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THROUGHPUT ANALYSIS OF ARQ SCHEMES USING STATE TRANSITION DIAGRAMS

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ABSTRACT

Automatic-Repeat-Request (ARQ) schemes have been widely used and various methods and representations have been used in their analysis. In this paper, the schemes are represented by z-transform based transition diagrams to yield easily retractable expressions. The approach has enabled the analysis of the Selective Repeat-Stutter 2 (SR-ST2) scheme having variable selective repeat mode retransmissions and the Selective Repeat-Stutter-Go Back N (SR-ST-GBN) scheme having variable stutter mode retransmissions. Using the SR-ST-GBN scheme, it is also shown that it is possible to choose the design parameters of ARQ-based communication systems in order to achieve maximum throughput.

Keywords: *Automatic-Repeat-Request, throughput, throughput efficiency*

INTRODUCTION

Automatic-Repeat-Request (ARQ) techniques have been widely used to enable the reduction of data losses arising from disturbances in communication channels. The schemes currently in use are based on three basic ARQ schemes - the Stop-and-Wait (SW), Go-Back-N (GBN) and Selective Repeat (SR) schemes. In the SW scheme, once the transmitter has transmitted a packet, it must wait for a response from the receiver before transmitting the next packet. On the other hand, in the GBN scheme the transmitter transmits continuously but has to retransmit an equivalent of N packets each time a transmission fails. In the SR scheme, the transmitter also transmits continuously, but only retransmits failed packets. An addition to these basic schemes is the Stutter (ST) scheme in which the transmitter also transmits continuously but retransmits any failed packet continuously until its transmission succeeds. A description of ARQ schemes has been given by Miller and Lin (1981) and Lin and Costello (2004). Given the success of ARQ protocols in improving the performance of conventional communication systems, their application has been extended to modern communication systems and other related areas. Examples include improving the transmission capacity of multi-user MIMO networks operating in severely fading channels (Shi *et al*., 2008), and enabling the use of maintenance-free battery operation in wireless sensor networks (Tacca *et al.*, 2007).

The main measure of performance in ARQ

schemes is the throughput efficiency, which gives the amount of correctly transmitted data per unit time as a fraction of the amount of data that could have been transmitted within the same time. Among the three basic ARQ schemes, the ideal SR scheme gives the highest throughout efficiency. The main limitation of the ideal SR scheme is the requirement for an infinite buffer storage size at the receiver. Modifications aimed at enabling the use of finite buffer capacity at the receivers while maximising the throughput have led to the SR-GBN, SR-ST1, SR-ST2 and SR-ST -GBN mixed mode schemes. Descriptions of these schemes are given later in this report while more thorough descriptions can be found in other reports (Miller and Lin, 1981; Lin *et al*., 1984). Using the throughput efficiency as the main measure of performance, the analysis of Miller and Lin (1981) and Lin *et al.* (1984) show that while the ideal SR scheme has the highest throughput efficiency and the SW scheme the lowest, the SR-ST schemes outperform the SR-GBN schemes, which in turn outperform the GBN scheme. The throughput efficiency of data communication systems is also increased by using forward error correction (FEC), which is often used in conjunction with ARQ. Generally, schemes which utilise both ARQ and FEC for error control have higher throughput efficiency at higher error rates and are therefore preferred at those error rates (Lin and Costello, 2004).

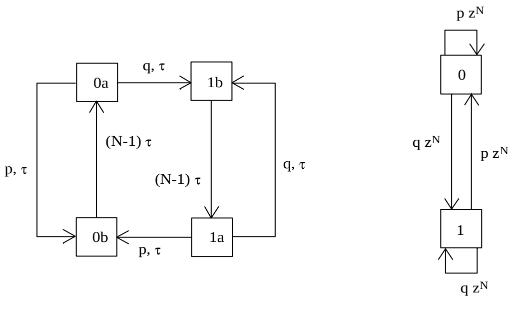
Various methods have been used in deriving the expressions for the measures of performance in ARQ schemes. These include time series analysis (Lin *et al*., 1984), signal flow graphs (Lu and Chang, 1989), markov chain models (Lu *et al*., 2004; Yun and Kaverad, 2005), and hidden markov models (Ausavapattannakum and Nostratinia, 2007). In this paper, we employ the approach used in the analysis of frame synchronized communication systems in which z transform based state transition diagrams were employed. In a frame synchronized communication system the information is transmitted serially as frames with frame control information called markers inserted between consecutive frames. The receiver of such a system then use the markers to detect the frames. Since errors can occur in the markers, the receiver in such a system has to verify the correctness of the received markers each time it receives a frame. The states of the receiver as it goes about detecting and verifying the markers can be represented by state transition diagrams from which the performance metrics can then be derived using time series analysis or transfer function analysis. The approach of using transfer function analysis based on the z-transform was used in (Kundaeli, 1993) where the synchronization efficiency was also initially used as the main measure of performance. This approach was subsequently applied to numerous types of frame synchronized communication systems leading to improved performance over adopted ones. An adequate review of the methods employed for each type of frame synchronized systems is beyond the scope of this report and interested readers are therefore advised to consult (Kundaeli, 1998; 2007) and references therein. A good treatment of applying the z-transform to state transition diagrams is also well treated in Kundaeli (2008). In the current analysis the method of using the synchronization efficiency is applied to ARQ systems to obtain the throughput efficiency instead of the delay times. Since some transmissions do not contribute to the throughput efficiency, they are neglected thereby leading to simple models and easily tractable performance expressions. We therefore employ this approach to derive the throughput efficiency for the SW, SR, GBN, ST, SR‑ST1, SR‑ST2, SR‑GBN and SR‑ST‑GBN ARQ schemes, and also show that the methods used in choosing the design parameters for frame synchronized communication systems can be used to choose the design parameters for ARQ schemes.

SYSTEM MODELS AND ANALYSIS

Our analysis is based on the assumption that the information is transmitted in terms of packets, with the transmission time of a packet being t seconds. Consequently, time is divided into slots of t seconds, and the transmitter-receiver round trip delay is assumed to equal the duration of N-1 = S packets. Thus, if a packet is transmitted at time $t = t_0$, its propagation and the associated responses (ACK/NACK) from the receiver are assumed to be processed in time for a new packet to be transmitted at time $t = t_0 + N$*í*. Since the emphasis in this report is on state transition diagrams, timing diagrams are not provided, but they can be found in most reports where they form the basis of analysis. In this analysis, the probability of a packet being received without errors at the receiver is taken as *p* while the probability of being in error is $q = 1 - p$. In keeping with common notation, the transmitter sends an ACK to the transmitter if a received packet is not in error and a NACK otherwise. Also, in the SR-ST and SR-GBN schemes, the transmitter switches to the ST or GBN modes after *v* consecutive SR-mode retransmissions of a failed packet have occurred. Similarly, in the SR-ST-GBN scheme, the transmitter switches to the GBN mode after *u* consecutive ST-mode retransmissions of a failed packet have occurred. In order to aid the derivations in this analysis, we use the following sample values of the parameters in the transition diagrams of the schemes: $N = 3$, $v = 2$ and $u = 2$.

We start the analysis with the SW scheme in which whenever the transmitter transmits a packet, it must first wait for the receiver response. If an ACK is received, the transmitter sends a new packet, otherwise it resends the NACKed packet. The operation of the SW scheme is represented by the transition diagrams in Fig. 1. Assuming that the transmitter is in state 0a and has just received an ACK from the receiver, it transits to state 0b with probability *p*.

This transition takes time í during which a new packet is also transmitted. The transmitter must then wait for the next response from the receiver, which takes the round trip delay time (*N-1)í*, and corresponds to the transition from state 0b to 0a. On the other hand, if the transmitter receives a NACK in state 0a, it transits to state 1b with probability *q*, taking time *í* and retransmitting the failed packet. Thereafter, it takes the round trip delay time (*N-1)í* to transit to state 1a, in time to receive the next response from the receiver. If that response is an ACK, the transmitter transits to state 0b with probability *p*, taking time *í* and transmitting a new packet. If the response is a NACK, however, the transmitter í transits to state 1b with probability *q*, taking time *í* and retransmitting the failed packet. The transition diagram in Fig. 1(a) can be represented by its z-transform equivalent as shown in Fig. 1(b) where state 0 in Fig 1 (b) corresponds to state 0a in Fig. 1(a) and state 1 to state 1a. Thus, as long as there are no packet failures the transmitter stays in state 0. If a packet fails, however, the transmitter goes to state 1 and does not return to state 0 until the failed packet has been successfully transmitted. The main states in this diagram and in the subsequent diagrams for the other ARQ schemes are therefore 0 and 1.



(a) (b)

Fig. 1: The transition diagram of the SW scheme.

We can obtain the transfer function from state 0 to 1 in Fig. 1(b) when the transmission *fails* as

$$\Phi_f(z) = \frac{q\, z^N}{1 - p\, z^N} \qquad (1)$$

and that from state 1 to 0 when there is a *recovery* from failure as

$$\Phi_r(z) = \frac{p\, z^N}{1 - q\, z^N} \qquad (2)$$

We can then obtain the transition time from state

0 to 1 as

$$L_f = \frac{d}{dz}\Phi_f(z)\bigg|_{z=1} = \frac{N}{q} \tag{3}$$

and that from state 1 to 0 as

$$L_r = \frac{d}{dz}\Phi_r(z)\bigg|_{z=1} = \frac{N}{p} \tag{4}$$

with the throughput efficiency given by

$$\eta = \frac{L_f}{(L_f + L_r)} = p \tag{5}$$

Noting that in one cycle time (*N* í seconds) only one packet is transmitted, the throughput in terms of packets is given by

$$\eta = \frac{L_f}{(L_f + L_r)N} = \frac{p}{N} \tag{6}$$

We also note that in digital communication, the raw data bits are often encoded in order to enable the receiver to both detect and correct errors in the received information. The encoding involves adding extra redundant but carefully chosen bits to each *k* data bits to produce a unique *n*-bit code. The set of codes thus generated is termed a rate (*n*,*k*) code. Therefore, although the transmission of extra redundant bits with the information lowers the throughput efficiency, it increases the channel utilisation because information that is corrupted is corrected at the receiver instead of being re-transmitted. Such encoding leads to the throughput in terms of bits as

$$\eta = \frac{L_f k}{(L_f + L_r)Nn} = \frac{kp}{Nn} \tag{7}$$

a result which can be obtained from (4) in Lin *et al*. (1984), and which reduces to (6) above when *k* = *n* which signifies the absence of encoding for error control.

In the SR scheme, contrary to the SW scheme, the transmitter transmits a packet in each slot. If it receives a NACK for any packet *i*, it only retransmits that packet and continues to send new packets. The transition diagram of the SR scheme is therefore given in Fig. 2 for *N* = 3. In this diagram, as in subsequent diagrams, squared states represent cases where ACK/NACK decisions for the packet of interest are made while circled states represent cases involving other packets. Therefore, if a packet of interest fails (with probability *q*) in state 0, the transmitter transits to state 1 while retransmitting that packet. It then transits to states 2 and 3 transmitting other different packets in each case. In state 3, if the transmitter receives an ACK (with probability *p*) for the packet of interest, it transits to state 0, otherwise if it receives a NACK, it transits to state 1 while retransmitting the packet of interest. It then transmits two other but different packets in states 1 and 2 before returning to state 3. Note therefore that the loop in state 3 involves transmitting the packet of interest and *N*-1 other packets, but as pointed out before, only the transitions involving the packet of interest bear the *z* parameter.

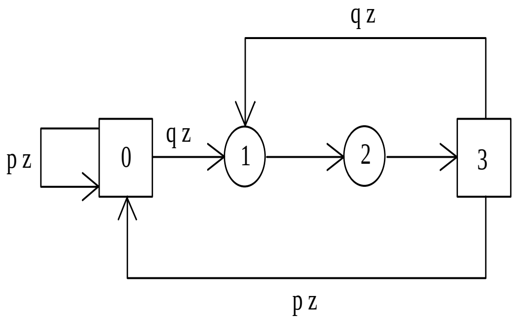


Fig. 2: The transition diagram of the SR scheme with *N* = 3.

The attendant transfer functions are then given by

$$\Phi_f(z) = \frac{qz}{1 - pz} \qquad \Phi_r(z) = \frac{pz}{1 - qz} \tag{8}$$

Using the procedure applied in the SW scheme, we obtain

$$L_f = \frac{1}{q} \qquad L_r = \frac{1}{p} \tag{9}$$

and

$$\eta = \frac{L_f}{L_f + L_r} = \frac{p}{p+q} = p \tag{10}$$

In the GBN scheme, as in the SR scheme, the transmitter transmits new packets continuously in each slot while checking the receiver response. If the transmitter receives a NACK for any packet *i* (transmitted *N* packets earlier), it resends packet *i* and all the subsequent *N*-1 packets that were sent after packet *i*, and repeats resending them until it receives an ACK for packet *i*. When it receives an ACK, it continues to transmit new packets from where it left off. The transition diagram of the GBN scheme is given in Fig. 3. Note that the difference between the transition diagram in Fig. 3 and that in Fig. 2 is the requirement for the transmitter to transmit the same *N* packets each time the packet of interest fails in the GBN scheme. For this reason, these transmissions consume *N*-1 extra slots each time a packet of interest fails and these must be taken into account, hence the *z* parameter after states 1 and 2 in Fig. 3.

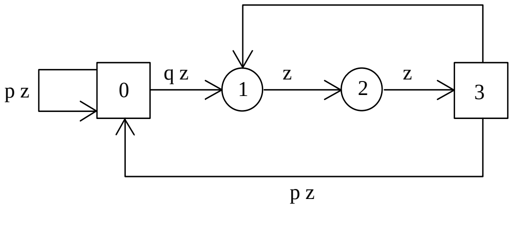


Fig. 3: The transition diagram of the GBN scheme with *N* = 3.

The transfer functions for this scheme are given by

$$\Phi_f(z) = \frac{qz}{1-pz} \qquad \Phi_r(z) = \frac{p\,z^N}{1-q\,z^N} \tag{11}$$

and the other parameters as

$$L_f = \frac{1}{q} \qquad L_r = \frac{N}{p} \tag{12}$$

and

$$\eta = \frac{L_f}{L_f + L_r} = \frac{p}{p+Nq} = \frac{p}{1+(N-1)q} = \frac{p}{1+Sq}. \tag{13}$$

In the ST scheme, the transmitter also transmits continuously but when it receives a NACK for any packet *i*, it resends that packet continuously until it receives an ACK for it. When it receives the ACK, it sends new packets from where it left off. The transition diagram for the ST scheme is therefore as given in Fig. 4. Comparing this diagram with that of the SR scheme in Fig. 2, it is seen that in the ST scheme all transitions after state 1 involve transmitting the packet of interest, and therefore all transitions bear the *z* parameter unlike in the SR scheme.

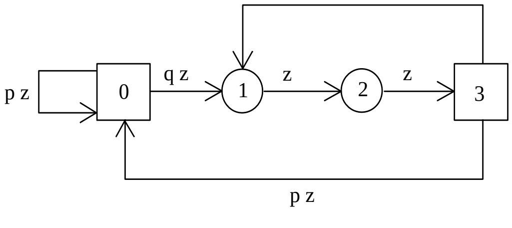


Fig. 4: The transition diagram of the ST scheme with *N* = 3.

The transfer functions for the scheme are then given by

$$\Phi_f(z) = \frac{qz}{1 - pz}$$

$$\Phi_r(z) = z^{N-1}\frac{pz}{1 - qz} = \frac{pz^N}{1 - qz} \tag{14}$$

and the other parameters by

$$L_f = \frac{1}{q}$$

$$L_r = \frac{1 + (N - 1)p}{p} \tag{15}$$

and

$$\eta = \frac{p}{p + \{1 + (N-1)p\}q} = \frac{p}{1 + (N-1)pq} = \frac{p}{1 + Spq}. \tag{16}$$

In the SR-ST1 scheme, the transmitter normally operates in the SR mode. If it receives v+1 consecutive NACKs for any packet i however, it switches to the ST mode and transmits packet i continuously until it receives an ACK for it. The transition diagram of the SR-ST1 scheme with $v = 2$ is given in Fig. 5. Using the descriptions given for the previous schemes it can be seen that if a packet (of interest) fails, the transmitter applies SR mode from state 0 to 3, and if a second failure occurs, it applies SR mode from state 3 to 6. If there is a third (v+1) failure, however, then the transmitter applies ST mode from state 6 to 9.

We therefore obtain the transfer functions as

$$\Phi_f(z) = \frac{qz}{1 - pz}$$

$$\Phi_r(z) = \sum_{m=0}^{v-1}(qz)^m pz + \frac{(qz)^v p z^N}{1 - qz} \tag{17}$$

which lead to

$$L_f = \frac{q}{(1-p)^2} = \frac{1}{q}$$

$$L_r = \sum_{m=0}^{v-1}(m+1)q^m p + \frac{q^v[1 + p(N+v-1)]}{p} = \frac{1 + (N-1)q^v p}{p} \tag{18}$$

and

$$\eta = \frac{p}{1 + (N - 1)q^{v+1}p} = \frac{p}{1 + S\,q^{v+1}p}. \tag{19}$$

In the SR-ST2 scheme, the transmitter uses a flag to control its operation and normally operates in the SR mode. It remains in the SR mode with the Flag-Not-Set (FNS) as long as the number of NACKs for any packet does not exceed v. If it receives v + 1 consecutive NACKs for any packet i, however, it sets the flag (FS) and switches to the ST mode. It then applies ST transmission to that packet and subsequently to all failed packets until they have all been received correctly. After that, the transmitter reverts to the FNS/SR mode. The transition diagram of the SR-ST2 scheme is given in Fig. 6 showing the paths

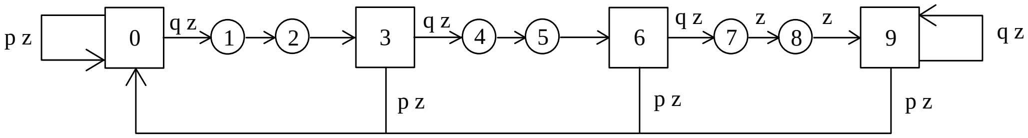


Fig. 5: The transition diagram of the SR-ST1 scheme with *N* = 3, *v* = 2.

taken depending on whether the transmitter is in the FS or FNS mode when the packet of interest fails. Thus, when the transmitter receives the first NACK for the packet of interest, it transits to state 1. If it is already in the FS mode, it applies ST mode to the packet of interest to state 11, otherwise it applies SR mode to state 3. If a second failure is detected (in state 3) and the FS condition is valid, the ST mode is applied to state 13, otherwise the SR mode is applied to state 6. If a third (v+1) failure is detected in state 6, the transmitter applies ST mode to state 9. Note that in this case the transmitter also sets the flag to the FS condition.

In order to determine P_{NN}, the probability that the transmitter is in the FS mode when the packet of interest fails, we proceed as follows.

Pr{a packet has suffered v+1 consecutive failures} = q^{v+1}.

Pr{a packet has not suffered v+1 consecutive failures} = 1-q^{v+1}

Pr{(N-1) packets have not suffered v+1 failures} = $(1\text{-}q^{v+1})^{N\text{-}1.}$

Pr{any of the (N-1) packets has suffered v+1 failures} = 1-(1-qv+1)N-1=P_{NN}.

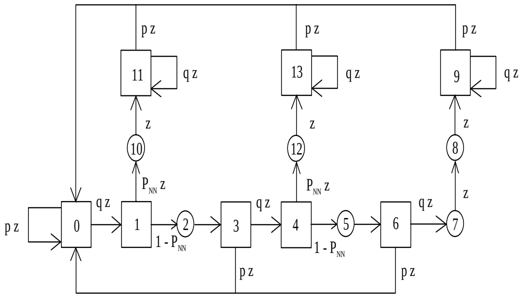


Fig. 6: The transition diagram of the SR-ST2 scheme with N = 3, v = 2.

Note that this value is the same as that given by (7) in Miller and Lin (1981). The transfer functions for the SR-ST2 scheme are then given by

$$\Phi_f(z) = \frac{qz}{1\text{-}pz}$$

$$\Phi_r(z) = \sum_{m=0}^{v\text{-}1}\{(1\text{-}P_{NN})qz\}^m \frac{P_{NN}pz^N}{1\text{-}qz} \qquad (20)$$

$$+\sum_{m=0}^{v\text{-}1}\{(1\text{-}P_{NN})qz\}^m\left(1\text{-}P_{NN}\right)pz + \frac{\{(1\text{-}P_{NN})qz\}^v pz^N}{1\text{-}qz}$$

from which we obtain

$$L_f = \frac{q}{(1-p)^2} = \frac{1}{q}$$

$$L_r = \frac{\begin{bmatrix}\{\{q+(N-1)p\}\{1-pq^v(1-P_{NN})^v\}\}P_{NN} \\ +\{(N-1)p^2+P_{NN}qp\}q^v(1-P_{NN})^v+p\end{bmatrix}}{(p+P_{NN}q)p} \qquad (21)$$

and subsequently

$$\eta = \frac{p(p+P_{NN}q)}{(p+P_{NN}q)+(N-1)pq\{P_{NN}+pq^v(1-P_{NN})^{v+1}\}}. \qquad (22)$$

Note that when P_{NN} = 0, the transition diagram of Fig. 9 reduces to that of Fig. 5 and (22) reduces to (19) for the SR-ST1 scheme. Likewise, when v = 1 (22) reduces to (8) in Miller and Lin (1981).

In the SR-GBN scheme, the transmitter normally operates in the SR mode. If it receives $v+1$ consecutive NACKs for packet i, however, it switches to the GBN mode thus transmitting packet i and the N-1 packets that followed packet i in the most current transmission continuously until it receives an ACK for it. The descriptions given for the previous schemes can be easily used to track the path taken by the transmitter in the transition diagram given in Fig. 7.

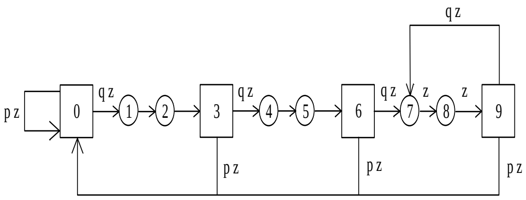


Fig. 7: The transition diagram of the SR-GBN scheme with N = 3, v = 2.

The transfer functions are then given by

$$\Phi_f(z) = \frac{qz}{1-pz}$$

$$\Phi_r(z) = \sum_{m=0}^{v-1} (qz)^m pz + \frac{(qz)^v p z^N}{1-q z^N} \tag{23}$$

from which we obtain

$$L_f = \frac{q}{(1-p)^2} = \frac{1}{q}$$

$$L_r = \sum_{m=0}^{v-1} (m+1) q^m p + \frac{q^v p(N+vp)}{(1-q)^2} = \frac{1+(N-1)q^v}{p} \tag{24}$$

and

$$\eta = \frac{p}{1+(N-1)q^{v+1}} = \frac{p}{1+S q^{v+1}}. \tag{25}$$

In the ST-SR-GBN scheme, the transmitter normally operates in the SR mode but when it receives a NACK, it transmits the failed packet *u* times (ST mode) and proceeds to transmit other new packets. If all *u* retransmissions are received in error, the transmitter switches to the GBN mode. The transition diagram of the SR-ST-GBN scheme is given in Fig. 8 where it is seen that if the transmitter receives a NACK for a packet (of interest), it transmits that packet u (= 2) times, thereby transiting to states 1 and 2. It then transmits another different packet and transits to state 3. If the u (= 2) packets transmitted in states 0 and 1 are received in error in both states 3 and 4, then the transmitter applies GBN mode to state 7.

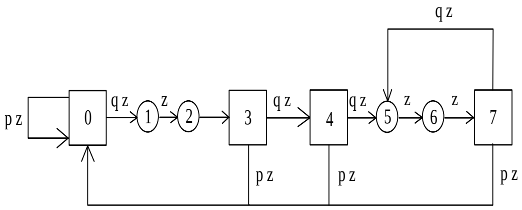


Fig. 8: The transition diagram of the SR-ST-GBN scheme with *N* = 3, *u* = 2.

The transfer functions for this scheme are given by

$$\Phi_f(z) = \frac{qz}{1-pz}$$

$$\Phi_r(z) = z^{u-1} \sum_{m=0}^{u-1} (qz)^m pz + \frac{z^{u-1}(qz)^u p z^N}{(1-q z^N)} \tag{26}$$

from which we obtain

$$L_f = \frac{1}{q}$$

$$L_r = \frac{u(1-q)+q+(N-1)q^u}{p} \tag{27}$$

And

$$\eta = \frac{p}{p+upq+q^2+(N-1)q^{u+1}}. \tag{28}$$

Note that when *u* = 1, the SR-ST-GBN reduces to the SR-GBN scheme with *v* = 1 thus giving

$$\eta = \frac{p}{p+pq+q^2+(N-1)q^2} = \frac{p}{1+(N-1)q^2} = \frac{p}{1+S q^2}. \tag{29}$$

RESULTS AND DISCUSSION

As seen in the previous section, the use of transition diagrams makes it easy to visualize the operation of the ARQ schemes. Although some of the presented transition diagrams can be presented in different ways, the reported ones were found to be most representative. In order to enable comparison with results presented in other reports, in this analysis the packet length has been fixed at 524 bits and the transmitter-receiver round trip delay at *N*-1 = *S* = 129 packet durations. Moreover, the variations of the throughput efficiency with the bit error rate, allowed retransmissions, packet length and round trip delay for the analyzed ARQ schemes have been presented in other reports (Miller and Lin, 1981; Lin *et al.*, 1984), and are not presented in this report. We therefore focus only on the results of the SR-ST2 and SR-ST-GBN schemes in comparison with the other ARQ schemes, and use the term throughput to refer to the throughput efficiency.

We first present the variation of the throughput with *v* for the ARQ schemes in Figs. 9 and 10. We see in Fig. 9 that the GBN scheme has the lowest throughput and the ideal SR (iSR) the highest, both of which do not vary with *v*. It is also seen that the throughput of the SR-ST2 scheme is much lower than that of the SR-GBN and SR-ST1 schemes. In all cases, however, the throughput does not exceed 0.6 and the value of *v* has to approach 10 in order for the throughput of the SR-ST2, SR-GBN and SR-ST1 schemes to approach this value. When the probability of error is decreased to 10^{-4}, the throughput of the GBN scheme increases to just a little above 0.1 while that of the iSR scheme jumps to 0.95. Also, a value of at least 4 is needed for *v* in the SR-ST2, SR-GBN and SR-ST1 schemes in order for the throughput to match that of the ideal SR scheme. Although not shown, it can be deduced from these two results that when the probability of error decreases further the throughputs of the SR-ST2, SR-GBN and SR-ST1 schemes increase towards 1 and thus approach that of the iSR scheme.

The behaviour of the SR-ST-GBN scheme is shown in Figs. 11 and 12, where it is also compared with that of the GBN, SR-GBN and iSR schemes. In these results, the value of $v = u$ was used for the SR-GBN scheme. First, it is seen from Figs. 11 and 12 that the iSR and SR-GBN schemes outperform the SR-ST-GBN scheme. It can then be seen from Fig. 11 that the throughput of the SR-ST-GBN scheme first increases with *u* and then decreases. It can be concluded from the plot that if the SR-ST-GBN scheme is used in channels imparting a bit error rate of 10^{-3}, then the value of *u* should be 6 for maximum throughput. As seen in Fig. 12, however, if the bit error rate is reduced to 10^{-4} then the value of *u* can be reduced to 2, and subsequently to 1 for lower values of the error rate. Although not shown, it was found that for error rates of 10^{-5} and below, the optimum value of *u* for the SR-ST-GBN scheme is 1, which is also the value of *v* at which the SR-ST-GBN scheme transforms into the SR-GBN scheme.

In previous work the synchronization efficiency (which corresponds to the throughput efficiency) of frame synchronized communication systems was used in determining the design parameters for the systems (Kundaeli, 1993; 1998; 2007). The same approach can also be used to determine such parameters for ARQ schemes as exemplified in the SR-ST-GBN scheme. Generally, the design parameters include the packet length, the data transmission rate, the type of ARQ scheme, and the values of *v* and *u*, for given transmitter-receiver round trip delay and probability of bit error in the channel. Note that once the transmission rate has been fixed, the packet length can be chosen to fix the number of packets in a transmission cycle. In this case, the strategy is to select the ARQ scheme having the highest throughput for the given packet length, probability of error, and round trip delay, constrained on *v* and *u*. The performance of ARQ-based communication systems is also governed by the backward channel errors, the packet delay and the buffer size required at the receiver. These factors are very essential in characterising the overall performance of the communication system (Lin and Costello, 2004). Since they have not been included in this analysis, determination of the design parameters without them will not give a true representative picture of the performance. A further analysis to include these effects is therefore necessary. Note also that in the analysis of Ausavapattanakum and Nosratinia (2007), the throughput of the GBN scheme in block fading channels has been analyzed, and the performance of other ARQ schemes in such channels, and with respect to other performance metrics, is still an area of interest.

CONCLUSION

The analysis of ARQ schemes has been undertaken using z-transform based state transition diagrams. The approach has enabled the derivation of easily tractable expressions for the throughput efficiencies of the SW, SR, GBN, ST, SR-ST1, SR-ST2, SR-GBN and SR-ST-GBN

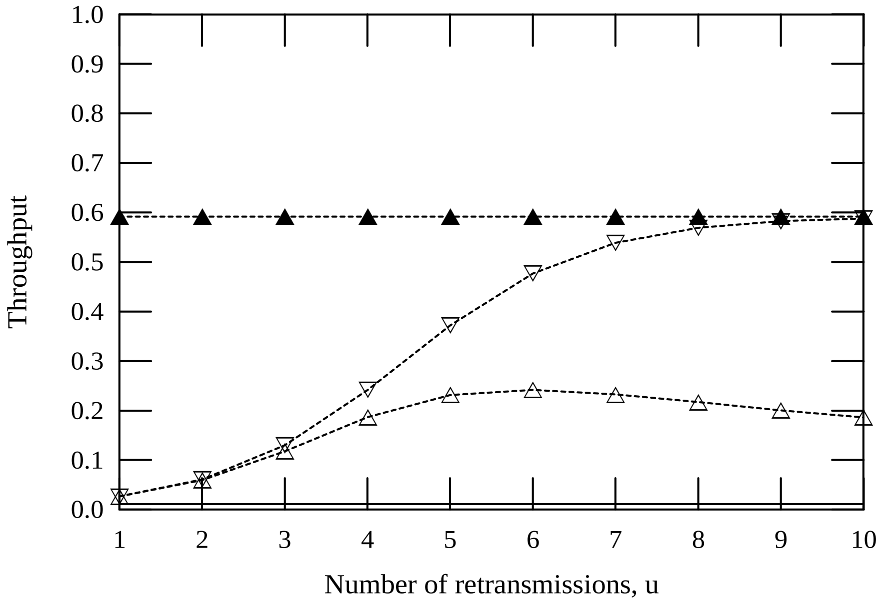


Fig 9: Variation of the throughput with *v* for the GBN (———), SR-ST2 (-△-△-), SR-GBN (-▽- -▽-), SR-ST1 (-▲- -▲-) and iSR (-▼-▼-) schemes at Pe = 10^{-3}.

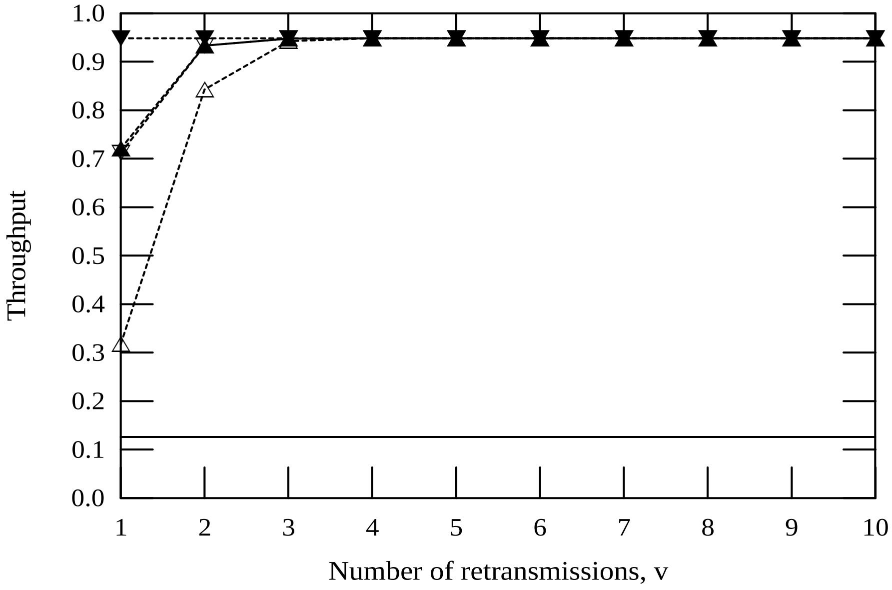


Fig. 10: Variation of the throughput with *v* for the GBN (———), SR-ST2 (-△-△-), SR-GBN (-△- -△), SR-ST1 (-▲- -▲-) and iSR (-▼-▼-) schemes at Pe = 10^{-4}.

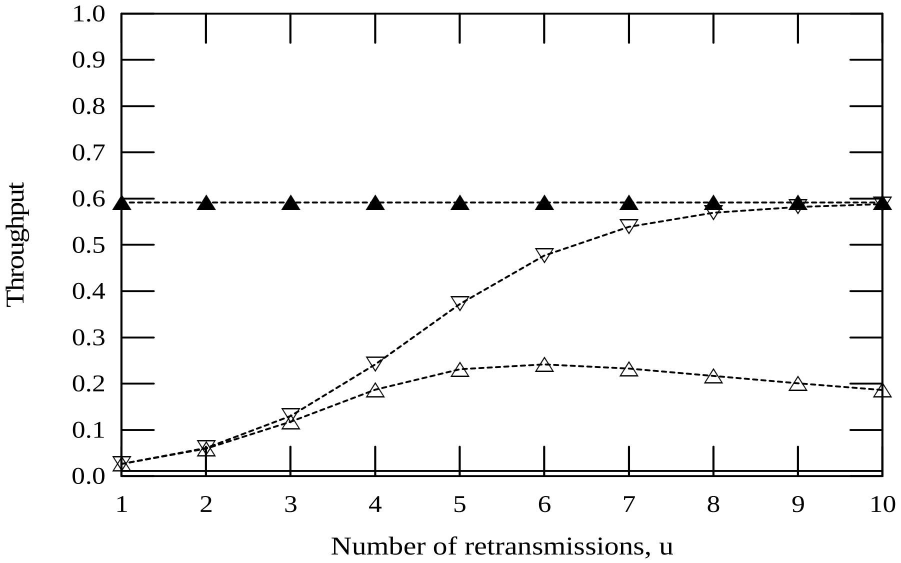


Fig. 11: Variation of the throughput with *u* for the GBN (———), SR-ST-GBN (-△-△-), SR-GBN (-▽- -▽-) and iSR (-▲- -▲-) schemes at Pe = 10^{-3}.

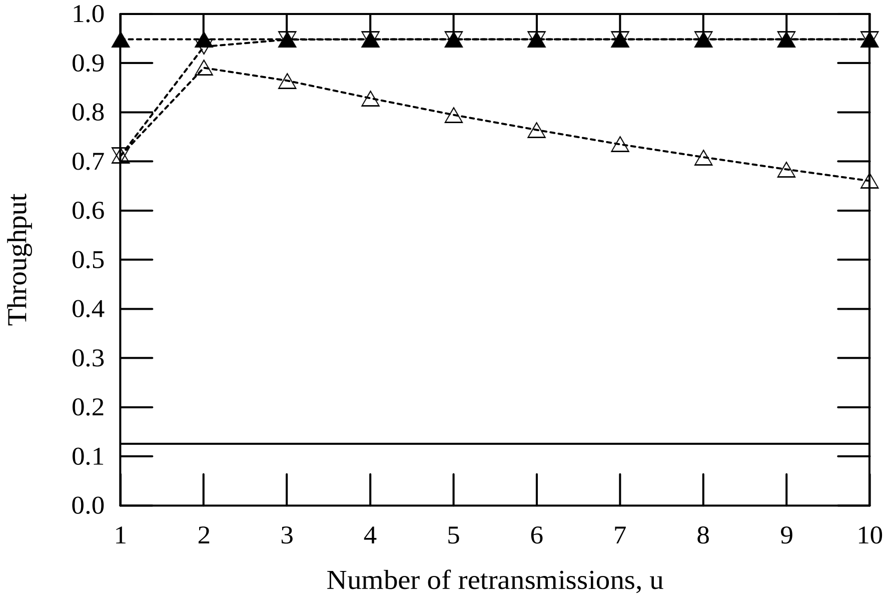


Fig. 12: Variation of the throughput with *u* for the GBN (———), SR-ST-GBN (-△-△-), SR-GBN (-▽- -▽-) and iSR (-▲- -▲-) schemes at Pe = 10^{-4}.

ARQ schemes. Moreover, general performance expressions for the SR-ST2 and SR-ST-GBN schemes were derived. The SR-ST2 scheme was found to outperform the SR-GBN but to be outperformed by the SR-ST1 schemes for all values of allowed retransmissions in the SR mode of operation. The SR-ST-GBN scheme was also found to be outperformed by the SR-GBN for all values of allowed retransmissions in the SR/ST modes of operation. It was also possible to determine the value of *u* required for maximum throughput efficiency for a given bit error rate for the SR-ST-GBN scheme. Using this result and following the approach of earlier work on frame synchronized communication systems, it is possible to determine the design parameters for ARQ-based communication systems. This, however, will need taking into account the effects of the backward channel error, packet delay and required buffer size at the receiver.

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