perri (2007), Singh *et al*. (2007), Gupta and Shabbir (2008), Sharma and Tailor (2010), Subramani and Kumarapandiyan (2012), Subramani and Kumarapandiyan (2013), Subramani and Kumarapandiyan (2014), Singh and Kumar (2011), Tailor *et al*. (2012), Lu (2013), Sharma and Singh (2014), Lu and Yan (2014), Verma *et al*. (2015), Ahmed and Singh (2015), Audu and Adewara (2017a,b), Audu and Muili (2019), Muili *et al*. (2020), Singh *et al*. (2020), Audu *et al*. (2020a,b,c,d), Audu *et al*. (2021a,b,c,d,e,f), Ahmed *et al.* (2016a,b,), Yunusa *et al*. (2021), Zaman *et al.* (2021), Audu and Singh (2021).

Regression estimator is often use to estimate the population characteristics such as population mean, total and variance when the regression line of y on **x** does not pass through the origin but makes an intercept along the y-axis. Many modifications of the regression type estimators have been done to improve their efficiency by using a number of known parameters of the auxiliary variable such as the coefficient of variation $C_x$, the coefficient of kurtosis $\beta_2(x)$, standard deviation $\delta_x$, the coefficient of skewness $\beta_1(x)$, and the correlation coefficient between the study variable and an auxiliary variable $\rho_{yx}$. Shabbir and Gupta (2010) proposed a regression ratio-type exponential estimator by combining Rao's and Bedi's estimators (Rao, 1991; Bedi, 1996). Following these works, Grover and Kaur (2011) introduced a regression exponential type estimator. Ozgul and Cingi (2014) proposed a new class of exponential regression cum ratio estimator using functions of any known population parameters of the auxiliary variable, such as standard deviation, coefficient of variation, coefficient of skewness, coefficient of kurtosis and coefficient of correlation of the auxiliary variable for the estimation of finite population mean. Several authors like Kadilar and Cingi (2004), Kadilar and Cingi (2006), Subramani and Kumarapandiyan (2016), Abid *et al.* (2016), Subzar *et al.* (2017), Subzar *et al*. (2018 a, b, c) have proposed some regression-based estimators which utilized known functions of auxiliary variables. However, these auxiliary parameters are sensitive to outliers or extreme values that do present in the population distributions. Outliers are observations that are distant from other observations in the population. They tend to inflate average deviation of the entire observations from central values. When there are outliers in data, the auxiliary functions like Kurtosis, Skewness, Coefficient of variation, standard deviation etc, will be affected and consequently the efficiency of the estimators which utilize these functions will drastically reduce. Some of the regression estimators in the above paragraph are function of these auxiliary functions. Similarly, regression slope in the regression estimators is also sensitive to outliers and its effect will decreases the efficiency of the estimators. Zaman and Bulut (2018), Zaman (2019) suggested robust regression slopes like Hampel M, Huber M, LTS and LAD methods proposed by Hampel (1971), Huber (1973), Fox (2002) and Nadia and Muhammad (2013), respectively, which are robust against outliers as an alternative to regression slope in the regression estimators of the previous authors. However, the problem of effects of outliers on auxiliary functions in the previous studies was not addressed. Similarly, Yadav and Zaman (2021) suggested non-conventional robust parameters of auxiliary variable which are robust against outliers. However, the problems of effects of outliers on regression slopes were not considered. This current study focused on the modification of robust regression estimators using robust non-conventional measures (Gini's mean, Downton's method, and probability weighted moment proposed by Gini, 1936, Downton, 1966 and Greenwood *et al*., 1979, respectively) and robust regression slopes simultaneously to address the effect of outliers on auxiliary functions and regression slopes respectively.

Several authors have proposed different regression-type estimators using different auxiliary information. The notable ones include Kadilar and Cingi (2004), Kadilar and Cingi (2006), Kadilar and Cingi (2007), Subramani and Kumarapandiyan (2016), Abid *et al*. (2016), Subzar *et al.* (2017), Subzar*et al*. (2018a), Subzar *et al*. (2018b), Subzar *et al*. (2018c), Zaman and Bulut (2018), Zaman (2019), Yadav and Zaman (2021). However, none of the estimators considered situations when the study variablesare associated with independentmulti-auxiliary variables like expenditure with salary and teacher-pupils ratio, GDP with inflation rate, export rate and import rate, obesity with body weight, height and blood pressure etc. Recently, Audu *et al.* (2020a) proposed regression-type class of estimators using multi-auxiliary variables. However, the estimator utilizes conventional measures of dispersion (Coefficients of Variation, Skewness and Kurtosis) which are sensitive to outliers or extreme values. Audu *et al.* (2020a) adopted transformation techniques to the work of Zaman (2019) and then proposed a general form of estimators as:

$$t_p = v\frac{\left[\bar{y}+\sum_{h=1}^{r}\alpha_{rbst(zb)h}(\bar{X}_h-\bar{x}_h)\right]}{\prod_{h=1}^{r}\bar{x}_h}\prod_{h=1}^{r}\bar{X}_h+(1-v)\frac{\left[\bar{y}+\sum_{h=1}^{r}\alpha_{rbst(zb)h}(\bar{X}_h-\bar{x}_h)\right]}{\prod_{h=1}^{r}(A_h\bar{x}_h+B_h)}\prod_{h=1}^{r}(A_h\bar{X}_h+B_h) \quad (1)$$

where $A_j$ and $B_j$ are either population coefficients of variation or kurtosis of $j^{th}$ independent auxiliary variables $X_j$, $j$ =1, 2,..,r, $A_h \neq B_h$

$$MSE(t_p)_{\min} = \frac{1-f}{n}\left( S_y^2 + \sum_{h=1}^{r}\left(\alpha_{rbst(zb)h} + R_h\varphi_h\right)\left(\left(\alpha_{rbst(zb)h} + R_h\varphi_h\right)S_{x_h}^2 - 2S_{yx_h}\right) - \frac{D_{yx}^2}{D_x}\right) \quad (2)$$

Where $D_{yx} = \sum_{h=1}^{r} R_h\left(1-\varphi_h\right)\left(S_{x_h}^2\left(\alpha_{rbst(zb)h} + R_h\varphi_h\right) - S_{yx_h}\right)$, $D_x = \sum_{h}^{r} S_{x_h}^2 R_h^2\left(1-\varphi_h\right)$

This study aimed at proposing some modified ratio-type estimators which are robust against outliers or extreme values and more efficient than related existing estimators using robust non-conventional measures.

## MATERIALS AND METHODS

### Proposed Estimators

Having studied the class of estimator by Audu *et al.* (2020a), using non-conventional measures of dispersion (Gini's Mean, Downton's Methods and Probability Weighted Moment) which are Robust against extreme values or outliers, the class of suggested estimator is as in (3)

$$t_{ik} = \theta\frac{[\bar{y} + \sum_{h=1}^{r}\alpha_{rbst(zb)h}(\bar{X}_h - \bar{x}_h)]}{\prod_{h=1}^{r}\bar{x}_h}\prod_{h=1}^{r}\bar{X}_h + (1-\theta)\frac{[\bar{y} + \sum_{h=1}^{r}\alpha_{rbst(zb)h}(\bar{X}_h - \bar{x}_h)]}{\prod_{h=1}^{r}(\eta_{ih}\bar{x}_h + \eta_{jh})}\prod_{h=1}^{r}(\eta_{ih}\bar{X}_h + \eta_{jh}) \quad (3)$$

$$\eta_i,\ \eta_j,\ ij = 1,2,3,\ \eta_i \neq \eta_j \in \left\{G\times n,\ D\times n,\ S_{pw}\times n\right\}\ k = 1,2,...,6$$

**Properties (Bias and MSE) of the Proposed Estimator $t_{ik}$**

To obtain the bias and mean squared error (MSE) of $t_{3k}$, the error terms $e_0 = \frac{\bar{y}-\bar{Y}}{\bar{Y}}$ and $e_h = \frac{\bar{x}-\bar{X}_h}{\bar{X}_h}$ are defined such that the expectations are given as:

$$\left.\begin{array}{l} E(e_o) = E(e_h) = 0, E(e_o^2) = \psi_{n,N}C_y^2, E(e_h^2) = \psi_{n,N}C_{x_h}^2, E(e_o e_h) = \psi_{n,N}\rho_{yx_h}C_yC_{x_h} \\ E(e_h e_k) = 0\ \ \forall\ \ h \neq k = 1,2,...,r, E(e_h^u e_0^m) = 0\ \forall\ \ u+m > 2,\ \psi_{n,N} = 1/n - 1/N \end{array}\right\} \quad (4)$$

**Theorem 1:** The bias of the suggested estimators $t_{3k}\left(t_{3k},\ k = 1,2,3,4,5,6\right)$ to first order of approximation is:

$$Bias\left(t_{3k}\right) = \frac{1-f}{n}\left(\sum_{h=1}^{r}C_{xh}^2\left(\bar{Y}\left(v + (1-v)\varphi_{ijh}^2\right) + \alpha_{rbst(zb)h}\bar{X}_h\varphi_{ijh}\right) - \bar{Y}\sum_{h=1}^{r}\rho_{yxh}C_yC_{xh}\left(v + (1-v)\varphi_{ijh}\right)\right) \quad (5)$$

**Proof:**

Express $t_{3k}$ in terms of $e_o$ and $e_h$, we have defined in section (3)

$$t_{3k} = v^*\frac{\left(\bar{Y}(1+e_o) - \sum_{h=1}^{r}\alpha_{rbst(zb)h}\bar{X}_h e_h\right)}{\prod_{h=1}^{r}(1+e_h)\bar{X}_h}\left(\prod_{h=1}^{r}\bar{X}_h\right) + \frac{(1-v^*)\left(\bar{Y}(1+e_o) - \sum_{h=1}^{r}\alpha_{rbst(zb)h}\bar{X}_h e_h\right)}{\prod_{h=1}^{r}\eta_{ih}(1+e_h)\bar{X}_h + \eta_{jh})}\prod_{h=1}^{r}(\eta_{ih}\bar{X}_h + \eta_{ih}) \quad (6)$$

$$\begin{aligned} t_{3k} &= v^*\left(\bar{Y}(1+e_o) - \sum_{h=1}^{r}\alpha_{rbst(zb)h}\bar{X}_h e_h\right)\prod_{h=1}^{r}(1+e_h)^{-1} \\ &\quad + (1-v^*)\left(\bar{Y}(1+e_o) - \sum_{h=1}^{r}\alpha_{rbst(zb)h}\bar{X}_h e_h\right)\prod_{h=1}^{r}(1+\varphi_{ijh}e_h)^{-1} \end{aligned} \quad (7)$$

Where $\varphi_{ijh} = \frac{\eta_{ih}\bar{X}_h}{\eta_{ih}\bar{X}_h + \eta_{jh}}$

Simplify (7) up to first order approximation, we have

$$\begin{aligned} t_{3k} &= v^*\left(\bar{Y} - \bar{Y}\sum_{h=1}^{r}e_j + \bar{Y}\sum_{h=1}^{r}e_h^2 + \bar{Y}\sum_{h=1}^{r}e_he_k + \bar{Y}e_o - \bar{Y}\sum_{h=1}^{r}e_oe_h - \sum_{h=1}^{r}\alpha_{rbst(zb)h}\bar{X}_he_h + \sum_{h=1}^{r}\alpha_{rbst(zb)h}\bar{X}_he_h^2\right) \\ &\quad + (1-v^*)\left(\bar{Y} - \bar{Y}\sum_{h=1}^{r}\varphi_{ijh}e_h + \bar{Y}\sum_{h\neq k=1}^{r}\varphi_{ijh}\varphi_k e_he_k + \bar{Y}e_o - \bar{Y}\sum_{h=1}^{r}\varphi_{ijh}e_oe_h - \sum_{h=1}^{r}\alpha_{rbst(zb)h}\bar{X}_he_h + \sum_{h=1}^{r}\alpha_{rbst(zb)h}\varphi_{ijh}\bar{X}_he_h^2\right) \end{aligned} \quad (8)$$

$$t_{3k}-\bar{Y}=\bar{Y}e_o-\sum_{h=1}^{r}\left(\bar{Y}\left(v^*+(1-v^*)\varphi_{ijh}\right)+\alpha_{rbst(zb)h}\bar{X}_h\right)e_h-\bar{Y}\sum_{h=1}^{r}\left(v^*+(1-v^*)\varphi_{ijh}\right)e_oe_h$$
$$+\sum_{h=1}^{r}\left(\bar{Y}\left(v^*+(1-v^*)\varphi_{ijh}^2\right)+\alpha_{rbst(zb)h}\bar{X}_h\varphi_{ijh}\right)e_h^2+terms\ with\ cross\ product\ of\ X_h^s \quad (9)$$

Take expectation of (9) and apply the results of (4), $Bias\left(t_{3k}\right)$ is obtained as in (5). Hence the proof

**Theorem 2:** The MSE of the suggested estimators $t_{3k}$ $\left(k=1,2,3,4,5,6\right)$ to first order of approximation is:

$$MSE\left(t_{3k}\right)=\frac{1-f}{n}\left(S_y^2+\sum_{h=1}^{r}S_{xh}^2\left(R_h^2\left(v^*+(1-v^*)\varphi_{ijh}\right)+\alpha_{rbst(zb)h}\right)^2-2\sum_{h=1}^{r}S_{yxh}\left(R_h\left(v^*+(1-v^*)\varphi_{ijh}\right)+\alpha_{rbst(zb)h}\right)\right) \quad (10)$$

where $R_h=\dfrac{\bar{Y}}{\bar{X}_h}$

**Proof:**

Square both sides of (9),

$$\left(t_{3k}-\bar{Y}\right)^2=\bar{Y}^2e_0^2+\sum_{h=1}^{r}\left(\bar{Y}\left(v+(1-v)\varphi_{ijh}\right)+\alpha_{rbst(zb)h}\bar{X}_h\right)^2e_h^2-2\bar{Y}\sum_{h=1}^{r}\left(\bar{Y}\left(v+(1-v)\varphi_{ijh}\right)+\alpha_{rbst(zb)h}\bar{X}_h\right)e_0e_h \quad (11)$$

Take expectation of (11) and apply the results of (4), $MSE\left(t_{3k}\right)$ is obtained as in (10). Hence the proof.

**Theorem 3:** The minimum MSE of $t_{3k}$ is

$$MSE(t_{3k})_{\min}=\frac{1-f}{n}\left(S_y^2+\sum_{h=1}^{r}\left(\alpha_{rbst(zb)h}+R_h\varphi_h\right)\left(\left(\alpha_{rbst(zb)h}+R_h\varphi_h\right)S_{x_h}^2-2S_{yx_h}\right)-\frac{E_{yx}^2}{E_x}\right) \quad (12)$$

Where $E_{yx}=\sum_{h=1}^{r}R_h(1-\varphi_{ijh})\left(S_{xh}^2(\alpha_{rbst(zb)j}R_h\varphi_{ijh})-S_{yx_h}\right)$, $\quad E_x=\sum_{h=1}^{r}S_{xh}^2R_h^2\left(1-\varphi_{ijh}\right)^2$

$i=1,2,3\ \ j=1,2,3\ \ i\neq j$

**Proof:**

Differentiate partially (10) with respect to v, equate to zero and solve for v. that is,

$$\frac{\partial\left(MSE\left(t_{3k}\right)\right)}{\partial v}=0 \quad (13)$$

$$v^*=-\frac{\sum_{h=1}^{r}R_h(1-\varphi_{ijh})\left(S_{xh}^2(\alpha_{rbst(zb)j}R_h\varphi_{ijh})-S_{yx_h}\right)}{\sum_{h=1}^{r}S_{xh}^2R_h^2\left(1-\varphi_{ijh}\right)^2} \quad (14)$$

Substitute (14) in (10), minimum $MSE(t_{3k})$ is obtained as in (12). Hence the proof.

**Data for Empirical Studies**

To investigate the robustness of the proposed estimators against outliers or extreme values, the data for the study variable were generated using cubic equation $Y = 5X_1 + 20X_2^2 + 10X_3^3 + e$ and the auxiliary variables were generated from non-normal discrete and continuous (poison and exponential) distributions as presented in Table 1.The simulation studies were conducted to assess the performance of the proposed estimators $t_{3k}$ with respect to *some* related existing estimators by mean squared error (MSE) and percentage relative efficiency (PRE) using data obtained from Table 1 and the results were presented in Tables 2-9.

**Table 1:** Populations used for empirical study on sample mean, $t_p$ and proposed estimators $t_{3k}$

| POPULATIONS | AUXILIARY VARIABLE (X) | METHODS FOR ROBUST REGRESSION SLOPE $\left(\alpha_{rbst(zb)}\right)$ | STUDY VARIABLE (Y) |
|---|---|---|---|
| I | $X_1 \sim pois(0.1)$<br>$X_2 \sim pois(0.2)$<br>$X_3 \sim pois(0.3)$ | Huber MM<br>Hampel M<br>Least Trimmed Squares (LTS)<br>Least Absolute Deviation (LAD) | $Y = 5X_1 + 20X_2^2 + 10X_3^3 + e,$<br>$where,\ e \sim (0,4)$ |
| II | $X_1 \sim \exp(0.1)$<br>$X_2 \sim \exp(0.2)$<br>$X_2 \sim \exp(0.3)$ | Huber MM<br>Hampel M<br>Least Trimmed Squares (LTS)<br>Least Absolute Deviation (LAD) | |

**RESULTS**

In this section, numerical results on the efficiency of the proposed estimators over sample mean and Audu *et al*. (2021a) estimators were presented. Table 2 shows the numerical results of MSEs and PREs of the proposed and some existing estimators using data generated from population I under Huber MM methods. The result revealed that the proposed estimators $t_{3k}, k = 1,2,3,4,5,6$ have minimum MSEs and higher PREs compared to sample mean and estimators $t_p, i = 1,2,...,16$. This implies that the proposed estimators are more efficient and have higher precision than other estimators in the study.

Table 3 shows the numerical results of MSEs and PREs of the proposed and some existing estimators using data generated from population I under Hampel M methods. The result revealed that the proposed estimators $t_{3k}, k = 1,2,3,4,5,6$ have minimum MSEs and higher PREs compared to sample mean and estimators $t_p, i = 1,2,...,16$. This implies that the proposed estimators are more efficient and have higher precision than other estimators in the study.

Table 4 shows the numerical results of MSEs and PREs of the proposed and some existing estimators using data generated from population I under LTS methods. The result revealed that the proposed estimators $t_{3k}, k = 1,2,3,4,5,6$ have minimum MSEs and higher PREs compared to sample mean and estimators $t_p, i = 1,2,...,16$. This implies that the proposed estimators are more efficient and have higher precision than other estimators in the study.

Table 5 shows the numerical results of MSEs and PREs of the proposed and some existing estimators using data generated from population I under LAD methods. The result revealed that the proposed estimators $t_{3k}, k = 1,2,3,4,5,6$ have minimum MSEs and higher PREs compared to sample mean and estimators $t_p, i = 1,2,...,16$. This implies that the proposed estimators are more efficient and have higher precision than other estimators in the study.

**Table 2:** MSEs and PREs of sample Mean, $t_p$ and proposed estimators using population I under Huber MM method

| ESTIMATORS | AUXILIARY PARAMETERS | MSEs | PREs |
|---|---|---|---|
| *Sample Mean | Not Applicable | 5622.305 | 100 |
| $t_1$ | $A_j = 1,\ B_j = S_{x_j}$ | 6803.234 | 82.64165 |
| $t_2$ | $A_j = 1,\ B_j = C_{x_j}$ | 205.9036 | 2730.553 |
| $t_3$ | $A_j = 1,\ B_j = \beta_1(x_j)$ | 196.4039 | 2862.624 |
| $t_4$ | $A_j = 1,\ B_j = \beta_2(x_j)$ | 364.4835 | 1542.54 |
| $t_5$ | $A_j = S_{x_j},\ B_j = C_{x_j}$ | 36.36093 | 15462.49 |
| $t_6$ | $A_j = S_{x_j},\ B_j = \beta_1(x_j)$ | 330.7688 | 1699.769 |
| $t_7$ | $A_j = S_{x_j},\ B_j = \beta_2(x_j)$ | 36.43422 | 15431.39 |
| $t_8$ | $A_j = C_{x_j},\ B_j = S_{x_j}$ | 6582.704 | 85.41027 |
| $t_9$ | $A_j = C_{x_j},\ B_j = \beta_1(x_j)$ | 27.87464 | 20169.96 |
| $t_{10}$ | $A_j = C_{x_j},\ B_j = \beta_2(x_j)$ | 262.6451 | 2140.647 |
| $t_{11}$ | $A_j = \beta_1(x_j),\ B_j = S_{x_j}$ | 5651.147 | 99.48962 |
| $t_{12}$ | $A_j = \beta_1(x_j),\ B_j = C_{x_j}$ | 44.85158 | 12535.35 |
| $t_{13}$ | $A_j = \beta_1(x_j),\ B_j = \beta_2(x_j)$ | 271.0474 | 2074.288 |
| $t_{14}$ | $A_j = \beta_2(x_j),\ B_j = S_{x_j}$ | 89010.03 | 6.316485 |
| $t_{15}$ | $A_j = \beta_2(x_j),\ B_j = C_{x_j}$ | 661.6049 | 849.798 |
| $t_{16}$ | $A_j = \beta_2(x_j),\ B_j = \beta_1(x_j)$ | 804.9148 | 698.4969 |
| **Proposed Estimator** $t_{3k}$ | | | |
| $t_{31}$ | $\eta_{ih} = G_h \times n,\ \eta_{jh} = D_h \times n$ | 25.12698 | 22375.57 |
| $t_{32}$ | $\eta_{ih} = G_h \times n,\ \eta_{jh} = S_{pw_h} \times n$ | 25.31404 | 22210.23 |
| $t_{33}$ | $\eta_{ih} = D_h \times n,\ \eta_{jh} = G_h \times n$ | 25.64438 | 21924.12 |
| $t_{34}$ | $\eta_{ih} = D_h \times n,\ \eta_{jh} = S_{pw_h} \times n$ | 16.08859 | 34945.91 |
| $t_{35}$ | $\eta_{ih} = S_{pw_h} \times n,\ \eta_{jh} = G_h \times n$ | 25.76267 | 21823.46 |
| $t_{36}$ | $\eta_{ih} = S_{pw_h} \times n,\ \eta_{jh} = D_h \times n$ | 25.48027 | 22065.32 |

* Adapted from Audu *et al.* (2020a); MSE =Mean squared error; PRE= Percentage relative efficiency; tp = Audu *et al.* (2020a) estimators

Table 6 shows the numerical results of MSEs and PREs of the proposed and some existing estimators using data generated from population II under Huber MM methods. The result revealed that the proposed estimators $t_{3k}, k = 1,2,3,5,6$ with exception of $t_{34}$ have minimum MSEs and higher PREs compared to sample mean and estimators $t_p, i = 1,2,...,16$. This implies that the proposed estimators are more efficient and have higher precision than other estimators in the study.

Table 7 shows the numerical results of MSEs and PREs of the proposed and some existing estimators using data generated from population II under Hampel M methods. The result revealed that the proposed estimators $t_{3k}, k = 1,2,3,5,6$ with exception of $t_{34}$ have minimum MSEs and higher PREs compared to sample mean and estimators $t_p, i = 1,2,...,16$. This implies that the proposed estimators are more efficient and have higher precision than other estimators in the study.
Table 8 shows the numerical results of MSEs and PREs of the proposed and some existing estimators using data generated from population II under LTS methods. The result revealed that the proposed estimators $t_{3k}, k = 1,2,3,4,5,6$ have minimum MSEs and higher PREs compared to sample mean and estimators $t_p, i = 1,2,...,16$. This implies that the proposed estimators are more efficient and have higher precision than other estimators in the study.

Table 9 shows the numerical results of MSEs and PREs of the sample mean, $t_p$ andproposed estimators using data generated from population II under LAD methods. The result revealed that the proposed estimators $t_{3k}, k = 1,2,3,5,6$ with exception of $t_{34}$ have minimum MSEs and higher PREs compared to sample mean and estimators $t_p, i = 1,2,...,16$. This implies that the proposed estimators are more efficient and have higher precision than other estimators in the study.

## DISCUSSION

Sample observations are often prone to be characterized by outliers or extreme values due to randomness in their selection. These outliers or extreme values affect the estimate of the estimators by either over-estimation or under-estimation and consequently the centrality characteristics of the corresponding population parameters are is violated. This is the case with Audu *et al.* (2021a) classes of estimators. This problem was addressed by using robust non-conventional statistics (Gini's Mean, Downton's Methods and Probability Weighted Moment) as alternative to conventional ones and robust regression slopes (Hampel M, Huber M, LTS and LAD) as alternative to conventional regression slope in Audu *et al*. (2021a) estimators. The properties (Biases and MSEs) of the proposed estimators were derived using first order approximation procedure and numerical results of the properties were obtained for efficiency comparison ton sample mean and Audu *et al*. (2021a) estimators $t_p, i = 1,2,...,16$ using data generated as presented in Tables 2-9. The assessments of the efficiency (proximity of the estimators to the population) ware done using MSEs of the estimators andtheir efficiency gains were assessed using PREs.

From all the numerical results obtained in Tables 2-9with exception of few cases under normal and non-normal distributions, the proposed estimators have smallest MSEs and largest PREs compared to sample mean and Audu *et al*. (2021a) estimators $t_p, i = 1,2,...,16$. These results mean higher gaining in efficiency of the new proposed estimators in both situations when sample data is characterized by outliers or extreme values and when not.

## CONCLUSION

From the foregone results, it was revealed that the suggested estimators have minimum MSE compared to other competing estimators considered in literature. Hence, the suggested estimators demonstrated high level of efficiency over the other estimators considered in the study. These results shows that the proposed estimators are highly robust in the estimation of population means when the sample observations obtained from the study variables are characterized by extreme values or outliers due improper use of sampling techniques, data collection instruments or inexperienced interviewers. The suggested estimators are recommended for use in the estimation of population means of any variable of interest especially when the study and auxiliary variables are highly associated or correlated.

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**Table 3:** MSEs and PREs of sample mean, $t_p$ and proposed estimators using population I under Hampel M method

| ESTIMATORS | AUXILIARY PARAMETERS | MSEs | PREs |
|---|---|---|---|
| *Sample Mean | Not Applicable | 5622.305 | 100 |
| $t_1$ | $A_j = 1,\ B_j = S_{x_j}$ | 6780.805 | 82.91501 |
| $t_2$ | $A_j = 1,\ B_j = C_{x_j}$ | 217.9533 | 2579.592 |
| $t_3$ | $A_j = 1,\ B_j = \beta_1(x_j)$ | 208.2082 | 2700.329 |
| $t_4$ | $A_j = 1,\ B_j = \beta_2(x_j)$ | 380.8541 | 1476.236 |
| $t_5$ | $A_j = S_{x_j},\ B_j = C_{x_j}$ | 41.18567 | 13651.12 |
| $t_6$ | $A_j = S_{x_j},\ B_j = \beta_1(x_j)$ | 346.075 | 1624.591 |
| $t_7$ | $A_j = S_{x_j},\ B_j = \beta_2(x_j)$ | 41.27859 | 13620.39 |
| $t_8$ | $A_j = C_{x_j},\ B_j = S_{x_j}$ | 6442.649 | 87.26697 |
| $t_9$ | $A_j = C_{x_j},\ B_j = \beta_1(x_j)$ | 31.88466 | 17633.26 |
| $t_{10}$ | $A_j = C_{x_j},\ B_j = \beta_2(x_j)$ | 276.681 | 2032.053 |
| $t_{11}$ | $A_j = \beta_1(x_j),\ B_j = S_{x_j}$ | 5527.406 | 101.7169 |
| $t_{12}$ | $A_j = \beta_1(x_j),\ B_j = C_{x_j}$ | 50.22644 | 11193.91 |
| $t_{13}$ | $A_j = \beta_1(x_j),\ B_j = \beta_2(x_j)$ | 285.2647 | 1970.908 |
| $t_{14}$ | $A_j = \beta_2(x_j),\ B_j = S_{x_j}$ | 87936.19 | 6.393619 |
| $t_{15}$ | $A_j = \beta_2(x_j),\ B_j = C_{x_j}$ | 637.3852 | 882.089 |
| $t_{16}$ | $A_j = \beta_2(x_j),\ B_j = \beta_1(x_j)$ | 777.515 | 723.1121 |
| **Proposed Estimator** $t_{3k}$ | | | |
| $t_{31}$ | $\eta_{ih} = G_h \times n,\ \eta_{jh} = D_h \times n$ | 25.12954 | 22373.29 |
| $t_{32}$ | $\eta_{ih} = G_h \times n,\ \eta_{jh} = S_{pw_h} \times n$ | 25.3271 | 22198.77 |
| $t_{33}$ | $\eta_{ih} = D_h \times n,\ \eta_{jh} = G_h \times n$ | 25.66177 | 21909.26 |
| $t_{34}$ | $\eta_{ih} = D_h \times n,\ \eta_{jh} = S_{pw_h} \times n$ | 16.0475 | 35035.4 |
| $t_{35}$ | $\eta_{ih} = S_{pw_h} \times n,\ \eta_{jh} = G_h \times n$ | 25.7869 | 21802.95 |
| $t_{36}$ | $\eta_{ih} = S_{pw_h} \times n,\ \eta_{jh} = D_h \times n$ | 25.49629 | 22051.46 |

* Adapted from Audu *et al.* (2020a); MSE =Mean squared error; PRE= Percentage relative efficiency; tp = Audu *et al.* (2020a) estimators

**Table 4:** MSEs and PREs of sample mean, $t_p$ and proposed estimators using population I under least trimmed squares (LTS) Method

| ESTIMATORS | AUXILIARY PARAMETERS | MSEs | PREs |
|---|---|---|---|
| ***Sample Mean** | Not Applicable | 5657.39 | 100 |
| $t_1$ | $A_j = 1,\ B_j = S_{x_j}$ | 6664.464 | 84.8889 |
| $t_2$ | $A_j = 1,\ B_j = C_{x_j}$ | 218.5775 | 2588.277 |
| $t_3$ | $A_j = 1,\ B_j = \beta_1\left(x_j\right)$ | 209.0238 | 2706.577 |
| $t_4$ | $A_j = 1,\ B_j = \beta_2\left(x_j\right)$ | 377.6542 | 1498.035 |
| $t_5$ | $A_j = S_{x_j},\ B_j = C_{x_j}$ | 43.64495 | 12962.3 |
| $t_6$ | $A_j = S_{x_j},\ B_j = \beta_1\left(x_j\right)$ | 343.7608 | 1645.735 |
| $t_7$ | $A_j = S_{x_j},\ B_j = \beta_2\left(x_j\right)$ | 43.7337 | 12936 |
| $t_8$ | $A_j = C_{x_j},\ B_j = S_{x_j}$ | 6123.623 | 92.38632 |
| $t_9$ | $A_j = C_{x_j},\ B_j = \beta_1\left(x_j\right)$ | 34.21597 | 16534.35 |
| $t_{10}$ | $A_j = C_{x_j},\ B_j = \beta_2\left(x_j\right)$ | 276.0345 | 2049.523 |
| $t_{11}$ | $A_j = \beta_1\left(x_j\right),\ B_j = S_{x_j}$ | 5249.783 | 107.7643 |
| $t_{12}$ | $A_j = \beta_1\left(x_j\right),\ B_j = C_{x_j}$ | 52.83651 | 10707.35 |
| $t_{13}$ | $A_j = \beta_1\left(x_j\right),\ B_j = \beta_2\left(x_j\right)$ | 284.4231 | 1989.075 |
| $t_{14}$ | $A_j = \beta_2\left(x_j\right),\ B_j = S_{x_j}$ | 84132.29 | 6.724399 |
| $t_{15}$ | $A_j = \beta_2\left(x_j\right),\ B_j = C_{x_j}$ | 594.9157 | 950.9567 |
| $t_{16}$ | $A_j = \beta_2\left(x_j\right),\ B_j = \beta_1\left(x_j\right)$ | 727.4799 | 777.6697 |
| **Proposed Estimator** $t_{3k}$ | | | |
| $t_{31}$ | $\eta_{ih} = G_h \times n,\ \eta_{jh} = D_h \times n$ | | 22478.1 |
| $t_{32}$ | $\eta_{ih} = G_h \times n,\ \eta_{jh} = S_{pw_h} \times n$ | 25.16845 | 22317.95 |
| $t_{33}$ | $\eta_{ih} = D_h \times n,\ \eta_{jh} = G_h \times n$ | 25.34906 | 22028.28 |
| $t_{34}$ | $\eta_{ih} = D_h \times n,\ \eta_{jh} = S_{pw_h} \times n$ | 25.6824 | 35176.69 |
| $t_{35}$ | $\eta_{ih} = S_{pw_h} \times n,\ \eta_{jh} = G_h \times n$ | 16.08278 | 21999.12 |
| $t_{36}$ | $\eta_{ih} = S_{pw_h} \times n,\ \eta_{jh} = D_h \times n$ | 25.71644 | 22214.87 |

* Adapted from Audu *et al.* (2020a); MSE =Mean squared error; PRE= Percentage relative efficiency; tp = Audu *et al.* (2020a) estimators

**Table 5:** MSEs and PREs of sample mean, $t_p$ and proposed estimators using population I under least absolute deviation (LAD) method

| ESTIMATORS | AUXILIARY PARAMETERS | MSES | PRES |
|---|---|---|---|
| ***Sample Mean** | Not Applicable | 5622.305 | 100 |
| $t_1$ | | 6798.448 | 82.69983 |
| $t_2$ | $A_j = 1,\ B_j = C_{x_j}$ | 208.493 | 2696.64 |
| $t_3$ | $A_j = 1,\ B_j = \beta_1(x_j)$ | 198.9319 | 2826.246 |
| $t_4$ | $A_j = 1,\ B_j = \beta_2(x_j)$ | 368.0625 | 1527.541 |
| $t_5$ | $A_j = S_{x_j},\ B_j = C_{x_j}$ | 37.36166 | 15048.33 |
| $t_6$ | $A_j = S_{x_j},\ B_j = \beta_1(x_j)$ | 334.1092 | 1682.774 |
| $t_7$ | $A_j = S_{x_j},\ B_j = \beta_2(x_j)$ | 37.43919 | 15017.17 |
| $t_8$ | $A_j = C_{x_j},\ B_j = S_{x_j}$ | 6551.429 | 85.81799 |
| $t_9$ | $A_j = C_{x_j},\ B_j = \beta_1(x_j)$ | 28.69575 | 19592.81 |
| $t_{10}$ | $A_j = C_{x_j},\ B_j = \beta_2(x_j)$ | 265.684 | 2116.163 |
| $t_{11}$ | $A_j = \beta_1(x_j),\ B_j = S_{x_j}$ | 5623.277 | 99.98272 |
| $t_{12}$ | $A_j = \beta_1(x_j),\ B_j = C_{x_j}$ | 45.9762 | 12228.73 |
| $t_{13}$ | $A_j = \beta_1(x_j),\ B_j = \beta_2(x_j)$ | 274.1325 | 2050.944 |
| $t_{14}$ | $A_j = \beta_2(x_j),\ B_j = S_{x_j}$ | 88752.91 | 6.334784 |
| $t_{15}$ | $A_j = \beta_2(x_j),\ B_j = C_{x_j}$ | 656.5924 | 856.2854 |
| $t_{16}$ | $A_j = \beta_2(x_j),\ B_j = \beta_1(x_j)$ | 799.3109 | 703.394 |
| **Proposed Estimator** $t_{3k}$ | | | |
| $t_{31}$ | $\eta_{ih} = G_h \times n,\ \eta_{jh} = D_h \times n$ | 25.13041 | 22372.52 |
| $t_{32}$ | $\eta_{ih} = G_h \times n,\ \eta_{jh} = S_{pw_h} \times n$ | 25.31999 | 22205.01 |
| $t_{33}$ | $\eta_{ih} = D_h \times n,\ \eta_{jh} = G_h \times n$ | 25.65152 | 21918.02 |
| $t_{34}$ | $\eta_{ih} = D_h \times n,\ \eta_{jh} = S_{pw_h} \times n$ | 16.08507 | 34953.56 |
| $t_{35}$ | $\eta_{ih} = S_{pw_h} \times n,\ \eta_{jh} = G_h \times n$ | 25.77115 | 21816.27 |
| $t_{36}$ | $\eta_{ih} = S_{pw_h} \times n,\ \eta_{jh} = D_h \times n$ | 25.48685 | 22059.63 |

* Adapted from Audu *et al.* (2020a); MSE =Mean squared error; PRE= Percentage relative efficiency; tp = Audu *et al.* (2020a) estimators

**Table 6:** MSEs and PREs of sample Mean, $t_p$ and proposed estimators using population II under Huber MM method

| ESTIMATORS | AUXILIARY PARAMETERS | MSEs | PREs |
|---|---|---|---|
| ***Sample Mean** | Not Applicable | 15577841 | 100 |
| $t_1$ | $A_j = 1,\ B_j = S_{x_j}$ | 15404221 | 101.1271 |
| $t_2$ | $A_j = 1,\ B_j = C_{x_j}$ | 18779513 | 82.95125 |
| $t_3$ | $A_j = 1,\ B_j = \beta_1(x_j)$ | 11261172 | 138.3323 |
| $t_4$ | $A_j = 1,\ B_j = \beta_2(x_j)$ | 10302385 | 151.2062 |
| $t_5$ | $A_j = S_{x_j},\ B_j = C_{x_j}$ | 19044512 | 81.79701 |
| $t_6$ | $A_j = S_{x_j},\ B_j = \beta_1(x_j)$ | 45586359 | 34.17215 |
| $t_7$ | $A_j = S_{x_j},\ B_j = \beta_2(x_j)$ | 18984165 | 82.05703 |
| $t_8$ | $A_j = C_{x_j},\ B_j = S_{x_j}$ | 12183912 | 127.8558 |
| $t_9$ | $A_j = C_{x_j},\ B_j = \beta_1(x_j)$ | 11301055 | 137.8441 |
| $t_{10}$ | $A_j = C_{x_j},\ B_j = \beta_2(x_j)$ | 10322031 | 150.9184 |
| $t_{11}$ | $A_j = \beta_1(x_j),\ B_j = S_{x_j}$ | 25866786 | 60.22333 |
| $t_{12}$ | $A_j = \beta_1(x_j),\ B_j = C_{x_j}$ | 82155012 | 18.96152 |
| $t_{13}$ | $A_j = \beta_1(x_j),\ B_j = \beta_2(x_j)$ | 13550323 | 114.9629 |
| $t_{14}$ | $A_j = \beta_2(x_j),\ B_j = S_{x_j}$ | 55665730 | 27.98462 |
| $t_{15}$ | $A_j = \beta_2(x_j),\ B_j = C_{x_j}$ | 252042525 | 6.18064 |
| $t_{16}$ | $A_j = \beta_2(x_j),\ B_j = \beta_1(x_j)$ | 33909924 | 45.93889 |
| **Proposed Estimator** $t_{3k}$ | | | |
| $t_{31}$ | $\eta_{ih} = G_h \times n,\ \eta_{jh} = D_h \times n$ | 8534916 | 182.519 |
| $t_{32}$ | $\eta_{ih} = G_h \times n,\ \eta_{jh} = S_{pw_h} \times n$ | 8508981 | 183.0753 |
| $t_{33}$ | $\eta_{ih} = D_h \times n,\ \eta_{jh} = G_h \times n$ | 8502274 | 183.2197 |
| $t_{34}$ | $\eta_{ih} = D_h \times n,\ \eta_{jh} = S_{pw_h} \times n$ | 15346305 | 101.5087 |
| $t_{35}$ | $\eta_{ih} = S_{pw_h} \times n,\ \eta_{jh} = G_h \times n$ | 8534666 | 182.5243 |
| $t_{36}$ | $\eta_{ih} = S_{pw_h} \times n,\ \eta_{jh} = D_h \times n$ | 8534916 | 182.4283 |

* Adapted from Audu *et al.* (2020a); MSE =Mean squared error; PRE= Percentage relative efficiency; tp = Audu *et al.* (2020a) estimators

**Table 7:** MSEs and PREs of sample mean, $t_p$ and proposed estimators using population II under Hampel M method

| ESTIMATORS | AUXILIARY PARAMETERS | MSEs | PREs |
|---|---|---|---|
| ***Sample Mean** | Not Applicable | 15577841 | 100 |
| $t_1$ | $A_j=1,\ B_j=S_{x_j}$ | 15417937 | 101.0371 |
| $t_2$ | $A_j=1,\ B_j=C_{x_j}$ | 17202827 | 90.55396 |
| $t_3$ | $A_j=1,\ B_j=\beta_1(x_j)$ | 10741086 | 145.0304 |
| $t_4$ | $A_j=1,\ B_j=\beta_2(x_j)$ | 9904268 | 157.2841 |
| $t_5$ | $A_j=S_{x_j},\ B_j=C_{x_j}$ | 17415425 | 89.44853 |
| $t_6$ | $A_j=S_{x_j},\ B_j=\beta_1(x_j)$ | 39108031 | 39.83284 |
| $t_7$ | $A_j=S_{x_j},\ B_j=\beta_2(x_j)$ | 17387140 | 89.59404 |
| $t_8$ | $A_j=C_{x_j},\ B_j=S_{x_j}$ | 11585570 | 134.459 |
| $t_9$ | $A_j=C_{x_j},\ B_j=\beta_1(x_j)$ | 10775818 | 144.563 |
| $t_{10}$ | $A_j=C_{x_j},\ B_j=\beta_2(x_j)$ | 9921650 | 157.0086 |
| $t_{11}$ | $A_j=\beta_1(x_j),\ B_j=S_{x_j}$ | 23593206 | 66.02681 |
| $t_{12}$ | $A_j=\beta_1(x_j),\ B_j=C_{x_j}$ | 69938696 | 22.27357 |
| $t_{13}$ | $A_j=\beta_1(x_j),\ B_j=\beta_2(x_j)$ | 12730680 | 122.3646 |
| $t_{14}$ | $A_j=\beta_2(x_j),\ B_j=S_{x_j}$ | 49237388 | 31.63824 |
| $t_{15}$ | $A_j=\beta_2(x_j),\ B_j=C_{x_j}$ | 208129611 | 7.484683 |
| $t_{16}$ | $A_j=\beta_2(x_j),\ B_j=\beta_1(x_j)$ | 29912641 | 52.07779 |
| **Proposed Estimator** $t_{3k}$ | | | |
| $t_{31}$ | $\eta_{ih}=G_h\times n,\ \eta_{jh}=D_h\times n$ | 8520909 | 182.819 |
| $t_{32}$ | $\eta_{ih}=G_h\times n,\ \eta_{jh}=S_{pw_h}\times n$ | 8505158 | 183.1576 |
| $t_{33}$ | $\eta_{ih}=D_h\times n,\ \eta_{jh}=G_h\times n$ | 8508830 | 183.0785 |
| $t_{34}$ | $\eta_{ih}=D_h\times n,\ \eta_{jh}=S_{pw_h}\times n$ | 12199128 | 127.6964 |
| $t_{35}$ | $\eta_{ih}=S_{pw_h}\times n,\ \eta_{jh}=G_h\times n$ | 8522052 | 182.7945 |
| $t_{36}$ | $\eta_{ih}=S_{pw_h}\times n,\ \eta_{jh}=D_h\times n$ | 8523564 | 182.762 |

* Adapted from Audu *et al.* (2020a); MSE =Mean squared error; PRE= Percentage relative efficiency; tp = Audu *et al.* (2020a) estimators

**Table 8:** MSEs and PREs of Sample Mean, $t_p$ and proposed estimators using population II under least trimmed squares (LTS) Method

| ESTIMATORS | AUXILIARY PARAMETERS | MSEs | PREs |
|---|---|---|---|
| *Sample Mean | Not Applicable | 15574466 | 100 |
| $t_1$ | $A_j = 1,\ B_j = S_{x_j}$ | 15483421 | 100.588 |
| $t_2$ | $A_j = 1,\ B_j = C_{x_j}$ | 14323793 | 108.7314 |
| $t_3$ | $A_j = 1,\ B_j = \beta_1\left(x_j\right)$ | 9796152 | 158.9855 |
| $t_4$ | $A_j = 1,\ B_j = \beta_2\left(x_j\right)$ | 9176541 | 169.7204 |
| $t_5$ | $A_j = S_{x_j},\ B_j = C_{x_j}$ | 14453062 | 107.7589 |
| $t_6$ | $A_j = S_{x_j},\ B_j = \beta_1\left(x_j\right)$ | 28247419 | 55.13589 |
| $t_7$ | $A_j = S_{x_j},\ B_j = \beta_2\left(x_j\right)$ | 14461568 | 107.6956 |
| $t_8$ | $A_j = C_{x_j},\ B_j = S_{x_j}$ | 10407421 | 149.6477 |
| $t_9$ | $A_j = C_{x_j},\ B_j = \beta_1\left(x_j\right)$ | 9821856 | 158.5695 |
| $t_{10}$ | $A_j = C_{x_j},\ B_j = \beta_2\left(x_j\right)$ | 9190076 | 169.4705 |
| $t_{11}$ | $A_j = \beta_1\left(x_j\right),\ B_j = S_{x_j}$ | 18985302 | 82.03433 |
| $t_{12}$ | $A_j = \beta_1\left(x_j\right),\ B_j = C_{x_j}$ | 48315906 | 32.23466 |
| $t_{13}$ | $A_j = \beta_1\left(x_j\right),\ B_j = \beta_2\left(x_j\right)$ | 11223581 | 138.7656 |
| $t_{14}$ | $A_j = \beta_2\left(x_j\right),\ B_j = S_{x_j}$ | 36400723 | 42.78614 |
| $t_{15}$ | $A_j = \beta_2\left(x_j\right),\ B_j = C_{x_j}$ | 132188334 | 11.78203 |
| $t_{16}$ | $A_j = \beta_2\left(x_j\right),\ B_j = \beta_1\left(x_j\right)$ | 22769246 | 68.40133 |
| **Proposed Estimator** $t_{3k}$ | | | |
| $t_{31}$ | $\eta_{ih} = G_h \times n,\ \eta_{jh} = D_h \times n$ | 8579578 | 181.5295 |
| $t_{32}$ | $\eta_{ih} = G_h \times n,\ \eta_{jh} = S_{pw_h} \times n$ | 8579946 | 181.5217 |
| $t_{33}$ | $\eta_{ih} = D_h \times n,\ \eta_{jh} = G_h \times n$ | 8600990 | 181.0776 |
| $t_{34}$ | $\eta_{ih} = D_h \times n,\ \eta_{jh} = S_{pw_h} \times n$ | 7034920 | 221.388 |
| $t_{35}$ | $\eta_{ih} = S_{pw_h} \times n,\ \eta_{jh} = G_h \times n$ | 8582475 | 181.4682 |
| $t_{36}$ | $\eta_{ih} = S_{pw_h} \times n,\ \eta_{jh} = D_h \times n$ | 8579730 | 181.5263 |

* Adapted from Audu *et al.* (2020a); MSE =Mean squared error; PRE= Percentage relative efficiency; tp = Audu *et al.* (2020a) estimators

**Table 9:** MSEs and PREs of sample mean, $t_p$ and proposed estimators using population II under least absolute deviation (LAD) Method

| ESTIMATORS | AUXILIARY PARAMETERS | MSEs | PREs |
|---|---|---|---|
| ***Sample Mean** | **NA** | 15577841 | 100 |
| $t_1$ | $A_j = 1,\ B_j = S_{x_j}$ | 15416497 | 101.0466 |
| $t_2$ | $A_j = 1,\ B_j = C_{x_j}$ | 17614813 | 88.43603 |
| $t_3$ | $A_j = 1,\ B_j = \beta_1(x_j)$ | 10877779 | 143.2079 |
| $t_4$ | $A_j = 1,\ B_j = \beta_2(x_j)$ | 10007816 | 155.6568 |
| $t_5$ | $A_j = S_{x_j},\ B_j = C_{x_j}$ | 17840700 | 87.31631 |
| $t_6$ | $A_j = S_{x_j},\ B_j = \beta_1(x_j)$ | 40834142 | 38.14906 |
| $t_7$ | $A_j = S_{x_j},\ B_j = \beta_2(x_j)$ | 17804959 | 87.49159 |
| $t_8$ | $A_j = C_{x_j},\ B_j = S_{x_j}$ | 11735546 | 132.7407 |
| $t_9$ | $A_j = C_{x_j},\ B_j = \beta_1(x_j)$ | 10913921 | 142.7337 |
| $t_{10}$ | $A_j = C_{x_j},\ B_j = \beta_2(x_j)$ | 10025835 | 155.377 |
| $t_{11}$ | $A_j = \beta_1(x_j),\ B_j = S_{x_j}$ | 24145436 | 64.51671 |
| $t_{12}$ | $A_j = \beta_1(x_j),\ B_j = C_{x_j}$ | 73051247 | 21.32454 |
| $t_{13}$ | $A_j = \beta_1(x_j),\ B_j = \beta_2(x_j)$ | 12945372 | 120.3352 |
| $t_{14}$ | $A_j = \beta_2(x_j),\ B_j = S_{x_j}$ | 50781262 | 30.67636 |
| $t_{15}$ | $A_j = \beta_2(x_j),\ B_j = C_{x_j}$ | 219235125 | 7.105541 |
| $t_{16}$ | $A_j = \beta_2(x_j),\ B_j = \beta_1(x_j)$ | 30949924 | 50.3324 |
| **Proposed Estimator** $t_{3k}$ | | | |
| $t_{31}$ | $\eta_{ih} = G_h \times n,\ \eta_{jh} = D_h \times n$ | 8525925 | 182.7115 |
| $t_{32}$ | $\eta_{ih} = G_h \times n,\ \eta_{jh} = S_{pw_h} \times n$ | 8507641 | 183.1041 |
| $t_{33}$ | $\eta_{ih} = D_h \times n,\ \eta_{jh} = G_h \times n$ | 8508691 | 183.0815 |
| $t_{34}$ | $\eta_{ih} = D_h \times n,\ \eta_{jh} = S_{pw_h} \times n$ | 12997279 | 119.8546 |
| $t_{35}$ | $\eta_{ih} = S_{pw_h} \times n,\ \eta_{jh} = G_h \times n$ | 8526730 | 182.6942 |
| $t_{36}$ | $\eta_{ih} = S_{pw_h} \times n,\ \eta_{jh} = D_h \times n$ | 8528976 | 182.6461 |

***** Adapted from Audu *et al.* (2020a); MSE =Mean squared error; PRE= Percentage relative efficiency; tp =Audu *et al.* (2020a) estimators

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