MODELLING OF HYDROPOWER SYSTEMS OF GHANA AT LOW LEVEL CONDITIONS OF AKOSOMBO DAM

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ABSTRACT
This paper presents a model of Hydropower systems in Ghana. It deals with conditions when the water level of the Akosombo dam is low. There are two hydropower plants in Ghana located at Akosombo and Kpong. The two plants are along Volta River with the Akosombo plant upstream of the Kpong plant. Thus, the discharge from the upstream plant creates the head for the downstream plant. The Akosombo plant is designed to operate at a minimum level of 75.59m (248ft). It has, on a number of occasions, recorded low elevation in the dam (i.e. below 75.59m). At the low level conditions, the two plants are mostly coupled than independently run. A model has been developed within these low elevation conditions. It was found that with average efficiencies of 84.5% and 93.5% at Akosombo and Kpong plants respectively, the model fits the system.

Keywords: *Hydropower, Low elevation, Model, Head, Simulation.*

INTRODUCTION
Ghana has a hydropower system made up of two plants and managed by Volta River Authority (VRA). The hydropower plants are located at Akosombo and Kpong and were commissioned in 1965 and 1982 respectively. The two stations account for 1,072 megawatts (MW) out of a total national power-generating capacity of 1,652 MW, with Akosombo providing 912 MW while Kpong provides 160MW. The plants are located on the Volta River with Akosombo at the upstream of Kpong.

The Akosombo plant consists of six turbine-generator units with original total design capacity of 912MW and operates under normal condition between 84.15m (276ft) maximum and 75.59m (248ft) minimum of headwater elevation. The Kpong plant comprises of four turbine-generator units with 40MW capacity per unit making a total of 160MW. The normal water level for operation of the Kpong plant is between 17.70m maximum and 14.50m minimum. The available head for the running of Kpong plant is built up from the discharge of Akosombo plant.

Since the installation of the Hydropower plants at Akosombo and Kpong, there have been re-

cords of low water level periods at the Akosombo dam, which were below the minimum design condition. Those periods included the unprecedented drought of 1982-83 with a record of 71.86m on 12th June 1984 (VRA, 1996), which compelled the rationing of electricity until 1986. In August 1994, the level of Volta Lake was 72.99m. The year 1998 also recorded a low water level of 72.21m. These levels were well below the 72.59m, which is the minimum level for generating power without risk of damaging the turbines. Figure 1 is a graph of headwater levels from the periods of 1982 to 1999 with *Monthly average.*

The objective of the paper is to develop a mathematical model for the hydropower systems in Ghana for the low (less than 75.59m) headwater elevation conditions of Akosombo plant reservoir.

HYDROELECTRIC PLANT MODEL
Let us consider a hydroelectric plant and focus on some overall aspect of the falling water as it travels from the reservoir through the penstock to inlet gates through the hydraulic turbine down the draft tube and out the tailrace at plant exit. The power that the water can produce is equal to the rate of the water flow times a conversion coefficient that takes into account the net head (the distance through which the water falls less the losses in head caused by the flow) times the conversion efficiency of the turbine generator. Conversion efficiencies of turbine generators are typically in the range of 85 to 90% at the best efficiency operating point for the turbine generator (Wood and Wollenberg, 1984).

The amount of energy available in a unit of stored water is equal to the product of the weight of the water stored times the height that the water will fall.

Power = (weight of the water stored)
(height of fall)   (1)

Height of fall or Nominal Head = Headwater level in the reservoir [m] - Tailwater Level [m]

$$P = \eta \rho g Q H \ [W] \tag{2}$$

This can be expressed as:
$$P = f(Q, H) \tag{3}$$

where:
η = total efficiency
Q = discharge in m^3/s
H = head in metre
g = acceleration due to gravity (equal 9,81 m/s^2)
ρ = density of water (kg/m^3)

Turbines are operated with sufficiently high efficiencies that under most favorable operating conditions fall within 0.94 - 0.95; under maximum load conditions turbine efficiencies are within 0.88 - 0.93 (Wood and Wollenberg, 1984).

For a hydroelectric power plant with 'n' number of turbine-generator units, total power, P_t, is given as

$$P_t = n \ \eta \rho g \ q \ H \tag{4}$$
$$P_t = f(n, q, H) \tag{5}$$

Where 'q' is the discharge per turbine for 'n' turbines.

HYDROPOWER SYSTEM OF GHANA
The Hydropower system in Ghana consists of two plants located at Akosombo and Kpong on the Volta River in a cascaded manner. The Akosombo plant is at the upstream. Figure 2 is the schematic representation of the plants. The plant parameters are as given in Tables 1 and 2 respectively for the Akosombo and Kpong plants.

The head for the Kpong plant is created mainly out of the discharge from the Akosombo plant making the number of operating units subject to amount of discharge or number of units running at the Akosombo plant.

When the head water level is below 75.59m Akosombo plant may be operated up to 71.93m. At this level there is the possibility of formation of vortex with air entrainment at the intake. The 71.93m level may be considered as a critical level.

Table 1: Akosombo plant parameters

Description	Parameter
Maximum operating level	84.15m (276ft)
Minimum operating level	75.60m (248ft)
Max. annual inflow ever recorded (1963)	3049m³/s
Min. annual inflow ever recorded (1983)	288m³/s
Number of units (Turbines)	6
Penstock diameter (each)	7.2m
Rated Discharge	263.6m³/s
Maximum power output per unit 1 to 4	147 MW
Maximum power output per unit 5 & 6	162MW
Lake area	8502km²

The Volta River Authority (June, 1996)

Table 2: Kpong Plant Parameters

Description	Parameter
Maximum operating level	17.70m
Minimum operating level	14.50m
Number of units (Turbines)	4
Maximum Discharge	384.9m³/s
Nominal power output per unit	40MW
Head pond area	12km²
Head pond volume	190000km³

The Volta River Authority (June, 1996)

SYSTEM MODELLING ASSUMPTIONS

The following assumptions are made in modelling of the system:

- Inflow to the pond of Kpong plant is solely dependent on discharge from Akosombo plant. This assumption is made because it would not be possible to spill water when the level is below the design level and that inflow due to rainfall taken to be negligible.

- Out flow from the Kpong pond is only through the turbine. This is because the pond water level is not expected to rise to cause a spill of water.

- Power outputs and flows through the turbines at each plant are the same for each unit.

Tail water levels remain constant within the period of estimate.

MODELLING

For a hydropower plant with 'n' number of identical units, from equation (4)

$$P_t = n\rho g q H \eta$$

Taking $g = 9.81 kg/m^2 s^2$, $\rho = 980 kg/m^3$ and $\eta = 0.93$

$$P_t \propto nqH \qquad (5)$$

Let subscript 'a' and 'k' refer to Akosombo and Kpong plants respectively (figure 3),

then

$$P_{ta} \propto n_a q_a H_a \qquad (6)$$

$$P_{tk} \propto n_k q_k H_k \qquad (7)$$

Where $n_a = 1, 2, ..., 6$ and $n_k = 1, 2, ..., 4$

but since H_k depends on Q_a then

$$H_k = f(n_a q_a) \quad \text{or} \quad H_k = f(Q_a)$$

Where $Q = nq$

$$\text{Head, } H = L_1 - L_2 \qquad (8)$$

Where L_1 = head water level

L_2 = tail water level

Therefore

$$H_a = L_{a1} - L_{a2} \tag{9}$$

$$H_k = L_{k1} - L_{k2} \tag{10}$$

$$P_a = \rho g \eta_a Q_a H_a \tag{11}$$

$$P_a = \gamma Q_a \tag{12}$$

Where $\gamma = \rho g \eta_a H_a$ for a given H_a

and

$$P_k = \rho g \eta_k Q_k H_k \tag{13}$$

The total power, P, is therefore

$$P = P_a + P_k$$

$$\Rightarrow \quad P = \rho g(\eta_a Q_a H_a + \eta_k Q_k H_k) \tag{14}$$

with $H_k = f(Q_a)$
From the diagram (Figure 4)

$$L_1 = L'_1 + x \tag{15}$$

Where 'x' is change in water level per unit time

$$Q_{in} = Q_{out} + \frac{Ax}{t} \tag{16}$$

and
where 'A' is the pond area in m^2

$$x = \frac{(Q_{in} - Q_{out})t}{A} \tag{17}$$

$\Rightarrow$ Therefore, putting (17) into (15) for time period 't' we have

$$L_1 = L'_1 + \left(\frac{Q_{in} - Q_{out}}{A} \right) t \tag{18}$$

Hence applying to the Kpong plant,
$$H_k = L_{k1} - L_{k2} \tag{19}$$

$$H_k = L'_{k1} + \left(\frac{Q_a - Q_k}{A_k} \right) t - L_{k2} \tag{20}$$

$\Rightarrow$

$$H_k = \left(L'_{k1} - L_{k2}\right) + \left(\frac{Q_a - Q_k}{A_k}\right)t$$

(21)

Putting (21) into (13)

$$\Rightarrow \quad P_k = \rho g \left\{ \eta_k Q_k \left(\left(L'_{k1} - L_{k2}\right) + \left(\frac{Q_a - Q_k}{A_k}\right)t \right) \right\}$$

$$\Rightarrow \quad P_k = \rho g \left(\left(L'_{k1} - L_{k2}\right)\eta_k Q_k + \frac{\eta_k t Q_a Q_k}{A_k} - \frac{\eta_k t Q_k^2}{A_k} \right)$$

(22)

$$\Rightarrow \quad P_k = \frac{\rho g \eta_k}{A_k} \left\{ A_k\left(L'_{k1} - L_{k2}\right) + t Q_a \right] Q_k - t Q_k^2 \right\}$$

$$\Rightarrow \quad P_k = \frac{\rho g \eta_k}{A_k} \left[A_k\left(L'_{k1} - L_{k2}\right) + t Q_a \right] Q_k - \frac{\rho g \eta_k t}{A_k} Q_k^2$$

For a given time, t, and discharge, Q_a

$$P_k = \alpha Q_k - \beta Q_k^2$$

(23)

where

$$\alpha = \frac{\rho g \eta_k}{A_k} \left[A_k\left(L'_{k1} - L_{k2}\right) + t Q_a \right]$$

(24)

$$\beta = \frac{\rho g \eta_k t}{A_k}$$

and

(25)

Now, Total power P is the sum of P_a and P_k

Therefore $P = P_a + P_k$

Hence $P = gH_aQ_a + aQ_k - bQ_k^2$

or $P = gn_aH_aq_a + an_kq_k - bn_k^2q_k^2$

SYSTEM SIMULATION AND RESULT

The simulation of the model equations of the system was down for different efficiencies. A computer program is developed in C++ code for the simulation. The model equation simulated in sequential with the two components treated individually.

The simulated graph, figures 5 to 8, compared with the operation data shows that Akosombo plant is operating within the low level at an ap-

proximate average efficiency of 84.5% and with this efficiency the operation fit the Akosombo plant component of the model equation. Also from the figures, the simulated graph as compared with the operation data shows that Kpong plant is operating at an approximate average efficiency of 93.5%. This value of efficiency indicates that the plant is operating within the design head range. This also indicates that the model equation fits the plant.

The corresponding model is therefore an approximate model for the VRA hydropower system.

CONCLUSION AND RECOMMENDATION

When the headwater elevation of the Akosombo Dam is below 75.59m (248ft), the running of the system comprising the Akosombo plant and the Kpong plant needed to be monitored. Within these low level conditions a model is developed. The model fits the hydropower system with average efficiencies of 84.5% and 93.5% at Akosombo and Kpong plants respectively. The model as seen is therefore an approximate representation of the system.

The model is recommended for use in the studies regarding the hydropower system in Ghana under the low elevation of the Akosombo headwater. Such study may include optimisation of the power generation. A further study may be carried out with regards to the assumptions.

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