TECHNICAL NOTE

UNIVERSAL SPECIFIC ENERGY CURVE FOR PARABOLIC OPEN CHANNELS

K.O. Aiyesimoju
Department of Civil Engineering
University of Lagos
Lagos, Nigeria

ABSTRACT

From the general relationship between specific energy and flow depth for all open channels, the specific relationship for parabolic open channels was obtained here. By introducing suitable scaling parameters of the same length dimensions as the specific energy and flow depth, a relationship was obtained between a dimensionless specific energy and a dimensionless flow depth, which is applicable to any parabolic open channel at any discharge. This is presented in both tabular and graphical forms which allows the flow depth to be obtained without iteration, for any given specific energy and discharge in any parabolic open channel.

Keywords: *parabolic channel, specific energy, flow depth*

INTRODUCTION

In steady open channel flow situations where use is to be made of the energy conservation principle (Chow, 1959; Streeter,1971), the engineer often knows what the specific energy should be as well as whether the flow will be subcritical or supercritical. What he needs to know is the flow depth that will yield that specific energy. This requires the use of a specific energy versus flow depth curve because the relationship between specific energy and flow depth is such that although the specific energy is known explicitly in terms of the flow depth, the reverse is not the case. Unfortunately, the specific energy versus flow depth curve depends on the given discharge and cross-sectional shape. Thus for any particular cross-section, a specific curve has to be produced for each given discharge. Alternatively, the depth is obtained by iteration (Chow, 1959). Either of this is time-consuming.

To avoid this in rectangular and triangular open channels, Aiyesimoju (2006, 2007) obtained tabular and graphical relationships between the specific energy and the flow depth which are applicable to these channels irrespective of the discharge, thus allowing the flow depth to be determined for any given specific energy and discharge, without iteration. Other recent work in this area has mostly been concerned with problems involving the specific force rather than the specific energy, as in hydraulic jumps. For example Alhamid and Negm (1996) considered hydraulic jumps in sloping and rough rectangular channels. Mossa (1999) also considered hydraulic jumps in rectangular channels but focused on the oscillating characteristics that accompany the phenomenon.

The aim of this paper is to obtain tabular and graphical relationships between specific energy and flow depth that are universally applicable to all parabolic open channels (illustrated in Figure 1), irrespective of discharge.

DERIVATION OF THE SPECIFIC ENERGY EQUATION FOR PARABOLIC OPEN CHANNELS

The specific energy E for any depth of flow y in any open channel can be written as (see Chow, 1959)

$$E = y + \frac{Q^2}{2gA^2} \qquad (1)$$

where A is the flow cross-sectional area, Q is the discharge and g is acceleration due to gravity. In parabolic open channels (illustrated in Figure 1) by definition, the relationship between flow depth y varies with the square of the water surface width b, i.e.

$$y = cb^2 \qquad (2)$$

where c is constant and is the channel breadth at the water surface for flow depth y

$$A = \frac{2}{3}by$$

(compared to $A = \frac{1}{2}by$ for a triangular open channel i.e. linear y versus b relationship) which from Equation 2 implies

$$A = \frac{2y^{3/2}}{3c^{1/2}}$$

Substituting this in Eqn. 1 yields

$$E = y + \frac{9cQ^2}{8gy^3} \qquad (3)$$

Thus for a given parabolic section (and hence a given c), different values of Q result in different E-y curves.

Each of the terms in Eqn. 3 is of length scale and thus let there be a scaling parameter a. Di-

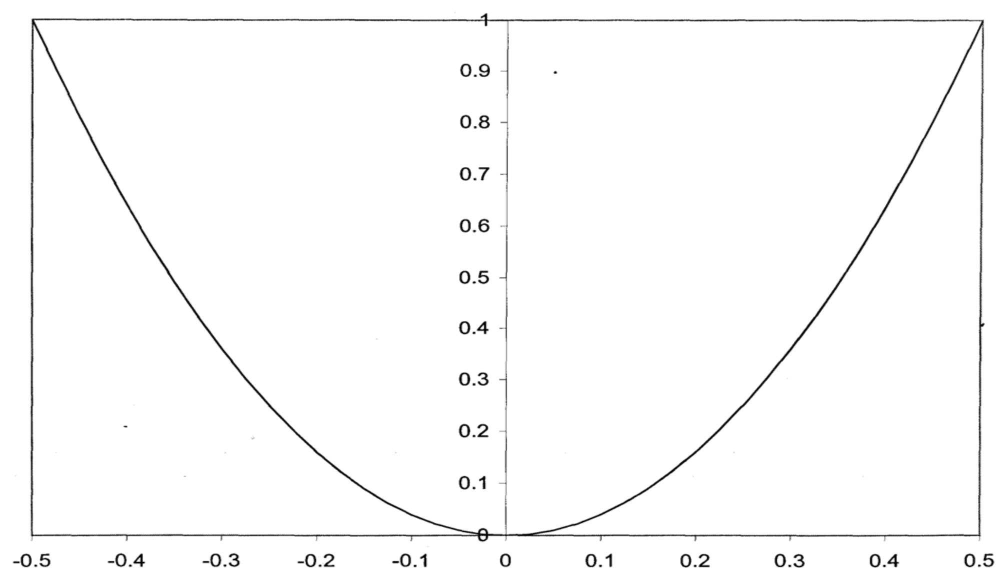

Fig. 1: Illustration of parabolic cross-sectional shape

viding through Eqn. 3 by the scaling parameter yields

$$\frac{E}{a} = \frac{y}{a} + \frac{9cQ^2}{8gay^3}$$

This can be written as

$$\frac{E}{a} = \frac{y}{a} + \frac{9cQ^2}{8ga^4\left(\frac{y}{a}\right)^3} \qquad (4)$$

Since the scaling parameter a is arbitrary, Eqn. 4 is simplified if we choose

$$\frac{9cQ^2}{8ga^4} = 1 \qquad (5)$$

which implies that Eqn. 4 can be written as

$$\frac{E}{a} = \left(\frac{y}{a}\right) + \frac{1}{\left(\frac{y}{a}\right)^3} \qquad (6)$$

Eqn. 5 implies that

$$a = \left(\frac{9cQ^2}{8g}\right)^{1/4} \qquad (7)$$

The critical depth is that at which the specific energy is minimum. Thus differentiating Eqn. 3 with respect to y and setting to zero at the critical depth y_c implies

$$1 + \frac{9cQ^2(-3)}{8gy_c^{\ 4}} = 0$$

which implies (from Eqn. 7)

$$1 - \frac{3a^4}{y_c^{\ 4}} = 0$$

which implies

$$a = y_c 3^{-1/4} \qquad (8)$$

Thus Equation 6 can be rewritten in terms of y_c to give

$$\frac{E}{y_c} = \left(\frac{y}{y_c}\right) + \frac{1}{3\left(\frac{y}{y_c}\right)^3} \qquad (9)$$

c is either known directly or alternatively if the overall channel depth Y corresponds to width B at top of channel, then from Equation 2

$$Y = cB^2$$

which implies

$$c = \frac{Y}{B^2} \qquad (10)$$

Using Eqn. 8 in Eqn. 7 gives

$$y_c = \left(\frac{27cQ^2}{8g}\right)^{1/4} \qquad (11)$$

and substituting Eqn. 10 in Eqn. 11 gives

$$y_c = \left(\frac{27Q^2Y}{8gB^2}\right)^{1/4} \qquad (12)$$

RESULTS AND DISCUSSION

From Eqn. 9, a relationship exists between E/y_c and y/y_c irrespective of the specific value of y_c and thus a plot of E/y_c *versus* y/y_c (say unit specific energy versus unit depth) can be produced.

This has been tabulated in Table 1 and plotted in Figure 2a. Figure 2b is an enlarged version for the larger depths of the supercritical region of Figure 2a i.e.

$$\left(0.5 \le \frac{y}{y_c} \le 1\right)$$

whilst Figure 2c covers the shallower depths (i.e.

$$\left(0.2 \le \frac{y}{y_c} \le 0.5\right)$$

It should be noted y_c in Equation 9 on which the table and figures are based, depends on both c and Q (as in Eqn. 11) and thus accounts for variation of parabolic channel shape as well as variation in discharge. Table 1 and Figures 2a to c are thus applicable to all parabolic open channels at all discharges.

For any parabolic open channel (any given value of c) for which the specific energy E is given and the flow depth y is required for any discharge Q, the critical flow depth y_c can be computed from Eqn. 11 or 12. From the specific energy E, E/y_c can be then be computed and the corresponding y/y_c read from either of Figures 2a, 2b, 2c or Table 1. Once y/y_c is known, flow depth y can then be obtained. Thus the flow depth can be obtained for any given specific energy and any given discharge in any parabolic open channel without iteration or any need to construct a specific energy versus flow depth curve for the specific channel at the given discharge. Aiyesimoju (2006, 2007) allow the same to be done but only for rectangular and triangular open channels respectively.

CONCLUSION

From the general relationship between specific energy and flow depth for all open channels, the specific relationship for parabolic open channels was obtained here. By normalizing the specific energy equation by a scaling parameter a and further obtaining the critical depth in terms of this scaling parameter, a relationship has been obtained between what can be termed unit specific energy E/y_c and unit depth y/y_c which is applicable to all parabolic open channels irrespective of the discharge. This has been tabulated in Table 1 and plotted in Figures 2a to 2c.

Use of either of Figures 2a, 2b, 2c or Table 1 allows the flow depth to be obtained for any given specific energy, without iteration or any

Table 1: Unit Energy versus Unit Depth for parabolic open channels

$\frac{E}{y_c}$	$\frac{y}{y_c}$
10.000	10.00
5.003	5.00
4.504	4.50
4.005	4.00
3.508	3.50
3.012	3.00
2.521	2.50
2.327	2.30
2.136	2.10
1.949	1.90
1.768	1.70
1.599	1.50
1.521	1.40
1.452	1.30
1.393	1.20
1.350	1.10
1.338	1.05
1.335	1.03
1.333	1.00
1.335	0.97
1.339	0.95
1.357	0.90
1.369	0.88
1.393	0.85
1.425	0.82
1.451	0.80
1.540	0.75
1.672	0.70
1.864	0.65
2.143	0.60
2.554	0.55
3.167	0.50
3.494	0.48
3.885	0.46
4.353	0.44
4.919	0.42
5.608	0.40
6.455	0.38
7.504	0.36
8.821	0.34
10.493	0.32
12.646	0.30
15.465	0.28
19.225	0.26
21.583	0.25
24.353	0.24
27.627	0.23
31.525	0.22
36.203	0.21
41.867	0.20

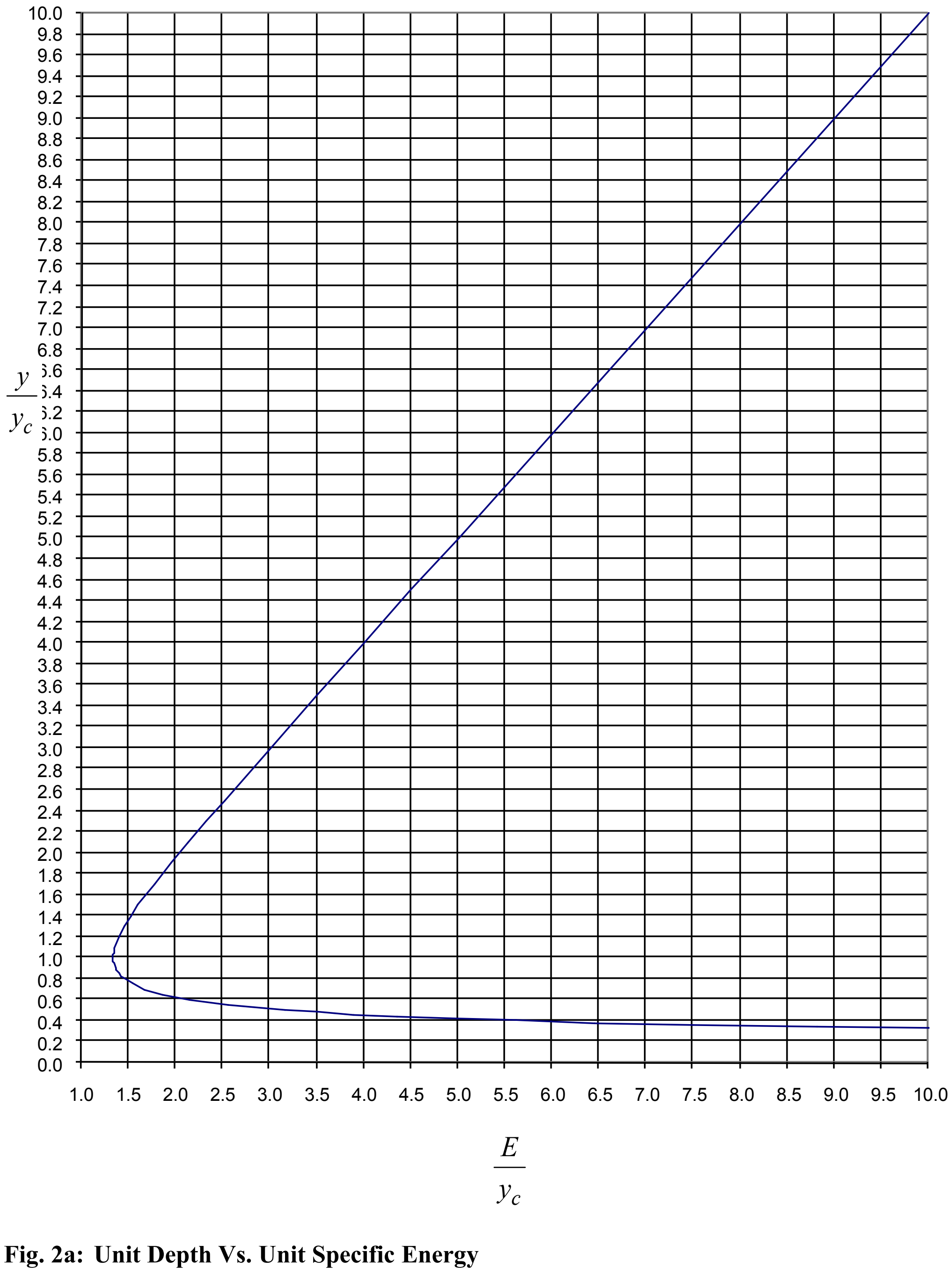

Fig. 2a: Unit Depth Vs. Unit Specific Energy

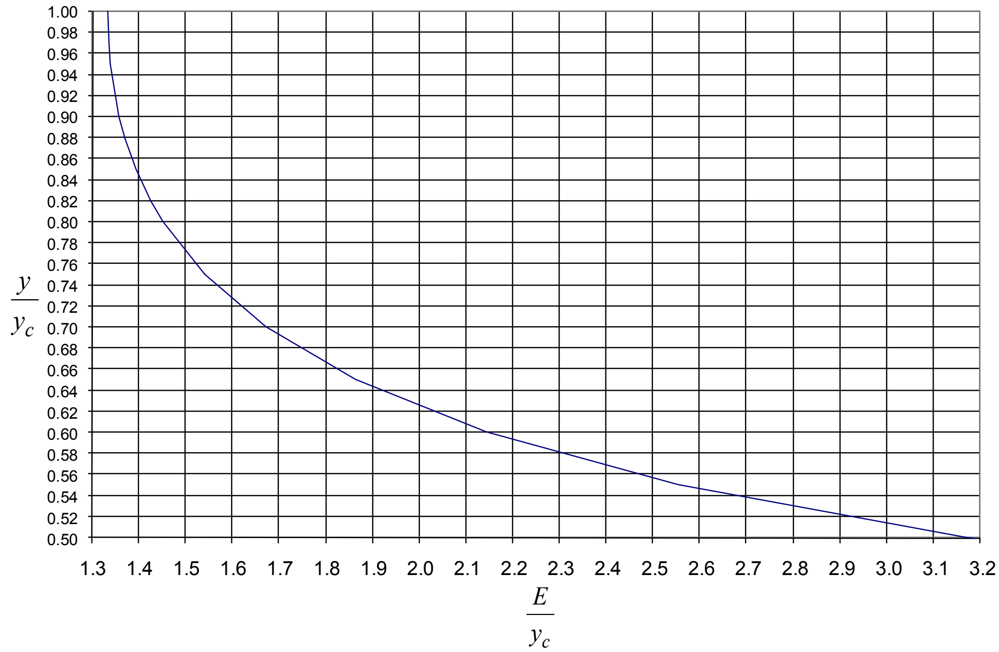

Fig. 2b: Unit Depth Vs. Unit Specific Energy (deep supercritical)

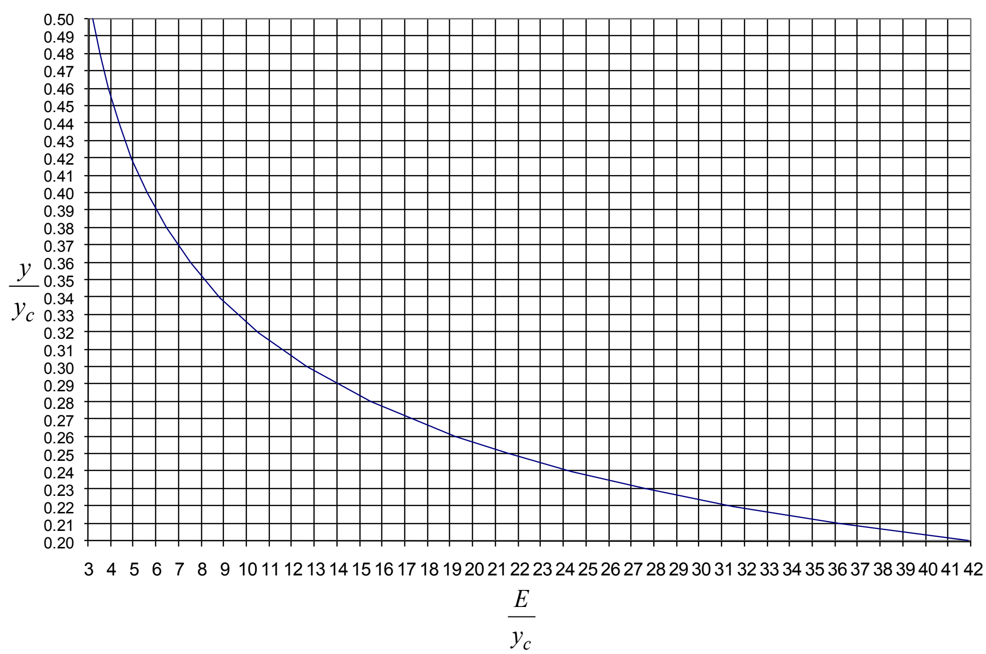

Fig. 2c: Unit Depth Vs. Unit Specific Energy (shallow supercritical)

need to construct a specific energy versus flow depth curve for the specific parabolic channel at the given discharge, as was hitherto the case.

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