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Optimizing Wireless Sensor Networks by Identifying Key Nodes Using Centrality Measures

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ABSTRACT

This study underscores the critical role of graph theory in optimizing the functionality of Wireless Sensor Networks (WSNs). Our research aims to enhance network efficiency by utilizing a variety of centrality metrics, including degree, betweenness, closeness, eigenvector, Katz, PageRank, subgraph, harmonic, and percolation centrality, to identify pivotal nodes. Employing an extended Barabási-Albert model graph of a 50-node network, our methodology focuses on pinpointing nodes crucial for optimal data processing, monitoring, and analysis in WSNs. This comprehensive approach deepens our understanding of sensor networks and significantly boosts operational efficiency by leveraging strategic node functionalities. The findings from our study are poised to revolutionize network management strategies, promoting the development of more robust and efficient WSN operations.

Keywords: Graph theory, Centralities, Wireless Sensor Networks, Node Ranks.

1. INTRODUCTION

In contemporary technology, Wireless Sensor Networks (WSNs) have emerged as foundational components across diverse domains, including environmental surveillance, urban smart infrastructure, and automated industrial processes. These networks, comprising spatially distributed, autonomous sensors, monitor various physical or environmental parameters such as temperature, acoustics, vibration, pressure, motion, and contaminant levels across different locales. Recent advancements in WSN applications highlight the growing need to enhance operational efficiency and reliability, a crucial area of research in recent studies (Anisi et al., 2011; Zytoune et al., 2010; Li et al., 2023; Dudin et al., 2014).

This paper introduces a novel approach to analyze WSNs by applying a spectrum of centrality measures—degree, betweenness, closeness, Eigenvector, Katz, PageRank, subgraph, harmonic, and percolation centrality. These metrics serve as tools to identify and examine nodes of paramount importance within networks, which are critical in data acquisition, dissemination, surveillance, and analysis. Such nodes significantly influence the network's

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resilience, operational efficiency, and data accuracy (Njotto, 2018; Mbiya et al., 2020; Ugurlu, 2022).

Our research uniquely focuses on methodically applying and examining these centrality metrics to pinpoint key nodes that sustain network integrity and functionality. By identifying these critical nodes, our study enables targeted enhancements and strategic interventions to improve network performance, reduce energy consumption, and extend network longevity. The insights and methodologies derived from this research contribute significantly to WSNs, promoting more efficient and durable network designs and operations. This work extends the current understanding of WSNs and sets the stage for future innovations, enhancing their efficacy and utility across a broad spectrum of applications.

2. OPTIMIZING WSNS: INSIGHTS FROM GRAPH THEORY CENTRALITY MEASURES

Graph theory, an essential branch of mathematics focused on analyzing interconnected systems through graphs, is particularly influential in studying and optimizing WSNs. In these networks, which consist of sensor nodes distributed over geographical areas, graphs depict nodes as sensors and edges as communication links, transforming complex network characteristics into manageable mathematical problems. This approach enables detailed network connectivity analysis, data pathways, and node roles, facilitating improved network design and operation (Klein, 2010).

The application of graph theory in WSNs leverages various centrality measures to offer insights into node significance, enhancing network performance and management. Degree centrality points out nodes with many direct connections, which are crucial for network integrity and information dissemination. Betweenness centrality identifies nodes as essential conduits for data flow, improving network cohesion and efficiency. Closeness centrality evaluates how quickly a node can communicate with all other nodes, enhancing monitoring and response speed. Eigenvector centrality highlights nodes that, through strategic connections, exert substantial influence on the network (Latora et al., 2017; Gómez, 2019).

Additional measures include Katz's centrality, which considers both direct and indirect connections, providing a broader view of node influence—vital in densely interconnected networks. PageRank assesses nodes based on the quality and quantity of their connections, focusing on their role in disseminating information. Subgraph centrality examines nodes' involvement in tightly knit groups, offering insights into local and comprehensive network dynamics. Harmonic Centrality, considering the inverse of shortest paths to all nodes, identifies influential nodes even in partially disconnected networks. Percolation Centrality evaluates the ability of nodes to maintain network connectivity under various conditions, highlighting their importance in network robustness and fluidity of information flow (Piraveenan et al., 2013; Deverashetti and Pradhan, 2018; Ma et al., 2010; Estrada and Rodríguez, 2005; Mbiya et al., 2020; Njotto, 2018).

This strategic integration of graph theory into WSNs advances network performance and reliability and provides a structured methodology for addressing challenges in deploying extensive sensor networks. Beyond mere technological enhancement, applying these mathematical models has broad implications across diverse fields such as environmental monitoring, security, healthcare, and more. The adaptability and efficiency brought by graph theory showcase its potential to reshape technology solutions, meeting contemporary challenges and pushing the boundaries of digital innovation in various sectors.

3. CENTRALITY MEASURES AND THEIR MATHEMATICAL FORMULAS IN GRAPH THEORY

1. Degree Centrality (DC): $C_D(v) = \frac{deg(v)}{N-1}$, where $C_D(v)$ is the degree centrality of node v, $deg(v)$ is the degree of node v (i.e., the number of edges connected to v), and $N-1$ represents the maximum possible degree of v in a network of N nodes.
2. Betweenness Centrality (BC): $C_B(v) = \sum_{s\neq v\neq t} \frac{\sigma_{st}(v)}{\sigma_{st}}$, where $C_B(v)$is the betweenness centrality of node v, σ_{st} is the total number of shortest paths from node s to node t, and $\sigma_{st}(v)$ is the number of those paths that pass through v.
3. Closeness Centrality (CC): $C_C(v) = \frac{N-1}{\sum_{u\neq v} d(v,u)}$, where $C_C(v)$ is the closeness centrality of node v, $d(v,u)$ is the shortest path distance between nodes v and u, and $N-1$ is the total number of other nodes in the network.
4. Eigenvector Centrality (EVC): $x_i^{(k)} = \sum_j a_{ij}\, x_j^{(k)}, x_j^{(0)} = 1,$ where $x_i^{(k)}$ is the eigenvector centrality of node i at the k^{th} iteration and a_{ij} are the entries of the adjacent matrix.
5. Katz Centrality (KC): $C_K(v) = \sum_{k=1}^{\infty} \sum_{j=1}^{N} \alpha^k\, (A^k)_{jv}$, where $C_K(v)$ is the Katz centrality of node v, α is an attenuation factor that penalizes paths according to their length, and $(A^k)_{jv}$ represents the number of paths of length k from node j to node v.
6. PageRank (PR): $PR(i) = \frac{1-d}{N} + d \sum_{j\in M(i)} \frac{PR(j)}{L(j)}$, where $PR(i)$ is the PageRank of node i, N is the total number of nodes in the network, and d is the damping factor, usually set to

around 0.85, which accounts for the probability that a node communicates directly with another node versus randomly linking to a node in the network. *M(i)* represents the set of nodes that link to node *i* (in the context of WSNs, these are the nodes that directly communicate with node *i*. *L(j)* is the number of links (or direct communication connections) that node *j* has. The term $\frac{1-d}{N}$ is a factor that ensures there is always a chance of reaching any node, providing a baseline importance level to every node.

7. Subgraph Centrality (SC): $SC(i) = \sum_{k=0}^{\infty} \frac{(A^k)_{ii}}{k!}$, where *SC(i)* is the subgraph centrality of node *i*, *A* is the adjacency matrix of the network, $\frac{(A^k)_{ii}}{k!}$ is the *i*-th diagonal element of the *k*-th power of the adjacency matrix *A*, representing the number of closed walks of length *k* starting and ending at node *i* and *k*! normalizes the count of walks based on their length, making longer walks contribute less to the centrality measure due to the factorial term in the denominator.
8. Harmonic Centrality (HC): $C_H(v) = \sum_{u \neq v} \frac{1}{d(u,v)}$, where $C_H(v)$ is the harmonic centrality of node *v*, *d(u,v)* represents the shortest path distance between node *u* and node *v*, and the summation is taken over all nodes *u* in the network except *v* itself. If *d(u,v)* = ∞ (i.e., if *u* and *v* are disconnected), then $\frac{1}{d(u,v)} = 0$.
9. Percolation Centrality (PC): $PC(v) = \frac{1}{N-2} \sum_{s \neq v \neq t} \frac{\sigma_{st}(v)}{\sigma_{st}} (x_s - x_t)$, where $PC(v)$ is the percolation centrality of node *v*, x_s and x_t are the states of nodes *s* and *t* respectively, with 1 indicating an active (functioning) state and 0 indicating an inactive (failed) state.

In this paper, we utilized Python programming to construct and analyze a network comprising 50 nodes, along with the implementation of various centrality measures to assess node significance within the network.

4. RESULTS

Our analysis is highlighted in figure 1, which presents a network modeled on an extended Barabási-Albert graph with 50 nodes. This choice is based on the original Barabási-Albert model, known for generating scale-free networks through preferential attachment, where nodes with higher degrees are more likely to acquire new connections (Zadorozhnyi and Yudin, 2014). Such a model mimics the degree distribution commonly observed in many real-world networks characterized by a power-law distribution. Consequently, this results in a network structure where a few nodes act as significant hubs with considerably higher degrees while most have lower degrees. Using 50 nodes allows for a manageable yet sufficiently complex network to effectively demonstrate the utility of various centrality metrics in identifying crucial nodes. These metrics include degree, betweenness, closeness, eigenvector, Katz, PageRank, subgraph, harmonic, and percolation centrality. As shown in table 1, our findings rank these nodes based on their centrality, providing a detailed examination of each node's unique role and importance. This analysis helps to elucidate their contributions to the overall architecture and functionality of the network, shedding light on how pivotal nodes influence network dynamics and efficiency.

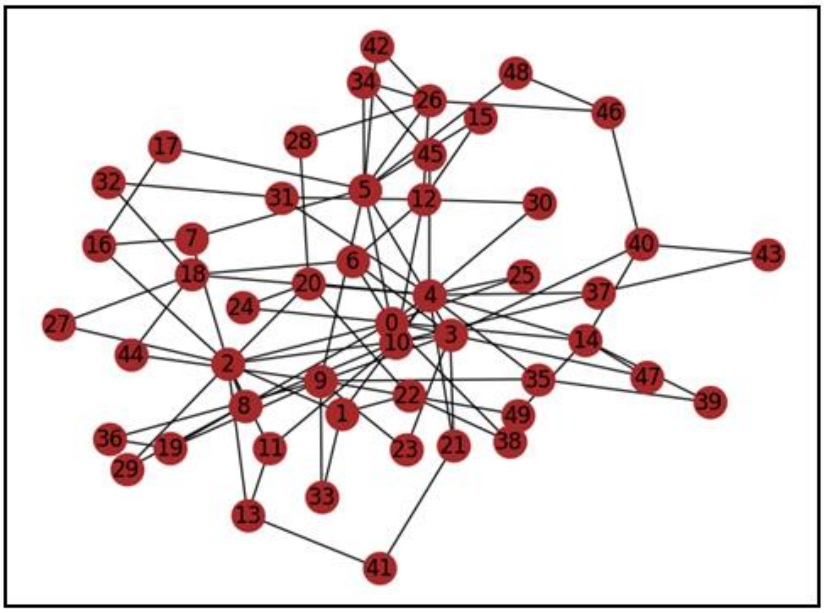

Figure 1. Graph Configuration of a 50-Node Network.

Table 1. Node Rankings: Network Centrality Metrics in 50-Node Network.

Rank	***DC***	***BC***	***CC***	***EVC***	***KC***	***PR***	***SC***	***HC***	***PC***
1	4	4	0	0	4	4	0	4	4
2	2	5	3, 4	4	0	2	4	0	5
3	0, 5	2		3	2	5	2	3	2
4		0	2, 5	2	3	0	3	2	0
5	3, 9	3		5	5	9	5	5	3
6		9	9	9	9	3	9	9	9
7	12, 14, 26	14	14	14	14	26	14	14	14
8		20	6, 20	8	8	14	8	20	20
9		18		6	20	12	26	8	18
10	8	26	8	20	26	18	10	10	26

5. CONCLUSIONS

In conclusion, our research presents a comprehensive analysis aimed at enhancing the efficiency and resilience of WSNs through the strategic identification of key nodes using a suite of centrality measures which computed through Python. By employing an extended Barabási-Albert model to represent a 50-node WSN, we have successfully pinpointed nodes critical for the network's optimal data processing, surveillance, and analytical functionalities. This identification is made possible by leveraging centrality metrics such as degree, betweenness, closeness, eigenvector, Katz, PageRank, subgraph, harmonic, and percolation centrality, each providing unique insights into node significance within the network's architecture. Our findings reveal that specific nodes play pivotal roles in maintaining network integrity, facilitating efficient data dissemination, and ensuring the robustness of the entire system. The rankings presented in Table 1 underscore the varied importance of nodes based on different centrality dimensions, offering a nuanced understanding of network dynamics that can inform targeted interventions for network enhancement.

6. ACKNOWLEDGEMENTS

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7. CONFLICT OF INTERESTS

There is no conflict of interests.

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