TECHNICAL NOTE

ACCURATE, EXPLICIT PIPE SIZING FORMULA FOR TURBULENT FLOWS

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ABSTRACT
This paper develops an explicit formula for computing the diameter of pipes, which is applicable to all turbulent flows. The formula not only avoids iteration but still estimates pipe diameters over the entire range of turbulent flows with an error of less than 4% in the worst cases. This is superior to (without requiring a higher level of difficulty in use) the only two previously developed relationships that are generally applicable to turbulent flows and do not require iteration. Their errors were up to 24% for Ranga Raju's method and up to 23% for Prabhata and Akalank's method. All the errors are relative to the ideal but iterative Colebrook-White formula.

Keywords: *Pipe sizing, turbulent flow, explicit*

INTRODUCTION
The importance of accurate sizing of pipes cannot be overemphasized in pipeline engineering since an overestimation implies a high initial cost of conduit installation whilst underestimation will lead to functional inadequacies.

Laminar flows are governed by the Hagen-Poiseuille equation, which shows a linear relationship between head loss, pipe discharge and pipe diameter. This enables pipe diameters to be estimated accurately and without iteration for any given head loss and discharge. Turbulent flows however are governed by more complex relationships. Aiyesimoju (2002) has compared the accuracy of the Hazen-Williams, Manning and Colebrook-White formulas which are the most commonly used ones of the numerous semi-empirical and fully empirical friction loss relationships for turbulent flows in water supply practice and confirmed that the Colebrook-White formula yields the best results.

The problem is that computation of pipe diameters with the formula involves iteration. Numerous graphical aids such as Moody's Chart (see Streeter, 1971) and Asthana's Diagram (Asthana, 1974) which were developed to simplify using this formula, are however often not available and when they are, the errors introduced in scaling the graphs are often significant.

Aiyesimoju (2008) recently developed an explicit formula for pipe sizing but this was specifically tailored to water distribution pipes, in which case, errors are within 2%. Other recent work in this area has been about assessment or proper use of existing methods. For example, Christensen et al. (2000) was concerned only with the proper use of the Hazen-Williams formula whilst Khatibi et al. (2000) focussed mainly on the estimation of friction factors for flow in nearly flat tidal channels. Although Aiyesimoju (2006) obtained an accurate equation for estimating turbulent head losses in water distribution pipes which avoids iteration, the equation does not avoid iteration if it is to be used to estimate the pipe diameter.

This paper is to develop an explicit formula to compute required pipe diameters, which is useful over the entire range of practical turbulent flow situations. The developed relationship as well as the only two previously developed relationships that are generally applicable to turbulent flows and that do not involve any iteration, will be compared with the most accurate Colebrook-White formula. The two methods are (1) Ranga Raju and Garde equation (Ranga Raju and Garde, 1971) and (2) Prabhata and Akalank equation (Prabhata and Akalank, 1976).

The comparison will be in the context of several examples of flow situations designed to reflect the full range of turbulent flows, as follows:

1. Pipe equivalent roughness height ε between 0.01mm (much less than the value for drawn tubing) and 10 cm (much larger than corresponding to riveted steel).
2. Pipe diameter D between 0.1mm and 50m.
3. Pipe relative roughness e/D up to 0.5 (over 10 times the limit in Moody's Chart).
4. Flow Reynold's Number R above 4000 (lower limit for turbulent pipe flow).
5. Minimum kinematic viscosity $n_{\min}$ of fluid will be taken as 10^{-8} m^2/s (order of magnitude less than benzene's) and maximum $n_{\max}$ will be taken as 10^{-2} m^2/s (order of magnitude greater than glycerol's).

PIPE DIAMETER ESTIMATION METHODS

Colebrook-White Formula

Friction loss in turbulent pipe flow is best estimated by the Darcy-Weisbach formula (Streeter, 1971)

$$S_f = f\frac{V^2}{2gD} \tag{1}$$

where the friction factor f is estimated with the Colebrook-White formula,

$$\frac{1}{\sqrt{f}} = -2\log_{10}\left(\frac{\varepsilon}{3.7D} + \frac{2.51}{R\sqrt{f}}\right) \tag{2}$$

In the above equations, S_f is the friction slope (head loss h over pipe length L), V is flow velocity, D is pipe diameter, g is acceleration due to gravity, ε is the pipe roughness height (a measure of the typical height of protrusions from the pipe material surface and R is pipe Reynold's number. The Colebrook-White formula is based on the results of experiments on sand-roughened pipes (see Streeter, 1971), with appropriate modification of the results to reflect observations of commercial pipes. The well known Moody's Chart is simply a graphical presentation of this. If Q is pipe discharge, n is the fluid kinematic viscosity and g is assumed as 9.8 m/s^2, then

$$V = \frac{4Q}{\pi D^2} \tag{3}$$

and

$$R = \frac{VD}{v} = \frac{4Q}{\pi v D} = \frac{1.273Q}{vD} \tag{4}$$

Eqn. 1 implies

$$f = \frac{2gDS_f}{V^2}$$

Using this in Eqn. 3 gives

$$f = \frac{12.1D^5 S_f}{Q^2} \tag{5}$$

Substituting Eqns. 4 and 5 in Eqn. 2 yields

$$Q = -6.57D^{\frac{5}{2}}\sqrt{S_f}\log_{10}\left(\frac{\varepsilon}{3.7D} + \frac{0.567v}{\sqrt{D^3 S_f}}\right) \tag{6}$$

Ranga Raju and Garde's method
RangaRaju and Garde (1971) proposed the following equations for pipe sizing

a) For $e^3 gS_f/n^2 \leq 10^{-2}$

$$\frac{Q}{\nu\varepsilon}\left[\frac{\varepsilon}{D}\right]^{2.633} = 3.39\left[\frac{\varepsilon^3 gS_f}{\nu^2}\right]^{0.54} \tag{7a}$$

which gives

$$D = \frac{\left[\frac{Q}{\nu}\right]^{0.38}\varepsilon^{0.62}}{1.59\left[\frac{\varepsilon^3 g S_f}{\nu^2}\right]^{0.205}} \tag{7b}$$

b) For $e^3 gS_f/n^2 > 10^{-2}$

$$\frac{Q}{3.479D^{2.5}\sqrt{S_f}} = -2\log_{10}\left(\frac{\varepsilon}{3.7D} + \frac{2.51}{R\sqrt{f_o}}\right)$$

$$\frac{Q}{\nu\varepsilon}\left[\frac{\varepsilon}{D}\right]^{2.633} = 2.85\left[\frac{\varepsilon^3 gS_f}{\nu^2}\right]^{0.502} \tag{7c}$$

which results in

$$D = \frac{\left[\frac{Q}{\nu}\right]^{0.38}\varepsilon^{0.62}}{1.488\left[\frac{\varepsilon^3 g S_f}{\nu^2}\right]^{0.191}} \tag{7d}$$

Prabhata and Akalank's method
Prabhata and Akalank (1976) proposed the following equation

$$D^* = 0.66(\varepsilon^{*1.25} + \nu^*)^{0.04} \tag{8}$$

where

D^* is the non-dimensional diameter $D(gS_f/Q^2)^{0.2}$
ε^* is the non-dimensional roughness $\varepsilon(gS_f/Q^2)^{0.2}$
ν^* is the non-dimensional roughness $\nu(1/gS_fQ)^{0.2}$

Proposed method
Eqn. 2 cannot be solved explicitly for f but experience suggests that the value of f on the left hand side of the equation is not very sensitive to the value of f on the right hand side. This suggests that reasonably accurate friction factors can be obtained by assuming a constant value for the f on the right hand side (say f_o). Thus from Equation 2, an explicit estimate for the friction factor f is

$$\frac{1}{\sqrt{f}} = -2\log_{10}\left(\frac{\varepsilon}{3.7D} + \frac{2.51}{R\sqrt{f_o}}\right) \tag{9a}$$

The best value of f_o over the entire range of flow of practical interest will be determined in this study. Substituting Eqns. 4 and 5 in Eqn. 9a and rearranging, yields

$$\frac{Q}{3.479D^{2.5}\sqrt{S_f}} = -2\log_{10}\left(\frac{\varepsilon}{3.7D} + \frac{2.51}{R\sqrt{f_o}}\right) \tag{9b}$$

To avoid iterating for D, its value on the right hand side of Eqn. 9b will be replaced by an estimate D' thus yielding

$$\frac{Q}{3.479D^{2.5}\sqrt{S_f}} = -2\log_{10}\left(\frac{\varepsilon}{3.7D'} + \frac{2.51}{R\sqrt{f_o}}\right) \tag{10}$$

A good estimate for D' can be obtained assuming a friction factor of f_o in Eqn. 5, thus giving

$$f_o = \frac{12.1D'^5 S_f}{Q^2} \tag{11}$$

which implies D' can be estimated as

$$D' = \left(\frac{f_o Q^2}{12.1S_f}\right)^{0.2} \tag{12}$$

Also, from Eqn. 4, an approximation can be obtained for the Reynolds Number, for use on the right hand side of Eqn. 10, as

$$R = \frac{1.273Q}{\nu D'}$$

Substituting the above in Eqn.10 yields

$$D = \left(\frac{Q}{-6.957\sqrt{S_f}\,\log_{10}\left(\dfrac{\varepsilon}{3.7D'} + \dfrac{1.972\nu D'}{Q\sqrt{f_0}} \right)} \right)^{0.4} \qquad (13a)$$

which from Eqn. 12 can be rewritten as

$$D = \frac{D'}{1.32 f_o^{0.2} \left(-\log_{10}\left(\dfrac{\varepsilon}{3.7D'} + \dfrac{1.972\nu D'}{Q\sqrt{f_0}} \right) \right)^{0.4}} \qquad (13b)$$

TEST PROBLEMS

Aiyesimoju (2002) has designed eight test problems which cover the extreme range of practical flow situations. These are applicable here and are thus adopted. The scenarios (where κ is ε/D) are as shown in Table 1.

The problem in each test case is to determine the pipe diameter for that case by the different methods under consideration. The performance of a method in a test problem will be measured by the factor (relative to Colebrook-White method) within which it is able to estimate the diameter for that test problem.

RESULTS

The specific parameters to ensure the earlier specified range practical turbulent flows are covered by the test problems described in Table 1, are shown in Table 2 for the ideal situation (Colebrook-White formula). For each test problem, the diameters as well as the factors of accuracy (the ratio of the larger of the computed diameter for the method and that by Colebrook-White formula to the smaller of the two) were computed for Prabhata and Akalank method, Ranga Raju and Garde method and the proposed method. These are all shown in Table 3.

DISCUSSION

The factor of accuracy as defined, is unity for a perfect result and the larger it is the less accurate the result is. It is of course never less than unity.

The computed factors of accuracy for the proposed method in Table 3 depend upon the value of f_o assumed (from Equation 13b). Setting the

Table 1: Scenarios of the test problems

Test No	1	2	3	4	5	6	7	8
k	low	low	large	large	low	low	large	large
R	low	large	low	large	low	large	low	large
D	large	large	large	large	small	small	small	small

Table 2: Specific flow parameters for different Test cases

	Colebrok-White (Ideal) Flow Parameters							
Case No.	**Rough height (m)**	**Pipe Diameter (m)**	**Relative Rough**	**Reynold's Number**	**Kinematic Viscosity**	**Discharge**	**Friction Factor**	**Friction slope**
	e	***D***	***k=e/D***	***R***	***n***	***Q***	***f***	S_f
1	1.E-08	50.0	2.0E-10	4.0E+03	1.E-05	1.6E+00	4.0E-02	2.58E-11
2	1.E-08	50.0	2.0E-10	1.0E+08	1.E-05	3.9E+04	5.9E-03	2.42E-03
3	1.E-01	50.0	2.0E-03	4.0E+03	1.E-05	1.6E+00	4.2E-02	2.71E-11
4	1.E-01	50.0	2.0E-03	1.0E+08	1.E-05	3.9E+04	2.3E-02	9.55E-03
5	1.E-08	1.0E-04	1.0E-04	4.0E+03	1.E-05	3.1E-06	4.0E-02	3.24E+06
6	1.E-08	1.0E-04	1.0E-04	1.0E+08	1.E-05	7.9E-02	1.2E-02	6.12E+14
7	5.E-05	1.0E-04	5.0E-01	4.0E+03	1.E-05	3.1E-06	3.3E-01	2.70E+07
8	5.E-05	1.0E-04	5.0E-01	1.0E+08	1.E-05	7.9E-02	3.3E-01	1.69E+16

Table 3: Computed pipe diameters and factors of accuracy for different test cases and different methods

Case No.	Ranga Raju		Prabhata and Akalank		Proposed	
	Pipe Diameter (eqn.7 D)	Factor of Accuracy	Pipe Diameter (eqn.8 D)	Factor of Accuracy	Pipe Diameter (eqn.14 D)	Factor of Accuracy
1	4.46E+01	1.120	4.88E+01	1.024	4.85E+01	1.031
2	4.87E+01	1.027	4.84E+01	1.033	4.84E+01	1.032
3	4.79E+01	1.044	4.88E+01	1.024	4.86E+01	1.029
4	5.16E+01	1.032	5.14E+01	1.027	4.88E+01	1.025
5	9.53E-05	1.050	9.76E-05	1.024	9.70E-05	1.031
6	1.02E-04	1.019	1.00E-04	1.005	9.75E-05	1.026
7	8.22E-05	1.217	8.18E-05	1.223	1.04E-04	1.036
8	8.07E-05	1.240	8.19E-05	1.221	1.04E-04	1.035

value of f_o to 0.125 gave the least value (of 1.036) for the worst factor of accuracy for the eight test problems. The displayed results for the proposed method in Table 3 are for this case. Although a kinematic viscosity coefficient of 10^{-5} m^2/s is shown in the Table 2, variation of this value from 10^{-8} m^2/s up to 10^{-2} m^2/s had no noticeable effect on the results.

A simple, accurate and explicit procedure for estimation of turbulent flow pipe diameters can thus obtained by substituting f_o of 0.125 in Equations 13b and 12 to give the proposed method as

$$D = \frac{1.15D'}{\left(-\log_{10}\left(\frac{\varepsilon}{3.7D'} + \frac{5.7\nu D'}{Q}\right)\right)^{0.4}} \tag{14}$$

where

$$D' = 0.397\left(\frac{Q^2}{S_f}\right)^{0.2}$$

From Table 3, the worst factors of accuracy were 1.240 for Ranga Raju's method, 1.223 for Prabhata and Akalank's method and 1.036 for the proposed method. Thus even in these extreme cases, the proposed method estimated diameters with less than 4% error as compared with up to 24% for Ranga Raju's method and up to 23% for Prabhata and Akalank's method.

CONCLUSION

An explicit formula has been developed to compute required pipe diameters for any given friction slope and discharge, without iteration and which is useful over the entire range of practical turbulent flow situations. The developed relationship (Equations 14 and 15) as well as the only two previously developed relationships that are generally applicable to turbulent flows and that do not involve any iteration (Ranga Raju's method and Prabhata and Akalank's method), were compared with the most accurate but iterative Colebrook-White formula.

Even in the extreme cases, proposed method estimated diameters with less than 4% error as compared with up to 24% for Ranga Raju's method and up to 23% for Prabhata and Akalank's method.

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