

Gaussian Plume Model Design of Effective Stack Height For Control of Industrial Emissions

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Abstract

The health impacts of exposure to generator exhaust fumes have long been identified by researchers as a major factor contributing to high morbidity and mortality rates in carcinogenic and cardiovascular related diseases. Notwithstanding, petrol and diesel generators are used frequently in augmenting the usual interrupted electric power supply experienced in most developing countries by individuals, institutions, and industries. Therefore, this research evaluated the quality of fume emitted by a diesel power plant and proffer solution for save dispersion in compliance with World Health Organisation (WHO) standards. This was achieved by employing the Gaussian plume dispersion model to design an effective stack height. The results showed that the existing stack height of the diesel power plants being 2.88m produced a maximum ground level SO₂ concentration of 450.46µg/m³ at 150m downwind. This is 800% above the maximum WHO emission limit of 50µg/m³. However, an effective stack height of 12.0m with an internal diameter of 150mm was designed for the power plant based on information about the emission, and was noted that it will produce a maximum ground level SO₂ concentration of 36.16µg/m³ during worst scenario at downwind distance of 650m thus, complying with WHO standard. Hence, it was concluded that the present installation of the diesel power plant with respect to stack height is a potential danger to the lives of humans and animals within the concerned area hence necessary recommendations were made.

Keywords: *Dispersion coefficient, Diesel generator, Emission, Exhaust, Fume.*

1.0 INTRODUCTION

The epileptic power supply in many developing countries have led some individuals, institutions, or industries to make use of private petrol and diesel engines in ensuring steady power supply. Some individuals even commercialize their private diesel engines by distributing the generated power to neighbours or entire community,

depending on the size of the engine. Most users or owners of such engines are actually aware that the smoke or air pollutants produced by the engines are toxic to humans but are ignorant about the phenomenon of *effective height of stack* in minimizing the concentrations of the pollutants. Hence, arbitrary heights of stacks are often installed which in most cases did not proffer solution to the problem.

Understanding downwind concentrations of emitted air pollutants will reveal whether the industries generating the pollutants meets standards set by regulatory bodies or not. However, information about the gas emission and nature of the atmosphere is vital in predicting the downwind concentrations (Bose and Chowdhury, 2023; Wu *et al.*, 2022; Hossain, 2022). The Gaussian plume model shown in Figure 1 is the most widely used among others in predicting downwind concentrations of air pollutants (Johnson, 2022; Khan and Hassan, 2020; Brusca *et al.*, 2016; Zanetti, 1990). It incorporates vital information about the emitted gas including effective stack height (summation of height of stack and height of plume rise) as well as nature of the atmosphere including wind velocity and atmospheric stability class.

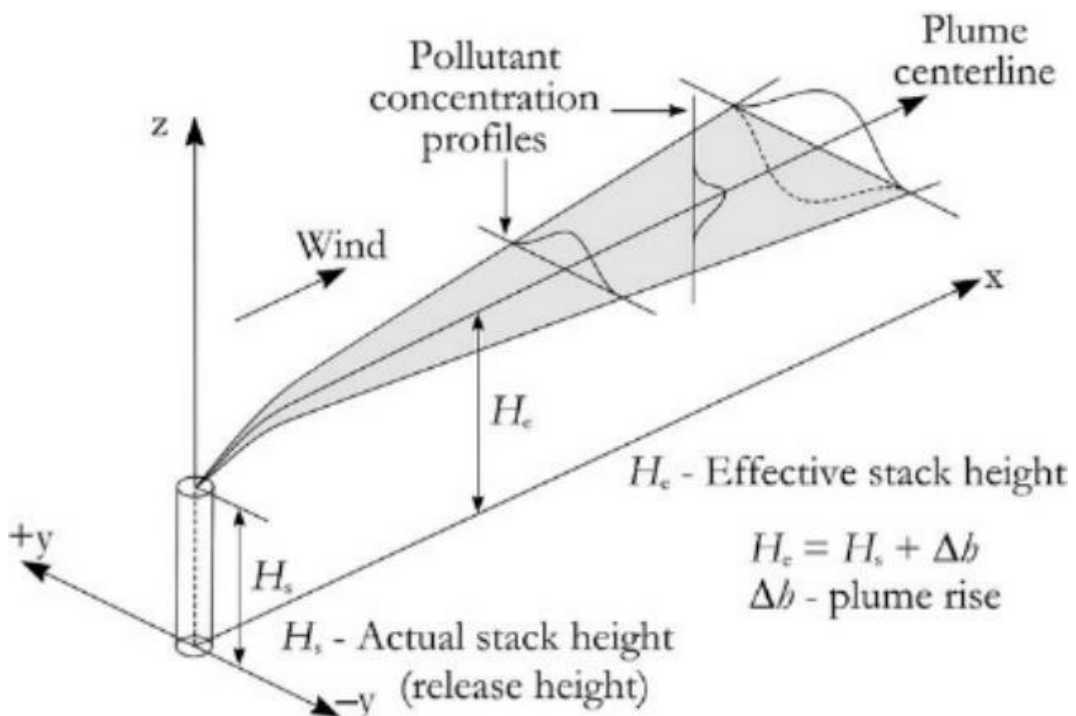


Figure 1: Schematic description of Gaussian plume model

Carbon monoxide (CO), ground level ozone (O_3), nitrogen dioxide (NO_2), particulate matter (PM) and sulphur dioxide (SO_2) have been identified as the main indicators of

air pollution (WHO, 2022). These gasses are well known to be toxic when inhaled by humans (Zadeh *et al.*, 2022; Manisalidis *et al.*, 2020; Chen *et al.*, 2007) hence both local and international regulatory bodies set permissible limits to checkmate their emissions by industries. However, Garg (2010) reported that the concentrations of PM and SO₂ are usually much higher than other air pollutant indicators in industrial gas emissions hence, in designing stack heights, priority are given to PM and SO₂.

Several researchers including Lelieveld *et al.* (2020) and Tong *et al.* (2019) have reported that one of the main factors contributing to excess global morbidity and mortality rate is inhalation of exhaust fume from diesel engines. Therefore, this research identified a diesel power plant that emits huge fume at low stack height in Yenagoa city (Nigeria) and proffer adequate stack height using the Gaussian plume model.

2.0 METHODOLOGY

The height of stack was estimated for the control of particulate matter and SO₂ through Equations (1) and (2) respectively, as reported in Garg (2010).

$$H_s = 74(Q_{PM})^{0.27} \quad (1)$$

$$H_s = 14(Q_{SO_2})^{0.33} \quad (2)$$

Where H_s is the height of stack in metres, Q_{PM} is the emission rate of particulate matter in tonne/hr while Q_{SO_2} is the emission rate of SO₂ in kg/hr. The higher value of H_s between Equations (1) and (2) was selected and added to the height of the plume rise above the stack (Δh) to obtain the effective height of stack (H_e). However, the rise in plume height (Δh) was gotten from Equation (3), known as the Holland's model.

$$\Delta h = \frac{v_s D}{u} \left[1.5 + 2.68 \times 10^{-3} P D \left(\frac{T_s - T_a}{T_s} \right) \right] \quad (3)$$

Where Δh is the rise of plume above stack in metre (m), v_s is the gas velocity inside stack in m/s, D is the internal diameter of stack at exit point measured in metre, u is the wind velocity in m/s, P is the atmospheric pressure in millibars (mBar) while T_s and T_a are the stack gas and ambient temperatures in Kelvin (K) respectively. Prior to the application of Equation (3), the gas velocity in the stack (v_s) was measured by means of a clamp-on ultrasonic gas flow meter (model: GC868) on the existing stack. This was achieved by using the device to record the flowrate of the flue gas and dividing it by the adopted internal cross-sectional area of stack at exit point. The stack gas and ambient temperatures (T_s and T_a) were known via a handheld flue gas analyser (model: 310 – TESTO). The average prevailing wind velocity (u) and atmospheric pressure (P) were obtained from the weather focus unit of a nearby meteorological station.

The general three-dimensional Gaussian plume model is given in Equation (4) as:

$$C(x, y, z); H_e = \frac{Q}{2\pi u \sigma_y \sigma_z} \left[e^{-\frac{y^2}{2\sigma_y^2}} \right] \left[e^{-\frac{(z-H_e)^2}{2\sigma_z^2}} + e^{-\frac{(z+H_e)^2}{2\sigma_z^2}} \right] \quad (4)$$

Where C is the concentration of concerned pollutants in g/m^3 , Q is pollutant emission rate in g/s , u is the wind velocity in m/s , x is the downwind distance from emission source in metres, y is the lateral or cross-wind distance from plume's centre line in metres, z is the vertical distance of plume's centre line above ground level in metres, σ_y and σ_z are the plume's standard deviations in crosswind and vertical directions respectively, while H_e is the effective height of stack in metres. Since the research is interested in determining the height of stack that will control the effect of the pollutant exposure to humans, it will be important to consider an effective height of stack that will checkmate maximum ground level concentration of the considered parameter. Hence, y and z in Equation (4) were set to be zero (i.e. directly under the plume centre line at ground level) to yield Equation (5) as;

$$C(x, 0, 0); H_e = \frac{Q}{\pi u \sigma_y \sigma_z} \left[e^{-\frac{1}{2} \left(\frac{H_e}{\sigma_z} \right)^2} \right] \quad (5)$$

Equation (5) will be maximum when the ratio of σ_z to H_e is 0.707 provided σ_z/σ_y is constant with x (Garg, 2010). In other words, at maximum ground level concentration, the relationship between H_e and σ_z can be expressed as shown in Equation (6) based on the assumption that σ_z/σ_y is constant with x .

$$\sigma_z = 0.707H_e \quad (6)$$

Hence, the estimated effective height was multiplied by 0.707 to determine the value of σ_z thereafter, the corresponding value of downwind distance (x) at $\sigma_z = 0.707H_e$ was recorded for atmospheric stability class F (stable) via vertical dispersion coefficient curve (Figure 2). This is because gaseous pollutant concentration increases with stable atmospheric conditions (Hu and Yoshie, 2020) hence the stability class F being stable was considered since it could lead to the possible maximum ground level concentration of the gaseous pollutant. The recorded downwind distance (x) was used in determining the corresponding value of σ_y for stability class F through horizontal dispersion coefficient curve (Figure 3). The pollutant emission rate Q as well as the wind velocity (u) were determined based on the field data obtained, and were substituted alongside other determined parameters into Equation (5) to know the possible maximum ground level pollutant concentration. This was compared with permissible limits set by WHO.

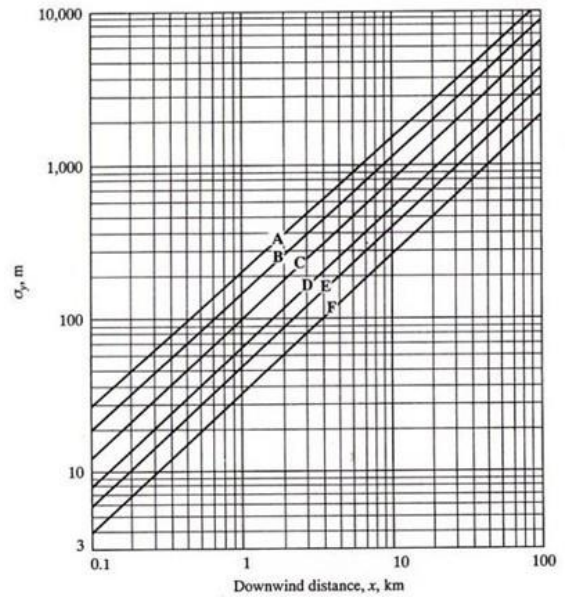
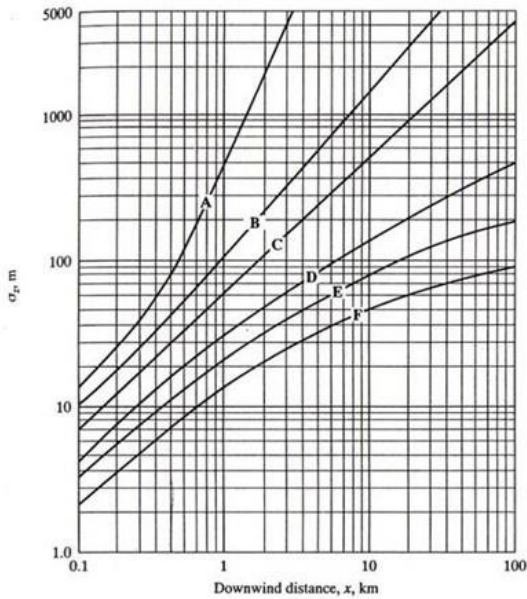


Figure 2: Dispersion coefficient curve for σ_z Figure 3: Dispersion coefficient curve for σ_y

3.0 RESULTS AND DISCUSSIONS

3.1 Emission Rate

The results of the field data used in calculating the emission rates for both particulate matters (PM_{10}) and sulphur dioxide (SO_2) are presented as follows.

Recorded concentration of PM_{10} = 0.0028ppm = 0.0028 mg/L

Recorded flowrate of gas in stack = 0.966 m³/s = 966 L/s

$$\therefore \text{Emission rate of } PM_{10} = 0.0028 \frac{mg}{L} \left(966 \frac{L}{s} \right) = 2.705 \text{ mg/s}$$

$$= 9.738 \times 10^{-6} \text{ ton/hr}$$

Observed quantity of diesel consumed = 156 L/hr (3/4 load of 1000KVA)

Recorded density of diesel = 866 kg/m³ = 0.866 kg/L

$$\text{Kilogram of diesel burnt per hour} = \frac{0.866kg}{L} \left(\frac{156L}{hr} \right) = 135 \text{ kg/hr}$$

Recorded sulphur content in diesel = 1322 mg/l = 1.322 × 10⁻³ kg/L

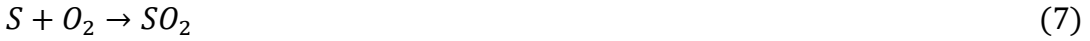
It implies 0.866kg of diesel contains 1.322 × 10⁻³ kg of sulphur

$$\therefore \text{Percentage of sulphur content in diesel} = \left(\frac{1.322 \times 10^{-3} kg}{0.866 kg} \right) 100\% = 0.15\%$$

$$\text{Sulphur produced per hour of burnt diesel} = 0.15\% \text{ of } 135 \text{ kg/hr}$$

$$= 0.203 \text{ kg/hr}$$

The formation of sulphur dioxide (SO₂) from the reaction between sulphur (S) and oxygen (O) is represented in Equation (7) as;



The molecular masses of both reactants are equal [i.e. 32 each; $S = 32, O_2 = 2(16) = 32$]. This implies that the reactant combined in ratio 1:1 to produce SO₂.

$$\begin{aligned} \Rightarrow 0.203\text{kg of } S \text{ per hour} + 0.203\text{kg of } O_2 \text{ per hour} \\ = 0.406\text{kg of } SO_2 \text{ per hour} \end{aligned}$$

\therefore Emission rate of SO₂ = **0.406 kg/hr**

3.2 Estimation of Effective Stack Height

The calculated rates of emission of PM₁₀ and SO₂ were substituted into Equations (1) and (2) to estimate the stack height (H_s).

$$H_s = 74(9.738 \times 10^{-6})^{0.27} = 3.3\text{m} \quad [\text{For particulate matter, PM}_{10}]$$

$$H_s = 14(0.406)^{0.33} = 10.4\text{m} \quad [\text{For sulphur dioxide, SO}_2]$$

Similarly, the height of plume rise (Δh) was estimated by employing Equation (3) for the relevant recorded data as follows.

$$\text{Recorded flowrate of gas in stack} = 0.966 \text{ m}^3/\text{s}$$

$$\text{Adopted internal diameter of stack (D)} = 150\text{mm} = 0.15\text{m}$$

$$\text{Cross – sectional area (A)of stack} = \pi D^2 = \pi(0.15^2) = 0.07\text{m}^2$$

$$\therefore \text{Velocity of stack gas (v}_s) = \frac{\text{flowrate}}{\text{cross-sectional area}} = \frac{0.966\text{m}^3/\text{s}}{0.07\text{m}^2} = 13.8 \text{ m/s}$$

$$\text{Average wind velocity (u)} = 2.16 \text{ m/s} \quad (\text{Obtained from meteorological station})$$

$$\text{Atmospheric pressure} = 1013\text{hPa} = 1013\text{mBar} \quad (\text{From meteorological station})$$

$$\text{Recorded stack gas temperature} = 125^\circ\text{C} = 398\text{K}$$

$$\text{Recorded ambient temperature} = 27^\circ\text{C} = 300\text{K}$$

$$\therefore \Delta h = \frac{13.8(0.15)}{2.16} \left[1.5 + 2.68 \times 10^{-3}(1013)(0.15) \left(\frac{398-300}{398} \right) \right] = 1.53\text{m}$$

As both required height of stack and emission rate of SO₂ were higher than their corresponding values for PM₁₀, the design was done based on SO₂ emission. Thus, estimated effective height (H_e) becomes;

$$\begin{aligned} H_e = \text{taller estimated } H_s + \Delta h = H_s \text{ for } SO_2 + \Delta h = 10.4\text{m} + 1.53\text{m} = 11.93\text{m} \\ \cong 12\text{m} \end{aligned}$$

3.3 Estimation of Maximum Ground Level Concentration

The estimated effective height (H_e), emission rate (Q) for SO₂ and wind velocity (u) were used in determining the possible maximum ground level concentration of pollutant as follows. By applying Equation (6), the value of σ_z at maximum ground level concentration becomes:

$$\sigma_z = 0.707H_e = 0.707(12.0) = 8.5m$$

$$\begin{aligned} \text{Corresponding value of } x \text{ at } \sigma_z = 8.5m \text{ for stability class } F = 0.65km \\ = 650m \end{aligned}$$

Value of σ_y at $x = 0.65km$ for stability class $F = 20m$

In other words, the parameters of Equation (5) are: $x = 0.65km = 650m$, $H_e = 12m$, $Q = 0.406kg/hr = 0.113g/s$, $u = 2.16m/s$, $\sigma_y = 20m$ and $\sigma_z = 8.5m$. Thus, maximum ground level concentration becomes;

$$\begin{aligned} C(650, 0, 0); 12 = \frac{0.113}{\pi(2.16)(20)(8.5)} \left[e^{-\frac{1}{2}\left(\frac{12}{8.5}\right)^2} \right] = 3.616 \times 10^{-5} g/m^3 \\ = 36.16 \mu g/m^3 \end{aligned}$$

Since the maximum ground level concentration ($36.16 \mu g/m^3$) is much lesser than the permissible limits set by WHO ($50 \mu g/m^3$), it implies the calculated effective stack height is satisfactory. This is because the concentrations of other air pollution indicators are usually much lesser than PM and SO₂ in diesel engine emissions (Garg, 2010). Hence, the results have shown that maximum ground level concentration of SO₂ being $36.16 \mu g/m^3$ will occur at a downwind distance of 0.65km or 650m from the stack or emission point. Prior to estimation of the effective height, the existing stack height of the diesel generator (H_s^*) was found to be 2.88m. Since the height of the plume rise (Δh) was calculated as 1.53m, it implies the total height (H_e^*) became 4.41m. Thus, on the basis of Gaussian plume model, the possible maximum ground level concentration of SO₂ (i.e. at atmospheric stability class F) for the existing installed stack height will occur at a downwind distance of 0.15km or 150m from the emission point with a concentration of $450.46 \mu g/m^3$. This is approximately 800% higher than the maximum permissible limit set by WHO ($50 \mu g/m^3$), which is not safe for the residents of the area. This buttresses the reports of Olu-Arotiowa, *et al.* (2022), Adeniran *et al.* (2019) and Otaru *et al.* (2013) who also noted that the ground level concentrations of certain toxic pollutants (including SO₂) emitted by some Nigerian industries exceeds the permissible limit. Hence, the designed or proposed stack height that yielded $36.16 \mu g/m^3$ which is 27.68% less than the WHO maximum emission limit, will certainly relief the health challenges experienced by people living closed to the industry.

4. CONCLUSION AND RECOMMENDATIONS

The research has shown that the fume emitted by the diesel power plant at the current installed stack height is extremely dangerous to humans and has provided an optimal stack height that will aid the dispersion of the fume to ground level concentration that conforms with WHO standard even at the worst scenario. All calculations were simplified as much as possible to serve as a guide to subsequent related researchers. Management of industries emitting stack gasses as well as relevant regulatory bodies especially in developing countries like Nigeria are hereby advised to ensure proper installation of stack with respect to effective height.

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