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Modified Classes of Regression-Type Estimators of Population Mean in the Presence of Auxiliary Attribute under Double Sampling Scheme

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ABSTRACT

In sample survey, reliable and efficient estimates are often obtained using information from auxiliary variables during estimation and designing stages. However, there are times when the auxiliary information is attribute-based. Some authors have proposed estimators using auxiliary attribute information when the population mean of auxiliary attribute is unknown. However, the estimators are ratio-based estimators which are less efficient when the bi-serial correlation between the study variable and the auxiliary attribute is negative. In this study, regression approach was used to modify estimator t_{ZKi}^{d} to produce estimators that can be used for both negative and positive correlation. In addition, another existing estimator was also modified to produce an estimator that is independent of an unknown population parameter. The Biases and Mean Squared Errors (MSEs) of the modified estimators were determined using the Taylor series approach up to the first order of approximation. The proposed estimators' efficiency conditions over some existing estimators were established. Empirical investigations were done using stimulation study and the results revealed that proposed estimators have the lowest MSEs and the highest PREs of all the competing estimators and therefore can give better estimates of the population mean. Therefore, it can be concluded that proposed estimators have better predictive power for estimating population mean under two-phase sampling scheme.

Keywords: Auxiliary attribute, Bias, Mean square error (MSE), Population mean

Notations:

Y: Study Variable

$\bar{Y} = \frac{1}{N}\sum_{i=1}^{N} Y_i$: *Population mean of variable Y.*

$\bar{y} = \frac{1}{n}\sum_{i=1}^{n} y_i$: *Sample mean base on sample of size* n

$n^{'}$: *Sample Size at first phase*

n: *Sample size at the second-phase*

$A = \sum_{i=1}^{N} \phi_i$: *Total number of unit in the population possessing attribute* ϕ

$P = \frac{A}{N}$: *Total number of unit in the population possessing attribute* ϕ

$a = \sum_{i=1}^{n} \phi_i$: *Total number of unit in the sample possessing attribute* ϕ

$p^{'} = \frac{a}{n}$: *Sample proportion at first phase*

$p = \frac{a}{n}$: *Sample proportion at second phase*

$S_y^2 = \frac{1}{N-1}\sum_{i=1}^{N}(Y_i - \bar{Y})^2$: *Population variance of Y*

$S_\phi^2 = \frac{1}{N-1}\sum_{i=1}^{N}(\phi_i - P)^2$: *Population variance of* ϕ

$C_y = \frac{S_y}{\bar{Y}}$: *Coefficient of population variation of the study variable*

$C_\phi = \frac{S_\phi}{\bar{X}}$: *Coefficient of variation of auxiliary attribute* ϕ

$f = \frac{n}{N}$: *Sampling fraction*

$\mu_r = \frac{1}{N-1}\sum_{i=1}^{N}(\phi_i - P)^r$: rth *Moment about the proportion*

$B_{2(\phi)} = \frac{\mu_4}{\mu_2^2}$: *Coefficient of population kurtosis of the form of the auxiliary attribute* ϕ

$\rho_{y\phi} = \frac{S_{y\phi}}{S_y S_\phi}$: *The population point of bi-serial correlation between the study variable y and auxiliary attribute* ϕ

$\lambda = \left(\frac{1}{n} - \frac{1}{N}\right)$, $\lambda^{'} = \left(\frac{1}{n^{'}} - \frac{1}{N}\right)$ *and* $p^{*'} = \frac{n^{'}p^{'} - np}{n^{'} - n}$

INTRODUCTION

The variances of estimators of population parameters such as population mean, median, variance, regression coefficient, and population correlation coefficient are greatly lowered when auxiliary information is employed effectively in probability sampling. Auxiliary data from the population elements can be successfully employed to construct an efficient sampling design and after sample selection, to boost the estimators' efficiency even more. It is feasible to enhance efficiency by employing ratio, product, dual-to-ratio, or regression methods of estimate if an auxiliary variable is available and significantly correlated with the study variable. These strategies can significantly enhance estimate accuracy, resulting in estimates that are near to the corresponding population values. Several authors have proposed estimators based on some well-known population parameters for the auxiliary variable(s). Cochran (1940) was the first to explore the problem of population mean estimation when auxiliary variables are present, and he presented the typical ratio estimator of population mean as a solution. The challenge of estimating the mean of a survey variable when auxiliary variables are provided has also been studied by other authors (Hartley and Ross,1954; Quenouille, 1956; Singh, 1965; Singh, 1967; Abu-Dayeh *et al*., 2003; Kadilar and Cingi, 2005; Khoshnevisan *et al.*, 2007; Perri, 2007; Singh *et al.* 2020; Singh and Kumar, 2011; Tailor *et al*., 2012; Lu, 2013, Sharma and Singh, 2014; Lu and Yan, 2014; Verma *et al.,* 2015; Audu and Singh, 2015; Audu *et al*., 2020a, b, c; Audu *et al*., 2021a, b, c, e, f, g; Audu and Adewara, 2017; Ahmed *et al*., 2016a, b, Yunusa *et al*., 2021; Zaman *et al.,* 2021; Singh *et al*., 2021). However, there are many practical situations in which auxiliary information is qualitative in nature, i.e., when auxiliary information is available in the form of an attribute, such as when a person's height is determined by whether the person is male or female, when a dog's efficiency is determined by the breed of the dog, when the yield of a wheat crop is determined by the variety of wheat grown, and so on.

Auxiliary attribute-based estimators have been proposed by a number of authors like Singh *et al*. (2007), Singh *et al.* (2013), Zaman and Kadilar (2019), Singh *et al.* (2020), Audu *et al.* (2020b) and Audu *et al*. (2021d) with the assumption that the population proportion of the attribute (P) is known or can be obtained. In some cases, the population proportion of the auxiliary attribute (P) in a sample survey is unknown. The two-phase sampling strategy can be used instead of the preceding strategies to generate an improved estimator. Neyman (1938) was the first to introduce the notion of two-phase sampling in population parameter estimation. Two-phase sampling is a practical and cost-effective procedure. This sampling strategy is used to gather information on the auxiliary variable for a low cost from a larger sample in the first phase and the auxiliary variable is relatively small sample in the second phase. Authors like Singh *et al*. (2007) and Zaman and Kadilar (2019) proposed estimators using auxiliary attribute information when the population mean of auxiliary attribute is unknown. However, their estimators are ratio-based which imply that they are less efficient when the correlation between the study and auxiliary attribute is negative. Therefore, the aim of this current study is to propose some novel estimators which are efficient and applicable for the estimation of population means irrespective of positive and negative correlations between the study variable and auxiliary attribute using regression transformation technique.

To describe some relevant existing estimators in literature, let consider a finite population Ω_N, let y and p be the study and auxiliary attribute, taking values y_i and p_i respectively, for the *i*th unit Ω_i. At the first phase, a sample S_1 (preliminary sample) of size n' observation be drawn with SRSWOR from the population to measure only the auxiliary attribute p' in order to formulate a good estimate of a population proportion P. At the second phase sampling, the selection of sample size n can be done in two cases, namely;

Case-I: The S_2 of a fixed size n is drawn from the first phase sample.

Case-II: The S_2 of a fixed size n is drawn directly from the population

The usual ratio estimator in the two-phase sampling is denoted by t_{NG}^{d} and given as in (1):

$$t_{NG}^{d} = \bar{y}\frac{p'}{p} \qquad (1)$$

The MSEs of t_{NG1}^{d} for Cases I and II respectively are as in (2) and (3) respectively.

$$MSE(t_{NG}^{d})_I = \bar{Y}^2\left(\lambda C_y^2 + (\lambda - \lambda')\left(C_\phi^2 - 2\lambda\rho_{y\phi}C_yC_\phi\right)\right) \qquad (2)$$

$$MSE(t_{NG1}^{d})_{II} = \bar{Y}^2\left(\lambda C_y^2 + (\lambda + \lambda')C_\phi^2 - 2\lambda\rho_{y\phi}C_yC_\phi\right) \qquad (3)$$

However, t_{NG}^{d} is less efficient when the point bi-serial correlation $(\rho_{y\phi})$ is negative and also when the difference between sample and population proportion is wide.

The Dual-to-Ratio estimator denoted by t_{NG}^{*d} is given as in (4):

$$t_{NG}^{*d} = \bar{y}\frac{p^{*'}}{p} \qquad (4)$$

The expression for MSEs of t_{NG1}^{*d} for Cases I and II respectively is as in (5) and (6) respectively.

$$MSE(t_{NG}^{*d})_I = \bar{Y}^2\left(\lambda C_y^2 + \frac{n}{n'-n}(\lambda - \lambda')\left(\frac{n}{n'-n}C_\phi^2 - 2\rho_{y\phi}C_yC_\phi\right)\right) \qquad (5)$$

$MSE(t_{NG}^{*d})_{II} = \bar{Y}^2\left(\lambda C_y^2 + \frac{n}{n^{'}-n}\left(\frac{n}{n^{'}-n}(\lambda - \lambda^{'})C_\phi^2 - 2\lambda\rho_{y\phi}C_yC_\phi\right)\right)$ (6)

However, t_{NG}^{*d} is less efficient when the point bi-serial correlation $(\rho_{y\phi})$is negative.

Singh *et al*. (2007) proposed a modified ratio exponential-type estimator denoted by t_{ST}^d as in (7):

$t_{ST}^d = \bar{y}\, exp\left(\frac{p^{'}-p}{p^{'}+p}\right)$ (7)

For Case-I and Case-II, the MSEs of the estimator are given as in (8) and (9) respectively:

$MSE(t_{ST}^d)_I = \bar{Y}^2\left(\lambda C_y^2 + (\lambda - \lambda^{'})\left(\frac{C_p^2}{4} - \rho_{y\phi}C_yC_\phi\right)\right)$ (8)

$MSE(t_{ST}^d)_{II} = \bar{Y}^2\left(\lambda C_y^2 + (\lambda - \lambda^{'})C_\phi^2 - \lambda\rho_{y\phi}C_yC_\phi\right)$ (9)

However, t_{ST}^d is less efficient when the point bi-serial correlation $(\rho_{y\phi})$is negative.

The Exponential dual-to-ratio estimator in two-phase sampling is as in (10) and denoted by:t_{SK}^d

$t_{SK}^d = \bar{y}\, exp\left(\frac{p^{'}*-p}{p^{'}*+p}\right)$ (10)

Where $p^{*'} = \frac{n^{'}p^{'}-np}{n^{'}-n}$

For Case-I and Case-II, the MSEs of dual-to-ratio estimators is as in (11) and (12) respectively:

$MSE(t_{SK}^d)_I = \bar{Y}^2\left(\lambda C_y^2 + \frac{n}{n^{'}-n}(\lambda - \lambda^{'})\left(\frac{n}{4(n^{'}-n)}C_\phi^2 - \rho_{y\phi}C_yC_\phi\right)\right)$ (11)

$MSE(t_{SK}^d)_{II} = \bar{Y}^2\left(\lambda C_y^2 + \frac{n^2}{4(n^{'}-n)^2}(\lambda + \lambda^{'})C_\phi^2 - \lambda\frac{n}{n^{'}-n}\rho_{y\phi}C_yC_\phi\right)$ (12)

However, t_{SK}^d is less efficient when the point bi-serial correlation $(\rho_{y\phi})$is negative.

Zaman and Kadilar (2019) suggested the family of ratio exponential estimators denoted by$\boldsymbol{t_{ZKi}^d}$.

$t_{ZKi}^d = \bar{y}\, exp\left(\frac{(kp^{'}+l)-(kp+l)}{(kp^{'}+l)+(kp+l)}\right)$ (13)

For Case-I and Case-II, the MSEs the estimators are respectively given in (14) and (15).

$MSE(t_{ZKi}^d)_I = \bar{Y}^2\left(\lambda C_y^2 + (\lambda - \lambda^{'})(\theta_i^2C_\phi^2 - 2\theta_i\rho_{y\phi}C_yC_\phi)\right) i = 1,2,\ldots\ldots\ldots 9$ (14)

$MSE(t_{ZKi}^d)_{II} = \bar{Y}^2(\lambda C_y^2 + (\lambda + \lambda^{'})\theta_1^2C_\phi^2 - 2\theta_i\lambda\rho_{y\phi}C_yC_\phi) i = 1,2,\ldots\ldots\ldots 9$ (15)

Where $\theta_1 = \frac{P}{2(P+\beta_2(\phi))}$, $\theta_2 = \frac{P}{2(P+C_\phi)}$, $\theta_3 = \frac{P}{2(P+\rho_{\phi y})}$; $\theta_4 = \frac{\beta_2(\phi)P}{2(\beta_2(\phi)P+C_\phi)}$; $\theta_5 = \frac{C_\phi P}{2(C_\phi P+\beta_2(\phi))}$; $\theta_6 = \frac{C_\phi P}{2(C_\phi P+\rho_{y\phi})}$; $\theta_7 = \frac{\rho_{y\phi}P}{2(\rho_{y\phi}P+C_\phi)}$; $\theta_8 = \frac{\beta_2(\phi)P}{2(\beta_2(\phi)P+\rho_{y\phi})}$; $\theta_9 = \frac{\rho_{y\phi}P}{2(\rho_{y\phi}P+\beta_2(\phi))}$

However, t_{ZKi}^d is less efficient when the point bi-serial correlation $(\rho_{y\phi})$is negative.

MATERIALS AND METHODS

Proposed Estimators

Having studied the estimators of Zaman and Kadilar (2019), the modified classes of estimators denoted by $t_{pi}$and t_{qi} for the population mean are proposed as follows.

$t_{pi} = \left(\bar{y} + b_\phi(p^{'} - p)\right) exp\left(\frac{(k_ip^{'}+l_i)-(k_ip+l_i)}{(k_ip^{'}+l_i)+(k_ip+l_i)}\right) i = 1,2,3,\ldots\ldots\ldots 9$ (16)

$t_{qi} = \frac{(\bar{y}+b_\phi(p^{'}-p))(k_ip^{'}+l_i)}{k_ip^{*}+l_i} exp\left(2\left(\frac{(k_ip^{*'}+l_i)-(k_ip^{'}+l_i)}{(k_ip^{*'}+l_i)+(k_ip^{'}+l_i)}\right)\right) i = 0,1,2,3,\ldots.9$ (17)

The members of the proposed estimators t_{pi} and t_{qi} are presented in Table 1 and Table 2 respectively.

Properties (Biases and MSEs) of t_{pi}And t_{qi}

Let S_1 (preliminary sample) be set of $n' -$ paired observation (y_i, ϕ_i) drawn with SRSWOR from Ω_N and S_2be a set of n paired observation (y_i, ϕ_i) drawn with SRSWOR from either S_1 or Ω_N .To obtain the properties of t_{pi} and t_{qi} the following error terms are defined for Case I and Case II.

$e_0 = \frac{\bar{y}-\bar{Y}}{\bar{Y}}, e_1 = \frac{p-P}{P}, e_2 = \frac{p^{'}-P}{P}$ Such that:

$$\bar{y} = (1 + e_0)\bar{Y}, p = (1 + e_1)P, p^{'} = (1 + e_2)P$$

Case I: If $S_2 \subset S_1$

Table 1: Members of the proposed estimator t_{pi}

i	Estimators	Values of $k and l$ k_i	l_i
1	$t_{p1}=\left(\bar{y}+b_\phi(p^{'}-p)\right)exp\left(\frac{p^{'}-p}{p^{'}+p+2\rho_{y\phi}}\right)$	1	$\beta_2(\phi)$
2	$t_{p2}=\left(\bar{y}+b_\phi(p^{'}-p)\right)exp\left(\frac{p^{'}-p}{p^{'}+p+2C_\phi}\right)$	1	C_ϕ
3	$t_{p3}=\left(\bar{y}+b_\phi(p^{'}-p)\right)exp\left(\frac{p^{'}-p}{p^{'}+p+2\rho_{y\phi}}\right)$	1	$\rho_{y\phi}$
4	$t_{p4}=\left(\bar{y}+b_\phi(p^{'}-p)\right)exp\left(\frac{\beta_2(\phi)(p^{'}-p)}{\beta_2(\phi)(p^{'}+p)+2C_\phi}\right)$	$\beta_2(\phi)$	C_ϕ
5	$t_{p5}=\left(\bar{y}+b_\phi(p^{'}-p)\right)exp\left(\frac{C_\phi(p^{'}-p)}{C_\phi(p^{'}+p)+2\beta_2(\phi)}\right)$	C_ϕ	$\beta_2(\phi)$
6	$t_{p6}=\left(\bar{y}+b_\phi(p^{'}-p)\right)exp\left(\frac{C_\phi(p^{'}-p)}{C_\phi(p^{'}+p)+2\rho_{y\phi}}\right)$	C_ϕ	$\rho_{y\phi}$
7	$t_{p7}=\left(\bar{y}+b_\phi(p^{'}-p)\right)exp\left(\frac{\rho_{y\phi}(p^{'}-p)}{\rho_{y\phi}(p^{'}+p)+2C_\phi}\right)$	$\rho_{y\phi}$	C_ϕ
8	$t_{p8}=\left(\bar{y}+b_\phi(p^{'}-p)\right)exp\left(\frac{\beta_2(\phi)(p^{'}-p)}{\beta_2(\phi)(p^{'}+p)+2\rho_{y\phi}}\right)$	$\beta_2(\phi)$	$\rho_{y\phi}$
9	$t_{p9}=\left(\bar{y}+b_\phi(p^{'}-p)\right)exp\left(\frac{\rho_{y\phi}(p^{'}-p)}{\rho_{y\phi}(p^{'}+p)+2\beta_2(\phi)}\right)$	$\rho_{y\phi}$	$\beta_2(\phi)$

Under case I, the expectation (E) of the error terms were obtained as in (18)

$$\left.\begin{array}{l} E(e_0)=E(e_1)=E(e_2)=0, E(e_0^2)=\lambda C_y^2, \\ E(e_0e_1)=\lambda\rho_{y\phi}C_yC_\phi, E(e_0e_2)=\lambda^{'}\rho_{y\phi}C_yC_\phi, \\ E(e_1e_2)=\lambda^{'}C_\phi^2 E(e_1^2)=\lambda C_\phi^2, E(e_2^2)=\lambda^{'}C_\phi^2 \end{array}\right\} \quad (18)$$

Case II: If $S_2\subset\Omega_N$

Under case II, the expectation (E) of the error terms were obtained as in (19)

$$\left.\begin{array}{l} E(e_0)=E(e_1)=E(e_2)=0, E(e_0^2)=\lambda C_y^2, \\ E(e_0e_1)=\lambda\rho_{y\phi}C_yC_\phi, E(e_0e_2)=0, E(e_1e_2)=0 \\ E(e_1^2)=\lambda C_\phi^2, E(e_2^2)=\lambda^{'}C_\phi^2 \end{array}\right) \quad (19)$$

Express t_3 and $t_4$in term of error termse_0, e_1 and e_2

$$t_{pi}=\left(\bar{Y}(1+e_0)+b_\phi P(e_2-e_1)\right)exp\left(\varpi_1(e_2-e_1)\left(1+\varpi_1(e_2+e_1)\right)^{-1}\right) \quad (20)$$

$$t_{qi}=\frac{\left(\bar{Y}(1+e_0)+b_\phi P(e_2-e_1)\right)}{(1+\varpi_2(J_1e_1-J_2e_2))}(1+\varpi_2e_2)\,exp\left(\frac{\varpi_2(J_1e_1-J_2e_2-e_2)}{(1-\varpi_1(J_1e_1-J_2e_2+e_2))}\right) \quad (21)$$

Where$\varpi_{1i}=\left(\frac{kP}{2(kP+l)}\right)$, $\varpi_{2i}=\left(\frac{kP}{kP+l}\right)$, $J_1=\frac{n}{n'-n}$and $J_2=\frac{n^{'}}{n^{'}-n}$

Table 2: Members of the proposed estimator $\boldsymbol{t_{qi}}$

$\boldsymbol{i}$	**Estimators**	**Values of $\boldsymbol{k}$ and $\boldsymbol{l}$** $\boldsymbol{k_i}$	$\boldsymbol{l_i}$
0	$t_{q0}=\frac{\bar{y}+b_\phi(p^{'}-p)(kp^{'})}{p^{*'}}exp\left(\frac{p^{*'}-p^{'}}{p^{'}+p^{*'}}\right)$	1	0
1	$t_{q1}=\frac{\left(\bar{y}+b_\phi(p^{'}-p)(p^{'}+\rho_{y\phi})\right)}{p^{*'}+\rho_{y\phi}}exp\left(\frac{p^{*'}-p^{'}}{p^{'}+p^{*'}+2\rho_{y\phi}}\right)$	1	$\beta_2(\phi)$
2	$t_{q2}=\frac{\left(\bar{y}+b_\phi(p^{'}-p)(p^{'}+C_\phi)\right)}{p^{*'}+C_\phi}exp\left(\frac{p^{*'}-p^{'}}{p^{'}+p^{*'}+2C_\phi}\right)$	1	C_ϕ
3	$t_{3}=\frac{\left(\bar{y}+b_\phi(p^{'}-p)(p^{'}+\rho_{y\phi})\right)}{p^{*'}+\rho_{y\phi}}exp\left(\frac{p^{*'}-p^{'}}{p^{'}+p^{*'}+2\rho_{y\phi}}\right)$	1	$\rho_{y\phi}$
4	$t_{q4}=\frac{\left(\bar{y}+b_\phi(p^{'}-p)(\beta_2(\phi)p^{'}+C_\phi)\right)}{\beta_2(\phi)p^{*'}+C_\phi}exp\left(\frac{\beta_2(\phi)(p^{*'}-p^{'})}{\beta_2(\phi)(p^{'}+p^{*'})+2C_\phi}\right)$	$\beta_2(\phi)$	C_ϕ
5	$t_{q5}=\frac{\left(\bar{y}+b_\phi(p^{'}-p)(C_\phi p^{'}+\beta_2(\phi))\right)}{C_\phi p^{*'}+\beta_2(\phi)}exp\left(\frac{C_\phi(p^{*'}-p^{'})}{C_\phi(p^{'}+p^{*'})+2\beta_2(\phi)}\right)$	C_ϕ	$\beta_2(\phi)$
6	$t_{q6}=\frac{\left(\bar{y}+b_\phi(p^{'}-p)(C_\phi p^{'}+\rho_{y\phi})\right)}{C_\phi p^{*'}+\rho_{y\phi}}exp\left(\frac{C_\phi(p^{*'}-p^{'})}{C_\phi(p^{'}+p^{*'})+2\rho_{y\phi}}\right)$	C_ϕ	$\rho_{y\phi}$
7	$t_{q7}=\frac{\left(\bar{y}+b_\phi(p^{'}-p)(\rho_{y\phi}p^{'}+C_\phi)\right)}{\rho_{y\phi}p^{*'}+C_\phi}exp\left(\frac{\rho_{y\phi}(p^{*'}-p^{'})}{\rho_{y\phi}(p^{'}+p^{*'})+2C_\phi}\right)$	$\rho_{y\phi}$	C_ϕ
8	$t_{q8}=\frac{\left(\bar{y}+b_\phi(p^{'}-p)(\beta_2(\phi)p^{'}+\rho_{y\phi})\right)}{\beta_2(\phi)p^{*'}+\rho_{y\phi}}exp\left(\frac{\beta_2(\phi)(p^{*'}-p^{'})}{\beta_2(\phi)(p^{'}+p^{*'})+2\rho_{y\phi}}\right)$	$\beta_2(\phi)$	$\rho_{y\phi}$
9	$t_{q9}=\frac{\left(\bar{y}+b_\phi(p^{'}-p)(\rho_{y\phi}p^{'}+\beta_2(\phi))\right)}{\rho_{y\phi}p^{*}+\beta_2(\phi)}exp\left(\frac{\rho_{y\phi}(p^{*'}-p^{'})}{\rho_{y\phi}(p^{'}+p^{*'})+2\beta_2(\phi)}\right)$	$\rho_{y\phi}$	$\beta_2(\phi)$

Simplify (22) and (23) up to the first order of approximation, to obtain

$$\left.\begin{aligned}t_{pi}&=\bar{Y}+\bar{Y}e_0-\left(\bar{Y}\varpi_1+b_\phi P\right)e_1+\left(\bar{Y}\varpi_1+b_\phi P\right)e_2+\left(\tfrac{1}{2}\bar{Y}\varpi_1^2+b_\phi P\varpi_1\right)e_1^2\\&-\left(\tfrac{1}{2}\bar{Y}\varpi_1^2+b_\phi P\varpi_1\right)e_2^2-\left(\bar{Y}\varpi_1^2+2b_\phi P\varpi_1\right)e_1e_2-\bar{Y}\varpi_1e_1e_0+\bar{Y}\varpi_1e_0e_2\end{aligned}\right\} \quad (22)$$

$$\left.\begin{aligned}t_{qi}&=\bar{Y}+\bar{Y}e_0-b_\phi Pe_1+b_\phi Pe_2+\left((1-\varpi_2)b_\phi PJ_1+J_1^2\bar{Y}\varpi_2\left(\tfrac{\varpi_2}{2}-\varpi_1\right)\right)e_1^2\\&+J_1J_2\bar{Y}\varpi_2(2\varpi_1-\varpi_2)e_1e_2+\left(\bar{Y}\tfrac{\varpi_2^2}{2}(J_2^2-1)-\bar{Y}\varpi_1\varpi_2(J_2^2-1)\right)e_2^2\end{aligned}\right\} \quad (23)$$

Subtract $\bar{Y}$ from both side of equation (22) and (23) to obtain (24) and (25) respectively

$$\left.\begin{aligned} t_{pi}-\bar{Y} &= \bar{Y}e_0-\left(\bar{Y}\varpi_1+b_\phi P\right)e_1+\left(\bar{Y}\varpi_1+b_\phi P\right)e_2+\left(\tfrac{1}{2}\bar{Y}\varpi_1^2+b_\phi P\varpi_1\right)e_1^2 \\ &-\left(\tfrac{1}{2}\bar{Y}\varpi_1^2+b_\phi P\varpi_1\right)e_2^2-\left(\bar{Y}\varpi_1^2+2b_\phi P\varpi_1\right)e_1e_2-\bar{Y}\varpi_1e_1e_0+\bar{Y}\varpi_1e_0e_2 \end{aligned}\right\} \quad (24)$$

$$\left.\begin{aligned} t_{qi}-\bar{Y} &= \bar{Y}e_0-b_\phi Pe_1+b_\phi Pe_2+\left((1-\varpi_2)b_\phi PJ_1+J_1^2\bar{Y}\varpi_2\left(\tfrac{\varpi_2}{2}-\varpi_1\right)\right)e_1^2 \\ &+J_1J_2\bar{Y}\varpi_2(2\varpi_1-\varpi_2)e_1e_2+\left(\bar{Y}\tfrac{\varpi_2^2}{2}(J_2^2-1)-\bar{Y}\varpi_1\varpi_2(J_2^2-1)\right)e_2^2 \end{aligned}\right\} \quad (25)$$

Take expectation of (24) and (25) and apply result of (18), the Bias of t_{pi} and $t_{qi}$are

$$\left.\begin{aligned} Bias\left(t_{pi}\right)_I &= \left(\tfrac{1}{2}\bar{Y}\varpi_1^2+b_\phi P\varpi_1\right)\lambda C_\phi^2-\left(\tfrac{1}{2}\bar{Y}\varpi_1^2+b_\phi P\varpi_1\right)\lambda' C_\phi^2 \\ &-\left(\bar{Y}\varpi_1^2+2b_\phi P\varpi_1\right)\lambda' C_\phi^2-\bar{Y}\varpi_1\lambda\rho_{y\phi}C_\phi C_y+\bar{Y}\varpi_1\lambda'\rho_{y\phi}C_\phi C_y \end{aligned}\right\} \quad (26)$$

$$\left.\begin{aligned} Bias\left(t_{qi}\right) &= \left((1-\varpi_2)b_\phi PJ_1+J_1^2\bar{Y}\varpi_2\left(\tfrac{\varpi_2}{2}-\varpi_1\right)\right)\lambda C_\phi^2+J_1J_2\bar{Y}\varpi_2(2\varpi_1-\varpi_2)\lambda' C_\phi^2 \\ &+\left(\bar{Y}\tfrac{\varpi_2^2}{2}(J_2^2-1)-\bar{Y}\varpi_1\varpi_2(J_2^2-1)\right)\lambda' C_\phi^2 \end{aligned}\right\} \quad (27)$$

Take expectation of (24) and (25) and apply result of (19), the Bias of t_{pi} and $t_{qi}$are

$$\left.\begin{aligned} Bias\left(t_{pi}\right)_{II} &= \left(\tfrac{1}{2}\bar{Y}\varpi_1^2+b_\phi P\varpi_1\right)\lambda C_\phi^2-\left(\tfrac{1}{2}\bar{Y}\varpi_1^2+b_\phi P\varpi_1\right)\lambda' C_\phi^2 \\ &-\bar{Y}\varpi_1\lambda\rho_{y\phi}C_\phi C_y+\bar{Y}\varpi_1\lambda'\rho_{y\phi}C_\phi C_y \end{aligned}\right\} \quad (28)$$

$$\left.\begin{aligned} Bias\left(t_{qi}\right) &= \left((1-\varpi_2)b_\phi PJ_1+J_1^2\bar{Y}\varpi_2\left(\tfrac{\varpi_2}{2}-\varpi_1\right)\right)\lambda C_\phi^2+ \\ &\left(\bar{Y}\tfrac{\varpi_2^2}{2}(J_2^2-1)-\bar{Y}\varpi_1\varpi_2(J_2^2-1)\right)\lambda' C_\phi^2 \end{aligned}\right\} \quad (29)$$

Square (24) and (25), take expectation and applied the result of (18), the MSE of t_{pi} and $t_{qi}$for Case I are obtained as

$$\begin{aligned} MSE\left(t_{pi}\right)_I &= \bar{Y}^2\lambda C_y^2+C_\phi^2\left(\bar{Y}^2\varpi_1^2(\lambda-\lambda')+2\bar{Y}b_\phi P\varpi_1(\lambda-\lambda')+b_\phi^2P^2(\lambda-\lambda')\right) \\ &-2\rho_{y\phi}C_yC_\phi\left(\bar{Y}^2\varpi_1(\lambda-\lambda')+\bar{Y}b_\phi P(\lambda-\lambda')\right) \quad (30) \end{aligned}$$

$$MSE\left(t_{qi}\right)_I = \bar{Y}^2\lambda C_y^2+b_\phi^2P^2C_\phi^2(\lambda-\lambda')-2\bar{Y}b_\phi P\rho_{y\phi}C_yC_\phi(\lambda-\lambda') \quad (31)$$

Square (24) and (25), take expectation and applied the result of (19), the MSE of t_{pi} and $t_{qi}$for Case II are obtained as

$$\begin{aligned} MSE\left(t_{pi}\right)_{II} &= \bar{Y}^2\lambda C_y^2+C_\phi^2\left(\bar{Y}^2\varpi_1^2(\lambda+\lambda')+2\bar{Y}b_\phi P\varpi_1(\lambda+\lambda')+b_\phi^2P^2(\lambda+\lambda')\right) \\ &-2\lambda\rho_{y\phi}C_yC_\phi\left(\bar{Y}^2\varpi_1+\bar{Y}b_\phi P\right) \quad (32) \end{aligned}$$

$$MSE\left(t_{qi}\right)_{II} = \bar{Y}^2\lambda C_y^2+b_\phi^2P^2C_\phi^2(\lambda+\lambda')-2\lambda\bar{Y}b_\phi P\rho_{y\phi}C_yC_\phi \quad (33)$$

Where $b_\phi = \rho\frac{S_y}{S_x}$

Theoretical Efficiency Comparison of t_{pi} and t_{qi}

The efficiency conditions of the proposed estimators t_{pi} and t_{qi} over some existing estimators of like Ratio Estimator, Dual-to-Ratio Estimator, Singh *et al.* (2007), Exponential dual-to-Ratio Estimator, and Zaman and Kadilar (2019) for both case-1 and case-2 were established.

Theoretical efficiency comparison for case-I is as follows:

i. $MSE(t_{pi})_I - MSE(t_{NG}^d)_I < 0 \Rightarrow |\rho_{y\phi}| > C_\phi\left(\left(\bar{Y}\varpi_i + b_\phi P\right) + \bar{Y}\right)/2\bar{Y}C_y$ (34)

$MSE(t_{qi})_I - MSE(t_{NG}^d)_I < 0 \Rightarrow |\rho_{y\phi}| > C_\phi\left(b_\phi P + \bar{Y}\right)/2\bar{Y}C_y$ (35)

ii. $MSE(t_{pi})_I - MSE(t_{NG}^{*d})_I < 0 \Rightarrow |\rho_{y\phi}| > C_\phi\left(\left(\bar{Y}\varpi_i + b_\phi P\right) + K\bar{Y}\right)/2\bar{Y}C_y$ (36)

$MSE(t_{qi})_I - MSE(t_{NG}^{*d})_I < 0 \Rightarrow |\rho_{y\phi}| > C_\phi\left(b_\phi P + K\bar{Y}\right)/2\bar{Y}C_y$ (37)

iii. $MSE(t_{pi})_I - MSE(t_{ST}^d)_I < 0 \Rightarrow |\rho_{y\phi}| > C_\phi\left(\left(\bar{Y} + \varpi_I b_\phi P\right) + \frac{\bar{Y}}{2}\right)/2\bar{Y}C_y$ (38)

$MSE(t_{qi})_I - MSE(t_{ST}^d)_I < 0 \Rightarrow |\rho_{y\phi}| > C_\phi\left(b_\phi P + \frac{\bar{Y}}{2}\right)/2\bar{Y}C_y$ (39)

iv. $MSE(t_{pi})_I - MSE(t_{SK}^d)_I < 0 \Rightarrow |\rho_{y\phi}| > C_\phi\left(\left(\bar{Y}\varpi_i + b_\phi P\right) + \frac{\bar{Y}K}{2}\right)/2\bar{Y}C_y$ (40)

$MSE(t_{qi})_I - MSE(t_{SK}^d)_I < 0 \Rightarrow |\rho_{y\phi}| > C_\phi\left(b_\phi P + \frac{\bar{Y}K}{2}\right)/2\bar{Y}C_y$ (41)

v. $MSE(t_{pi})_I - MSE(t_{ZKi}^d)_I < 0 \Rightarrow |\rho_{y\phi}| > C_\phi\left(\left(\bar{Y}\varpi_i + b_\phi P\right) + \bar{Y}\theta_i\right)/2\bar{Y}C_y$ (42)

$MSE(t_{qi})_I - MSE(t_{ZKi}^d)_I < 0 \Rightarrow |\rho_{y\phi}| > C_\phi\left(b_\phi P + \bar{Y}\theta_i\right)/2\bar{Y}C_y$ (43)

If conditions (3.4), (36), (3.8), (40), (42) and (35), (37), (39), (41), (43) are satisfied respectively, then the proposed estimators $\boldsymbol{t_{pi}}$ and $\boldsymbol{t_{qi}}$ are more efficient than the existing estimators of Ratio Estimator, Dual-to-Ratio-Estimator, Singh *et al.* (2007), Exponential-dual-to-Ratio Estimator, and Zaman and Kadilar (2019) for case-1 respectively.

Theoretical efficiency comparison for case-II is as follows:

$MSE(t_{pi})_{II} - MSE(t_{NG}^d)_{II} < 0 \Rightarrow |\rho_{y\phi}| > \lambda^{**}C_\phi\left(\left(\bar{Y}\varpi_i + b_\phi P\right) + \bar{Y}\right)/2\lambda\bar{Y}C_y$ (44)

i. $MSE(t_{qi})_{II} - MSE(t_{NG}^d)_{II} < 0 \Rightarrow |\rho_{y\phi}| > \lambda^{**}C_\phi\left(b_\phi P + \bar{Y}\right)/2\lambda\bar{Y}C_y$ (45)

ii. $MSE(t_{pi})_{II} - MSE(t_{NG}^{*d})_{II} < 0 \Rightarrow |\rho_{y\phi}| > \lambda^{**}C_\phi\left(\left(\bar{Y}\varpi_i + b_\phi P\right) + K\bar{Y}\right)/2\lambda\bar{Y}C_y$ (46)

$MSE(t_{qi})_{II} - MSE(t_{NG}^{*d})_{II} < 0 \Rightarrow |\rho_{y\phi}| > \lambda^{**}C_\phi\left(b_\phi P + K\bar{Y}\right)/2\lambda\bar{Y}C_y$ (47)

iii. $MSE(t_{qi})_{II} - MSE(t_{ST}^d)_{II} < 0 \Rightarrow |\rho_{y\phi}| > \lambda^{**}C_\phi\left(\left(\bar{Y}\varpi_i + b_\phi P\right) + \frac{\bar{Y}}{2}\right)/2\lambda\bar{Y}C_y$ (48)

$MSE(t_{qi})_{II} - MSE(t_{ST}^d)_{II} < 0 \Rightarrow |\rho_{y\phi}| > \lambda^{**}C_\phi\left(b_\phi P + \frac{\bar{Y}}{2}\right)/2\lambda\bar{Y}C_y$ (49)

iv. $MSE(t_{pi})_{II} - MSE(t_{SK}^d)_{II} < 0 \Rightarrow |\rho_{y\phi}| > \lambda^{**}C_\phi\left(\left(\bar{Y}\varpi_i + b_\phi P\right) + \frac{\bar{Y}K}{2}\right)/2\lambda\bar{Y}C_y$ (50)

$MSE(t_{qi})_{II} - MSE(t_{SK}^d)_{II} < 0 \Rightarrow |\rho_{y\phi}| > \lambda^{**}C_\phi\left(b_\phi P + \frac{\bar{Y}K}{2}\right)/2\lambda\bar{Y}C_y$ (51)

v. $MSE(t_{pi})_{II} - MSE(t_{ZKi}^d)_{II} < 0 \Rightarrow |\rho_{y\phi}| > \lambda^{**}C_\phi\left(\left(\bar{Y}\varpi_i + b_\phi P\right) + \bar{Y}\theta_i\right)/2\lambda\bar{Y}C_y$ (52)

$MSE(t_4)_{II} - MSE(t_{ZKi}^d)_{II} < 0 \Rightarrow |\rho_{y\phi}| > \lambda^{**}C_\phi\left(b_\phi P + \bar{Y}\theta_i\right)/2\lambda\bar{Y}C_y$ (53)

Where $\lambda^* = (\lambda - \lambda'), \lambda^{**} = (\lambda + \lambda')$

If conditions (44), (46),(48), (50), (52) and (45), (47),(49),(51), (53) are satisfied, then the proposed estimators $\boldsymbol{t_{pi}}$ and $\boldsymbol{t_{qi}}$ are more efficient than the existing estimators of Ratio Estimator, Dual-to-Ratio-Estimator, Singh *et al.* (2007), Exponential-dual-to-Ratio Estimator and Zaman and Kadilar (2019) for case-II respectively.

Consistency Test for the Estimators $\boldsymbol{t_{pi}}$ and $\boldsymbol{t_{qi}}$

The consistency of the proposed estimators was tested as

$$\lim_{n\to N}(t_{pi}) = \left(\lim_{n\to N}\bar{y} + b_\phi\left(p' - \lim_{n\to N}p\right)\right)exp\left(\frac{(kp'+l) - \left(k\lim_{n\to N}p + l\right)}{(kp'+l) + \left(k\lim_{n\to N}p + l\right)}\right) \quad (54)$$

$$\lim_{n\to N}(t_{qi}) = \frac{\left(\lim_{n\to N}\bar{y} + b_\phi\left(p' - \lim_{n\to N}p\right)\right)(kp'+l)}{k\lim_{n\to N}p^* + l}exp\left(2\left(\frac{\left(k\lim_{n\to N}p^{*'} + l\right) - (kp'+l)}{\left(k\lim_{n\to N}p^{*'} + l\right) + (kp'+l)}\right)\right) \quad (55)$$

As $n \to N$, $\lim_{n\to N}\bar{y} = \bar{Y}$, $\lim_{n\to N}p = p'$, $\lim_{n\to N}p^{*'} = p'$ Therefore,

$$\lim_{n\to N}(t_{pi}) = \bar{Y} \text{ and } \lim_{n\to N}(t_{qi}) = \bar{Y} \quad (56)$$

Hence the estimators $\boldsymbol{t_{pi}}$ and $\boldsymbol{t_{qi}}$ is consistent.

Data for Empirical Study

In this section, stimulation studies were conducted to assess the performance of the proposed estimators over other estimators considered in the study. Data of size 500 units were generated for study population using functions defined in Table 3. Auxiliary attribute ψ was generated from a Bernoulli distribution with parameter $p = 0.7$, the study variable Y is generated with $Y = 10 + 40X + e$ and $Y = 10 - 40X + e$ when it is positively and negatively correlated with ψ respectively and e (error term) follows normal distribution with parameters 0 and 4. A preliminary large sample of size 100 and a secondary sample of size 50 were selected by method of Simple Random Sampling without replacement (SRSWOR) 1000 times. The Biases, MSEs and PREs of the considered estimators were computed for both case-I and case-II using (57), (58) and (59) respectively.

$$\boldsymbol{Bias(T)} = \frac{\mathbf{1}}{\mathbf{1000}}\sum_{\boldsymbol{j=1}}^{\mathbf{1000}}(\boldsymbol{T} - \bar{\boldsymbol{Y}}) \qquad (57)$$

$$\boldsymbol{MSE(T)} = \frac{\mathbf{1}}{\mathbf{1000}}\sum_{\boldsymbol{j=1}}^{\mathbf{1000}}(\boldsymbol{T} - \bar{\boldsymbol{Y}})^{\mathbf{2}} \qquad (58)$$

$$\boldsymbol{PREs(T)} = \left(\frac{\boldsymbol{MSE(t_O)}}{\boldsymbol{MSE(T)}}\right) \times \mathbf{100} \qquad (59)$$

Where $\boldsymbol{T}$ are any of the exiting or proposed estimators for both case-I and case-II, MSE of $t_0 = \lambda S_y^2$

Table 3: Population used for Simulation Study

Model	Auxiliary variable (x)	Study Variable (y)
I	$\psi \sim rbern(500,0.7)$	$Y = 10 + 40X + \mathrm{e}$,
II		$Y = 10 - 40X + \mathrm{e}$, where $\mathrm{e} \sim N(0,4)$

RESULTS

In this section, numerical results on the efficiency of the proposed estimators t_{3i} and t_{4i} over sample mean, Naik and Gupta (1996) t_{NG1}, Dual-to-Ratio t_{NG1}^{*d}, Singh *et al (2007)* t_{ST}^{d}, Exponential-dual-to-Ratio t_{SK}^{d}, Zaman and Kadilar (2019) t_{ZK1i} estimators were presented in Tables 4-7

Table 4 shows the numerical results of biases, MSEs and PREs for the proposed and that of the existing estimators using stimulated data obtained from models I under case I. The efficiency was tested for positive correlation between the study and auxiliary variables. The results revealed that the proposed estimators have minimum MSEs and higher PREs compared to conventional and other related estimators considered in the study. These results indicated that the proposed estimators are more efficient and have higher precision than other existing estimators considered in the study.

Table 5 shows the numerical results of biases, MSEs and PREs for the proposed and that of the existing estimators using stimulated data obtained from models II under case I. The efficiency was tested for negative correlation between the study and auxiliary variables. The results revealed that the proposed estimators with exceptions of t_{37} and t_{47} have minimum MSEs and higher PREs compared to conventional and other related estimators considered in the study. These results indicated that the proposed estimators with exception of two are more efficient and have higher precision than other existing estimators considered in the study.

Table 6 shows the numerical results of biases, MSEs and PREs for the proposed and that of the existing estimators using stimulated data obtained from models I under case II. The efficiency was tested for positive correlation between the study and auxiliary variables. The results revealed that the proposed estimators have minimum MSEs and higher PREs compared to conventional and other related estimators considered in the study. These results indicated that the proposed estimators are more efficient and have higher precision than other existing estimators considered in the study.

Table 7 shows the numerical results of biases, MSEs and PREs for the proposed and that of the existing estimators using stimulated data obtained from models II under case II. The efficiency was tested for negative correlation between the study and auxiliary variables. The results revealed that the proposed estimators with exceptions of t_{37}, t_{38}, t_{47} and t_{48}, have minimum MSEs and higher PREs compared to conventional and other related estimators considered in the study. These results indicated that the proposed estimators with exception of four are more efficient and have higher precision than other existing estimators considered in the study.

Table 4: Biases, MSEs and PREs of the proposed and some existing estimators using model 1 with correlation ($\rho_{y\phi}$ =0.9775665) under Case 1

Estimators	Biases	MSEs	PREs	Estimators	Biases	MSEs	PREs
Sample Mean $\bar{y}$	1.58	63.81	100	t_{33}	1.42	28.51	223.84
t_{NG1}	1.52	33.80	188.78	t_{34}	1.56	28.53	223.67
t_{NG1}^{*d}	1.46	140.47	45.43	t_{35}	1.30	28.48	224.01
t_{ST}^{d}	176.22	31.75	200.97	t_{36}	1.37	28.50	223.92
t_{SK}^{d}	0.61	33.67	189.50	t_{38}	1.48	28.51	223.77
Members oft_{ZK1i}				t_{38}	1.50	28.52	223.74
t_{ZK11}	1.36	50.76	125.70	t_{39}	1.35	28.49	223.95
t_{ZK12}	1.28	43.51	146.66	**Proposed Estimator** t_{4i}			
t_{ZK13}	1.31	46.50	137.22	t_{40}	1.16	28.47	224.18
t_{ZK14}	1.26	39.95	159.73	t_{41}	1.16	28.46	224.22
t_{ZK15}	1.40	53.71	118.80	t_{42}	1.16	28.46	224.22
t_{ZK16}	1.35	49.74	128.27	t_{43}	1.16	28.46	224.22
t_{ZK17}	1.28	43.69	146.05	t_{44}	1.16	28.46	224.21
t_{ZK18}	1.27	42.54	150.00	t_{45}	1.16	28.46	224.23
t_{ZK19}	1.36	50.94	125.25	t_{46}	1.16	28.46	224.22
Proposed Estimator t_{3i}				t_{47}	1.16	28.46	224.22
t_{31}	1.35	28.49	223.94	t_{48}	1.16	28.46	224.22
t_{32}	1.48	28.52	223.77	t_{49}	1.16	28.49	224.22

Table 5: Biases, MSEs and PREs of the proposed and some existing estimators using model 2 with correlation ($\rho_{y\phi}$ =-0.9791357) under case 1

Estimators	Biases	MSEs	PREs	Estimators	Biases	MSEs	PREs
Sample Mean $\bar{y}$	-0.18	68.10	100	t_{33}	1.62	31.87	213.71
t_{NG1}	-0.02	31.21	218.20	t_{34}	-0.51	30.78	221.24
t_{NG1}^{*d}	0.124	58.69	116.03	t_{35}	-0.20	30.94	220.07
t_{ST}^{d}	-150.95	41.79	162.94	t_{36}	0.43	31.27	217.78
t_{SK}^{d}	0.57	31.78	214.28	t_{37}	-11.50 x10^{+6}	2.64 x10^{+15}	2.58 x 10^{-12}
Members oft_{ZK1i}				t_{38}	-3.09	33.07	205.89
t_{ZK11}	-0.03	59.45	114.55	t_{39}	0.26	31.19	213.71
t_{ZK12}	0.055	52.36	130.06	**Proposed Estimator** t_{4i}			
t_{ZK13}	-5.57	250.05	27.23	t_{40}	-0.09	31.01	219.63
t_{ZK14}	0.08	48.62	140.07	t_{41}	-0.09	31.01	219.63
t_{ZK15}	-0.06	61.89	110.03	t_{42}	-0.09	31.01	219.62
t_{ZK16}	-1.08	103.61	65.73	t_{43}	-0.23	31.23	218.07
t_{ZK17}	-6.50x10^{+7}	8.44x10^{+16}	8.07x10^{-14}	t_{44}	-0.09	31.01	219.62
t_{ZK18}	-6.18	122.39	55.64	t_{45}	-0.09	31.01	219.63
t_{ZK19}	-0.72	90.68	75.09134	t_{46}	-0.09	31.01	219.63
Proposed Estimator t_{3i}				t_{47}	2.73 x10^{+17}	3.71 x10^{+36}	1.84 x10^{-33}
t_{31}	-0.25	30.92	220.26	t_{48}	-2.29	107.08	63.59
t_{32}	-0.41	30.83	220.87	t_{49}	-0.09	31.01	219.63

Table 6: Biases, MSEs and PREs of the proposed and some existing estimators using model 1 with correlation ($\rho_{y\phi}$ =0.9770976) under case 11

Estimators	Biases	MSEs	PREs	Estimators	Biases	MSEs	PREs
Sample Mean $\bar{y}$	0.37	65.39	100	t_{33}	1.06	27.28	239.71
t_{NG1}	1.37	62.21	105.11	t_{34}	1.40	27.52	237.65
t_{NG1}^{*d}	2.37	404.61	16.16	t_{35}	0.76	27.14	240.90
t_{ST}^{d}	169.98	20.70	115.86	t_{36}	0.92	27.21	240.35
t_{SK}^{d}	-0.91	60.24	108.55	t_{38}	1.19	27.36	238.98
Members of t_{ZK1i}				t_{38}	1.25	27.40	238.62
t_{ZK11}	0.13	43.56	150.12	t_{39}	0.87	27.18	240.54
t_{ZK12}	0.07	32.66	200.19	**Proposed Estimator** t_{4i}			
t_{ZK13}	0.08	36.99	176.74	t_{40}	0.43	26.81	243.93
t_{ZK14}	0.07	27.92	234.17	t_{41}	0.40	27.07	241.53
t_{ZK15}	0.17	48.30	135.37	t_{42}	0.40	27.05	241.72
t_{ZK16}	0.12	41.96	155.85	t_{43}	0.40	27.06	241.61
t_{ZK17}	0.07	32.92	198.64	t_{44}	0.41	27.02	241.96
t_{ZK18}	0.06	31.32	208.78	t_{45}	0.40	27.08	241.50
t_{ZK19}	0.13	43.85	149.13	t_{46}	0.40	27.07	241.54
Proposed Estimator t_{3i}				t_{47}	0.40	27.054	241.71
t_{31}	0.87	27.19	240.52	t_{48}	0.40	27.05	241.77
t_{32}	1.20	27.37	238.92	t_{49}	0.40	27.071	241.53

DISCUSSION

In this study, estimators proposed by Zaman and Kadilar (2019) and Zaman (2020) using regression were modified using regression approach to obtained new estimators which are applicable for both positive and negative bi-correlations between the study variable and auxiliary attribute. The properties (Biases and MSEs) of the proposed estimators were derived using first order approximation procedure and numerical results of the properties were obtained for efficiency comparison to sample mean, Naik and Gupta (1996) t_{NG1} , Dual-to-Ratio t_{NG1}^{*d} , Singh *et al (2007)* t_{ST}^{d} , Exponential-dual-to-Ratiot_{SK}^{d}, Zaman and Kadilar (2019) using data generated as presented in Tables 4-7. The assessments of the efficiency (proximity of the estimators to the population) ware done using MSEs of the estimators and their efficiency gains were assessed using PREs. From all the numerical results obtained in Tables 4-7, the proposed estimators have smallest MSEs and largest PREs other estimators in the study.

Tables 4 and 5 depicted the results of the efficiency and precision comparisons among the proposed and some existing estimators using stimulated data obtained from models I and II respectively, under case I. The results revealed that the proposed estimators have minimum MSEs and higher PREs compared to conventional and other related estimators considered in this study with exception of few cases. The results of Tables 4 and 5 revealed that the proposed estimators have precision gain between 23% - 124% and 7% -121% respectively over the existing estimators. These results indicated that the proposed estimators are more efficient and have higher precision than other existing estimators considered in the study.

Similarly, Tables 6 and 7 depicted the results of the efficiency and precision comparisons among the proposed and some existing estimators using stimulated data obtained from models I and II respectively, under case II. The results revealed that the proposed estimators have minimum MSEs and higher PREs compared to conventional and other related estimators considered in this study with exception of few cases. The results of Tables 6 and 7 revealed that the proposed estimators have precision gain between 21% -143% and 7% -101% respectively over the existing estimators. These results indicated that the proposed estimators are more efficient and have higher precision than other existing estimators considered in the study.

Table 7: Biases, MSEs and PREs of the proposed and some existing estimators using model 2 with correlation ($\rho_{y\phi}$ =-0.9791357) under case 11

Estimators	Bias	MSEs	PREs	Estimators	Biases	MSEs	PREs
Sample mean $\bar{y}$	2.23	67.40	100	t_{33}	3.99	37.40	180.21
t_{NG1}	1.91	25.36	265.77	t_{34}	-0.96	34.53	195.22
t_{NG1}^{*d}	1.58	52.12	129.33	t_{35}	-0.32	33.89	198.88
t_{ST}^{d}	-62.5	38.18	176.52	t_{36}	1.77	33.89	198.89
t_{SK}^{d}	2.99	26.72	252.21	t_{37}	8.80 x10^{+14}	7.73 x10^{+31}	8.72 x10^{-29}
Members of t_{ZK1i}				t_{38}	2.72 x10^{+10}	5.30 x10^{+22}	1.27 x10^{-19}
t_{ZK11}	2.36	56.73	118.80	t_{39}	1.47	33.70	200.03
t_{ZK12}	2.41	51.22	131.60	**Proposed Estimator t_{4i}**			
t_{ZK13}	-1.24	205.12	32.86	t_{40}	0.10	33.51	201.15
t_{ZK14}	2.41	48.31	139.52	t_{41}	0.11	33.63	200.40
t_{ZK15}	2.34	58.99	114.26	t_{42}	0.12	33.62	200.46
t_{ZK16}	1.31	110.92	60.77	t_{43}	0.96	427.57	15.76
t_{ZK17}	-2.46 x10^{+15}	6.05 x10^{+32}	1.11 x10^{-29}	t_{44}	0.11	33.61	200.53
t_{ZK18}	-1.39 x10	8.08 x10^{+23}	8.34 x10^{-21}	t_{45}	0.11	33.63	200.39
t_{ZK19}	1.54	101.47	66.42	t_{46}	0.08	33.82	199.31
Proposed Estimator t_{3i}				t_{47}	3.63 x10^{+27}	1.32 x10^{+57}	5.12 x10^{-54}
t_{31}	-0.44	33.99	198.30	t_{48}	2.66 x10^{+18}	3.53 x10^{+38}	1.91 x10^{-35}
t_{32}	-0.78	34.31	196.47	t_{49}	0.10	33.72	199.88

CONCLUSION

From the results of the theoretical and empirical studies, it can be concluded that the proposed estimators outperformed other competing estimators considered in study. Hence, the suggested estimators demonstrated high level of efficiency over the other estimators considered in the study. In the other words, the suggested estimators have higher chance of producing estimate that is closer to the true value of the population mean than other estimators considered in the literature of this study and therefore recommended for estimation of population mean under two-phase sampling scheme.

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