A robust approach to position fixing: Two-dimensional coordinate conformity in engineering surveying

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Abstract

Geomatics instruments have algorithms which derive two-dimensional and three-dimensional coordinates in a local system based on the equipment. The collected coordinates undergo coordinate transformation in order to model the space in question. However, Resident Engineers and geomatics engineers in the Malawian construction industry are still inclined to physical inspection of datasets which may expose some errors. In Malawi, so little use is made of adjustment and error detection in engineering surveys, except in datum transformation and the reconciliation of past and new surveys in cadastral surveying. Thus, this paper implemented a two-dimensional robust approach to fixing the alignment master-points through the implementation of the least squares mathematical approach. Position fixing was carried out by formulating the planimetric transformation between the design coordinates and the as-built survey coordinates using the Lilongwe Western By-Pass Road as a case study. This was validated by performing a re-survey of the stations. The scale factor of the sampled section equated to unity and the positions affected by a rotational angle of $354^{\circ}\ 46'\ 32''$. The observation quality for the Northings and Eastings was 69 mm and 11 mm, respectively. Outlier detection revealed that 29% of the alignment master stations were geometrically in their actual positions while 71% were out representing a merit in the approach which was impossible with physical inspection. The results also indicated that the approach have profound technical advice to engineering surveyors on the determination of residuals from two sets of coordinates in different coordinate systems.

Keyword: *Coordinate Conformity, Geomatics Instruments, Position Fixing, Robust Approach, Two-Dimensional.*

1.0 Introduction

Engineering surveying data are both homogeneous and heterogeneous in nature. When a survey engineer measures a single set of observations, for instance distances only, a homogeneous dataset is obtained. Most of all, the measurements are a combination of numerous kinds of observations, hence heterogeneous. A position is defined by coordinates, either rectangular (grid) or polar. In position fixing, the coordinates are obtained from a vector of observations. Engineering surveyors employ terrestrial, satellite surveys, or photogrammetric approaches in assigning coordinates to positions on the ground. Normally, survey engineers use coordinates certified to be true based on reference points or any datum to control the alignment of highways. This forms a design, a control survey, such that the construction surveys undertaken on this site has to comply. The compliance is based on the tolerance with which all the survey positions are to be fixed as indicated in the technical specifications for the project. As indicated by Allan (2007), there is a distinction between industrial tolerances which are in micrometers and civil engineering tolerances which are in millimetres.

1.1 Interpretation of the design on the ground

The survey engineer ensures that the surveys executed are within acceptable limits. The success of this lies in checking the surveyed positions against those indicated in the design in terms of coordinates and parameters. Highway engineers or survey engineers process drawings which foremen and the entire construction team translate on the ground. The designs are Computer-Assisted Drawings (CAD) which just like cadastral maps contain errors as a result of production and deformation (Sisman, 2014 a). The CAD drawings are printed on different sheet sizes for use on site. The foreman extracts some measurements from the CAD drawing and modifies others based on the circumstances on the ground. Furthermore, the survey team from the contractor carries out all construction surveys under the supervision of the Resident Engineer's (RE) representative (Uren and Price, 1994). The consultant surveyor verifies the surveys, including as-built surveys, done by the contractor or performs a joint survey with the contractor's survey team. Construction projects make use of maps and CAD drawings which require accurate geo-positioning and connection in order to have a uniform coordinate system.

1.2 The rigorous and non-rigorous method

Least squares is a rigorous method of control that plays a pertinent role in unifying coordinates by identifying outliers in both direct and indirect survey measurements. The least squares approach utilizes the same kind of fundamental principles regardless of varying geometric figures or configuration of polygons surveyed. Ghilani and Wolf (2010) define the least squares estimate as those which minimise a specified quadratic form of residuals. The method is applicable when the measurements contain only random errors.

Besides that, least squares method permits both pre-analysis of survey work during (office reconnaissance) and post-adjustment in order to determine the magnitude and pattern of errors in the data for polygons and networks. The non-rigorous methods of adjustment do not minimise the sum of the squares of residuals. Non-rigorous adjustments are different for all kinds of survey tasks. For instance, levelling networks require the Rise-and-Fall (RF) or Height of Plane of Collimation (HPC) method in reducing elevations, whereas traverses use the Compass Rule (CR) or the Transit Rule (TR). More details about RF, HPC, CR the TR can be found in Schofield and Breach (2007).

1.3 Coordinate transformation

Coordinate transformation is simply a mathematical operation that takes the coordinates of a point in one coordinate system and returns the coordinates of the same point in a second coordinate system. This process requires at least two control points in order to determine the coordinates of the other system. To obtain a unique solution, corrections are applied to data in order to restore consistency in the data. This process of making the data consistent such that the unknown parameters can be determined uniquely is known as adjustment. Least squares is the optimal technique of adjusting redundant observations.

In coordinate transformation, the method of least squares has been used in computing transformation parameters: scale factor, rotation angle and translation. Coordinates transformation is applied in industrial measurement, photogrammetry, and geodesy and cadastral studies. Sisman (2014 a) examines the 2D transformation parameters for analog cadastral maps using least squares and determines the erroneous measurements using least absolute value. In a similar study Sisman (2014 b) applies a full factorial method to determine the transformation parameters between the cadastral maps. Yang (1999) estimates the transformation parameters between Global Navigation Satellite System (GNSS) network and the corresponding geodetic network in China using a stochastic model. Furthermore, transformation parameters have also been derived between World Geodetic System 1984 (WGS84) and local coordinate systems for various countries (for instance in Abidin et al., 2005; Ziggah et al., 2013, and Lwangasi, 1993). Whether the transformation involves maps prepared in different coordinate systems or coordinates collected with different datums, either case assists in deducing transformation parameters and the transformed coordinates.

Greenfeld (1997) categories transformation into two broad kinds: planimetric (2D) and spatial (3D) of which the second type is based on the functional model applied. Either kind can be direct or reverse transformation. Based on the functional model, the transformation methods differ in number of parameters. For instance, the planimetric is four-parameter transformation also known as the 2D conformal coordinate transformation (Ghilani and Wolf, 2010). Various forms of 2D transformation are: 2D affine that computes parameters between map sheets and ground points; 2D conformal (similarity) transformation that transforms a survey in one coordinate system into a survey with a different coordinate system (Ghilani and Wolf, 2010; Congalton et al., 2001); and the 2D projective employed in transforming photograph coordinates to ground coordinates (Saeedi et al., 2009). The similarity transformation returns the shape of objects. The affine differs from the conformal in that in affine transformation the Eastings and Northings have distinct scales. Straight lines and parallelism are preserved in affine transformation whereas angles are not while the projective transformation maps lines to lines (and does not necessarily preserve parallelism).

The spatial transformation, for example, the seven-parameter transformation is used in defining datums (Hofmann-Wellenhof and Moritz, 2006). Congalton et al. (2001) call the seven-parameter transformation by other optional names namely: Helmert transformation or 3D similarity transformation or linear conformal transformation in three dimensions. The seven-parameter transformation has been applied in modelling geodetic datum transformation in geodetic surveying (Wu, et al., 2016), and it has also been used for expressing collinearity equations in photogrammetry (Moffitt and Mikhail, 1980). In the context of geodesy and geosciences, other functional models are defined: the nine-parameter (Awange, et al., 2008) and the ten-parameter (Hong-sic, 2004). Other transformation methods with profound application in geodesy are the Molodeskii formulae, multiple regression formulae, and geocentric translation.

In construction surveying, just like in any engineering survey, survey engineers encounter orthogonal and conformal coordinates. Except when a map projection is used, the grids form squares and the project area has no scale distortion (Allan, 2007). With no scale distortion, it is obvious that the true shape of the project area is also preserved. This presents one significant trait in engineering survey work that calls for 2D conformal coordinate transformation. The process of transforming coordinates determines transformation parameters and aids in error when based on least squares.

1.4 Transformation and error analysis

In civil engineering projects, the Resident Engineer (RE) ensures that the as-built surveys are in conformity with the CAD drawings and the specifications. The RE, through the consultant surveyor, establishes the truth by confirming the origin and configuration of the coordinates by comparing the coordinates. This coordinate comparison is simply a physical inspection method which may not reveal some geometric errors in the dataset. The least squares estimation of 2D transformation parameters computes the parameters and errors between any two datasets with coordinate system disparities. In Malawi, so little use is made of transformation and error detection in engineering surveys, except in datum transformation and the reconciliation of past and new surveys in cadastral surveying. Thus, the scope of this paper was to model and remove the coordinate system differences by determining the 2D coordinate transformation parameters for the Lilongwe Western By-Pass master points. The transformed coordinates for the master points were also checked against the specification and the re-surveyed station coordinates. Finally, the contribution of the least squares approach to the engineering surveying discipline was given.

2.0 Methodology

In Malawi, either in cadastral surveys or engineering surveys, the GNSS remains the more widely used geomatics measurement instrument than the Total Station (TS). The 2D approach to fixing the alignment master-points for the Lilongwe Western By-Pass road was examined using two datasets. The survey of this section was repeated on the basis of coordinate system differences. One set of 2D coordinate observations were captured with the TS (Table 1) and the other set was collected with GNSS receiver (Table 2).

The GNSS defines positions in 3D as Easting (E), Northing (N) and ellipsoidal Height (H). To be worthwhile in construction surveying, the ellipsoidal height has to be converted to orthometric height (elevation) using the geoid model. From the geoid model, the geoid undulation plays a greater role in generating elevations (Nahavandchi and Sjöberg, 2001). The geoid model for Malawi is not defined and hence, construction engineers substitute ellipsoidal heights for reduced levels collected by spirit-levelling or TS. As a result, all the elevations in Table 1 were collected with a dumpy level.

Table 1: Alignment master points: design (Local)

No.:	Element Type	Chainage	Radius	Easting(m)	Northing(m)	Bearing (Degrees)	Height(m)
1	S	0+000.000	infinite	579876.488	8455235.71	193.29293	1107.038
2	R	0+370.630	-1090	579915.463	8454867.14	193.29293	1103.635
3	S	0+886.857	infinite	580089.771	8454381.23	162.85330	1093.573
4	R	1+153.662	710	580236.905	8454158.66	162.85330	1088.415
5	S	1+379.631	infinite	580329.628	8453952.59	183.20870	1084.329
6	R	3+512.885	-490	580885.788	8451893.11	183.20870	1049.973
7	S	3+826.220	infinite	581059.908	8451632.61	141.77181	1048.563

There were coordinate inconsistencies between the 2D coordinates in Table 1 and Table 2 in that their origin and orientation were not matching. The requirement was to eliminate the coordinate disparities before checking the data in Table 2 against the specification (design). This was achieved by physical inspection of the coordinates in which two sets coordinates were compared in order to determine the differences. Then, the 2D transformational parameters ($a, b, \Delta E$ and ΔN) were determined using least squares method.

Table 2: Alignment master points: as-built (WGS84)

Chainage	Easting(m)	Northing(m)
0+000.000	579779.073	8455198.312
0+370.630	579851.445	8454834.816
0+886.857	580069.270	8454366.797
1+153.662	580235.859	8454158.532
1+379.631	580347.158	8453961.780
3+512.885	581088.521	8451961.492
3+826.220	581285.636	8451717.925

2.1 Computation of 2D unknown parameters

Least squares requires a good number of observations (Sorenson, 1970) of which a minimum of three are necessary between the two datasets (Ghilani and Wolf, 2010) in order to get the solution. Measurements and observations contain errors; this explains why Gauss and Davis (1963) note the necessity of having more observations than required to determine the unknown.

The vector of observations, g; was 2D GNSS observations in the assumed coordinate system (the as-built coordinates), and d was the vector of coordinates extracted from the CAD drawing (design coordinates). Then, the reverse transformation parameter was defined by:

- Translation in E (ΔE);
- Translation in N (ΔN);
- Scale factor (k), and
- Rotation angle (θ).

The vector of 2D GPS coordinates was (E_g, N_g) and the 2D TS coordinates was (E_d, N_d). For the ith position, the 2D GPS position was (E_{g_i}, N_{g_i}) and the 2D TS position was (E_{d_i}, N_{d_i}). To solve for the unknowns, two observation equations were generated from each alignment master point (Allan, 2007), i.e.:

$$E_{g_i} = a.E_{d_1} + bN_{d_i} + \Delta E \quad (1)$$

$$N_{g_i} = -b.E_{d_1} + aN_{d_i} + \Delta N \quad (2)$$

Where

$$a = k.Cos\theta \quad (3)$$

$$b = k.Sin\theta \quad (4)$$

In equations 3 and 4, κ was computed by:

$$k = \sqrt{\left(a^2\right)+\left(b^2\right)} = 1 \quad (5)$$

The rotational angle was calculated by the following relationship:

$$\theta = \tan^{-1}\left(\frac{b}{a}\right) \quad (6)$$

In matrix form equation (1) and (2) simplifies to

$$\begin{bmatrix} E_{g_i} \\ N_{g_1} \end{bmatrix} = \begin{bmatrix} E_{d_i} & N_{d_i} & 1 & 0 \\ N_{d_i} & -E_{d_i} & 0 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ \Delta E \\ \Delta N \end{bmatrix} \quad (7)$$

2.2 The least squares theory

Implementing the least squares theory, equation 5 becomes:

$$\left(A^T WA\right)\hat{\ell} = A^T Wb + v \quad (8)$$

In equation 6 is the normal equation where A^T is the transpose of Jacobian Matrix (coefficient matrix); W is the variance-covariance matrix (weight matrix); $\hat{\ell}$ is the least squares solution (vector of unknowns); b is the vector of Observed values minus Computed values (O-C), v is the vector of residuals.

The paper tested the alignment master points from chainage 0+000 to 3+837.709, a stretch with seven fixed positions. In this mathematical model, these points were explicit in matrix A^T and A. The seven fixed points, occupy 14 rows in the model (${}_{14}A^4$) with four columns containing the 2D coordinates and ones and zeros (in any order) depending on the definition of the translation. On the other hand, the as-built coordinates were plugged in vector b, ${}_{14}b^1$, for the seven fixed points.

2.3 Unweighted least squares

The precision of the instrument according to the instrument manufacture was not specified. For this reason, the design survey that led to the processing of the CAD drawing and the as-built survey were given an equal weight of unity, i.e, $W = 1$. Hence, equation 6 was simplified to:

$$\begin{bmatrix} E_1 & N_1 & \cdots & E_n & N_n \\ N_1 & -E_1 & \cdots & N_n & -E_n \\ 1 & 0 & \cdots & 1 & 0 \\ 0 & 1 & \cdots & 0 & 1 \end{bmatrix} \begin{bmatrix} E_1 & N_1 & 1 & 0 \\ N_1 & -E_1 & 0 & 1 \\ \vdots & \vdots & \vdots & \vdots \\ E_n & N_n & 1 & 0 \\ N_n & -E_n & 0 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ \Delta E \\ \Delta N \end{bmatrix} = \begin{bmatrix} E_1 & N_1 & \cdots & E_n & N_n \\ N_1 & -E_1 & \cdots & N_n & -E_n \\ 1 & 0 & \cdots & 1 & 0 \\ 0 & 1 & \cdots & 0 & 1 \end{bmatrix} \begin{bmatrix} E_1 \\ N_1 \\ \vdots \\ E_n \\ N_n \end{bmatrix}$$

$$+ \begin{bmatrix} v_{E_1} \\ v_{N_1} \\ \vdots \\ v_{E_1} \\ v_{N_n} \end{bmatrix} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (9)$$

Implementing the multi-dimensional case of 14 equations and plugging in the values from tables 1 and 2 in (7), the alignment coordinates (Eastings and Northings) were loaded and coded in MATLAB R2017b to compute the four parameters $\left(\begin{bmatrix} a & b & \Delta E & \Delta N \end{bmatrix}^T\right)$. The Cholesky decomposition was applied and $\hat{\ell}$ was reduced to:

$$\hat{\ell} = (N)^{-1} d = \begin{bmatrix} a \\ b \\ \Delta E \\ \Delta N \end{bmatrix} \tag{10}$$

Where:

$$N = \left(A^T A\right) \tag{11}$$

$$d = \left(A^T b\right) \tag{12}$$

Here, the least squares solution (b) and the vector of residuals were computed from equations (13) and (14), respectively. The computed 2D transformation parameters were used to re-compute the original coordinates using the forward 2D similarity coordinate transformation.

$$v = A\hat{\ell} - b \tag{14}$$

2.4 Quality assessment and outlier detection

The Root Mean Square Error (RMSE) was performed to determine the 2D (for the Eastings and Nothings) quality of the section from 0+000 and 3+826.220 and was given by: Using where n is the number of occupied stations (which is seven) and E_i and N_i is the Easting and Northing in the i th row. The $\mathrm{RMSE}_{\mathrm{E}}$ land $\mathrm{RMSE}_{\mathrm{N}}$ for each survey station was computed and then compared to the specified technical specifications of 2 cm

$$\mathrm{RMSE}_{\mathrm{E}} = \sqrt{\Sigma\left[\frac{\left(v_{E_i}\right)^2}{n}\right]} \tag{15}$$

$$\mathrm{RMSE}_{\mathrm{N}} = \sqrt{\Sigma\left[\frac{\left(v_{N_i}\right)^2}{n}\right]} \tag{16}$$

The outliers were computed from the square root of equation 15 and 16 for each (ith) chainage by equation (17). The outliers were used for deducing the positional errors for the stations. Regardless of orientation, linear error is simply the hypotenuse which equates to the square root of the sum of squares of the residuals in Eastings and Northings (i.e., $\mathrm{RMSE}_{\mathrm{E}}$ and $\mathrm{RMSE}_{\mathrm{N}}$). Further details on the definition linear error can be sourced from most geomatics books, for example Kavanagh (2007) and other literatures on traverse adjustment.

$$e = \left[\left(RMSE_{E_i} \right)^2 + \left(RMSE_{N_i} \right)^2 \right]^{\frac{1}{2}} \tag{17}$$

3.0 Results and discussion

Through the physical inspection, all the stations (100%) were denied which resulted in redoing the survey for the whole stations with the TS without clear picture of misplaced chainages.

Employing equation (10) $\hat{\ell}$ was deter100mined as follows.

$$\hat{\ell} = (N)^{-1} d = \begin{bmatrix} a \\ b \\ \Delta E \\ \Delta N \end{bmatrix} = \begin{bmatrix} 0.996 \\ -0.091 \\ 772205.419 \\ -17727.115 \end{bmatrix} \tag{18}$$

The integrity of engineering surveying relies on performing independent checks. The two parameters a and b in (14) were used as an independent check. From equations (3) and (4), $a = 0.996$ and $b = 0.091$, and the scale factor (k) equated to unity. The computed scale factor of unity means that the survey measurements were orthogonal; there was no scale distortion in both dimensions over the survey area (section).

The rotation angle was deduced from simple trigonometry according to (6) as $354°46'32''$. The rotational angle provided the first evidence of shifting of the master stations because it presented a solution to the probable geometric locations of the stations with respect to the design. In addition, if θ were zero then it would mean zero translation in both dimensions.

The four parameters in $\hat{\ell}$ were as a result of the shift of the survey depicted in Table 2. To validate the argument, the translations, rotational angle and the scale factor were used back in formula (7) and the solution matched the coordinates. At this stage, the method may be used to detect coordinate differences but may require a thorough error analysis.

3.1 Residual analysis

To determine the residual, the solution for *b* was computed first as represented by the vector (19). The residuals computed residuals are depicted in equation (20).

$$b = \begin{bmatrix} 579779.032 \\ 8455198.316 \\ 579851.407 \\ 8454834.821 \\ 580069.235 \\ 8454366.801 \\ 580236.025 \\ 8454158.556 \\ 580347.126 \\ 8453961.786 \\ 581088.503 \\ 8451961.499 \\ 581285.620 \\ 8451717.933 \end{bmatrix} \qquad [19]$$

$$v = \begin{bmatrix} 579779.073 \\ 8455198.312 \\ 579851.445 \\ 8454834.816 \\ 580069.270 \\ 8454366.797 \\ 580235.859 \\ 8454158.532 \\ 580347.158 \\ 8453961.780 \\ 581088.521 \\ 8451961.492 \\ 581285.636 \\ 8451717.925 \end{bmatrix} - \begin{bmatrix} 579779.032 \\ 8455198.316 \\ 579851.407 \\ 8454834.821 \\ 580069.235 \\ 8454366.801 \\ 580236.025 \\ 8454158.556 \\ 580347.126 \\ 8453961.786 \\ 581088.503 \\ 8451961.499 \\ 581285.620 \\ 8451717.933 \end{bmatrix} = \begin{bmatrix} 0.041 \\ -0.004 \\ 0.039 \\ -0.004 \\ 0.035 \\ -0.005 \\ -0.165 \\ -0.024 \\ 0.032 \\ -0.006 \\ 0.018 \\ -0.008 \\ 0.016 \\ -0.008 \end{bmatrix} \quad [20]$$

3.2 Observation quality for master points

The 2D quality for the master points were computed from equations (14) and (15) as $0.069\,meters$ and $0.011\,meters$ for Eastings and Northings, respectively. The accuracy of the Northings was six-times better than that of the Eastings. Mihajlović and Cvijetinović (2017) performed a similar study for the 3D uncorrelated datum observations. In this study, seven transformation parameters were determined from non-linear dataset.

Table 3: Outlier detection for the master points

Chainage	Residual (E_i (cm))	Residual (N_i (cm))	Linear Error (cm)	Tolerance (2 cm)	Discrepancy	Condition
0+000.000	4.108	-0.427	4.130	2.000	2.130	Outside the specification
0+370.630	3.866	-0.413	3.888	2.000	1.888	Outside the specification
0+886.857	3.528	-0.478	3.560	2.000	1.560	Outside the specification
1+153.662	-16.547	-2.380	16.717	2.000	14.717	Outside the specification
1+379.631	3.228	-0.594	3.283	2.000	1.283	Outside the specification
3+512.885	1.821	-0.756	1.972	2.000	-0.028	Within allowable tolerance
3+826.220	1.632	-0.847	1.839	2.000	-0.161	Within allowable tolerance

With the specification of 2 cm, table 3 presents the outlier detection results. As it can be seen in table 3, the values in column four have to be less than those in column five for the master station to be within the specified tolerance. Based on this argument, only two master positions (3+512.885 and 3+826.220) were within acceptable limits being characterised by discrepancy values of -0.028 cm and -0.161cm, respectively. This follows that 29% of the alignment master stations were geometrically in their actual positions in their coordinate system and 71% were out by the discrepancies demonstrated in table 3 for this dataset and figure 2. Figure 1 illustrates the reference points within allowable tolerance and those outside.

The chainages 3+512.885 and 3+826.220 were not identified to be within the specification of 2 cm with the physical inspection method neither were the rest of the chainages. The 2D transformation adjustment managed to expose the discrepancy between the dataset due to the least squares approach. This method enabled all the master stations to be simultaneously incorporated in the adjustment in order to determine the precision of the adjusted quantities.

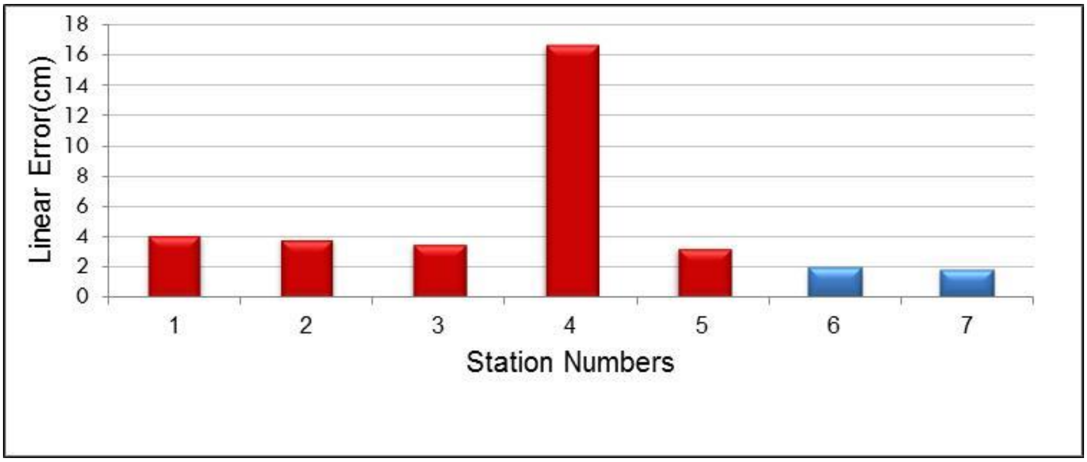

Figure 1: Master points within allowable tolerance and those outside

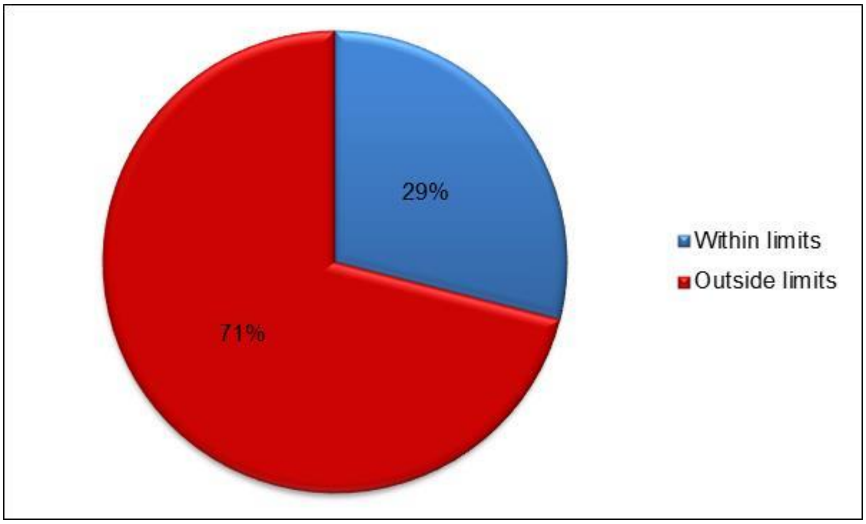

Figure 2: Alignment points within acceptable limits and those outside

The accepted stations in Table 3 were within their spatial locations as checked with the TS re-survey whereas the other five chainages were adjusted on the ground based on the transformed coordinates. Other than that, the computed discrepancies were also within the same range to the newly fixed positions on the ground. This follows that a mathematical difference on the design may not always be a problem on the ground. The 2D coordinate transformation technique may, apart from being used in deducing transformation parameters, also be utilized in preliminary planning of engineering surveys. In addition to that, the technique may also mitigate the burden of revisiting the ground when the problem is only on paper. This can save time and resources at the construction site and the survey team can concentrate on other tasks instead on repeating the job as a result of coordinate system differences. The strength of this technique is grounded on the detection of errors in observations using the mathematical theory of probability in that it is applied when random errors only exist in observations. The method would as well detect the differences if the sample size were increased.

3.3 Conclusion

Through physical inspection, the origin and orientation of the coordinate system used in analysing the spatial location of the seven master-points was not matching those on the design drawings. Instead, of the points were rejected and the section re-resurveyed. It has thus, been proved that physical inspection of survey data may increase the field workload of engineering surveyors. Furthermore, repeating the survey in the field may be a waste of time and resources when the disagreement is on the paper.

The least squares 2D coordinate transformation approach was validated with the TS as-built survey. The survey marks which were out of tolerance as a result of the model application were really out on the ground. Incorporating different numbers of chainages in the analysis never changed the outcome. The approach has also proved that coordinate system differences be removed first before comparing with the project specifications. Hence, the method demonstrates its robustness to position fixing for the alignment master points, centre line-left side of dual carriageway: the Lilongwe Western By-pass Road. This section had coordinate mismatch due to differences in coordinate systems and the study has unveiled the discrepancies using mathematical models. The paper has also provided profound technical advice to engineering surveyors on the determination of residuals from two sets of coordinates in different coordinate systems.

It can therefore be discerned that the engineering surveyor can re-survey station numbers one to 5 in order to establish the geometric relationship between the two sets of coordinates. The necessity of redoing the survey is to reduce the linear errors so that the discrepancy becomes negative (as the accepted ones). In the course of the re-survey, the geomatics engineer should also undertake an independent check (for example, using different control points) of those stations within acceptable limits. Then, the station coordinates for both sets: those within limits and outside should be included in the analysis to for outlier detection.

The analysis in this research involved only the seven chainages because it was surveyed twice as a result of physical inspection of the datasets. For this reason, the results are just restricted to this stretch of the road. Further research will involve larger sample size and parameters will be determined using the Cholesky decomposition and the centroid method (not discussed in this paper). The computed rotation angle, the scale factor and the translations will be used in weighted least squares for outlier detection.

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