APPROXIMATE MODELS FOR FLOOD ROUTING

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ABSTRACT

For rapid calculation of the downstream effects of the propagation of floods due to the collapse of a dam or intense rainfall, two approximate flood routing models are presented in this paper. A nonlinear technique for flood routing is developed that respects the physics of floods in waterways. The models are developed based on the assumption that the change in flow rate relative to the initial uniform flow rate is moderate. A nonlinear kinematic model and a nonlinear convection-diffusion model are extracted from a normalized form of the St. Venant equations, and applied to the analysis of translatory waves usually encountered in rivers and water-courses due to the propagation of floods. The qualitative analyses presented shows that the approximate flood routing models could be used instead of the complete hydrodynamic equations when the perturbation in the initial normal flow condition is moderate.

Keywords: *approximate models, nonlinear kinematic model, nonlinear convection-diffusion model, St. Venant equations, and watercourses*

INTRODUCTION

Historically civilization has been known to occur along riverbanks. This is due to the easy access to water for domestic, industrial, and agricultural purposes. Modern societies are showing similar trends of development along riverbanks. Typical examples are found in some flood plains of the Accra and the Kumasi metropolises. The concern here is with the movement of an abnormal amount of water along a river or watercourse. Such an event may be generated by intense rainfall on a catchment or may be the result of a manmade disaster such as, the collapse of a dam or the opening of a spillway to release large amounts of water as a flood control management decision. The phenomenon being considered may occur in small or large streams, in natural and artificial watercourses [Price. 1994].

In any of the above-mentioned cases lives and properties at the downstream side are at risk. These are emergency problems that the hydraulic engineer will continuously contend with. The engineer has to predict quickly and with some degree of accuracy the stage/depth of the flow and the expected time of arrival of this flow. This is necessary to help management decisions such as (i) the time required for evacuation of downstream dwellers and (ii) the portions of the channel dykes that are prone to over topping, and hence needing increase in height.

The magnitude of the problem and the risk involved in such unexpected occurrences precludes experimentation on the field. Thus this area of research depends very much on numerical modelling of open channel flow. The advent of the high-speed computer has contributed immensely to the development of this area of study.

Nonlinear approximate models are frequently used for overland flow and channel routing applications because of their simplicity and ease of solution. Two approximate flood routing models, the nonlinear kinematic model [Odai, 1999a] and the nonlinear convection-diffusion model [Onizuka and Odai, 1998] are proposed for this purpose.

In practice, since flow rates are difficult to measure directly, they are often estimated using measured flow depths. Thus expressing the approximate models in terms of flow depth allows for direct usage of available data. Hence these models may be especially suitable for flood routing when available data consists of observed upstream water depths or hydrographs on the reach under consideration [Cappelaere, 1997].

The qualitative analyses performed under subcritical flow conditions, show that the models perform comparably with the St. Venant equations. For an instantaneous collapse of a dam, the upstream flow depth is known, hence the approximate flow rates/depths at downstream sections are readily obtained. Special features of the approximate flood routing models are that they are simple to apply, and they require far less computer time than the St. Venant equations [Odai, 1999b]. Therefore they may be useful for the rapid calculations of flood propagation during flood forecasting.

The Complete Flood Model

A normalized version of the St. Venant equations [Onizuka and Odai, 1998; Odai, 1999b] with only one parameter, the Froude number of initial uniform flow, is used in this study. It is given, respectively, by the Continuity and the Dynamic equations

$$\frac{\partial h}{\partial t}+\frac{\partial (uh)}{\partial x}=0 \qquad (1)$$

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+\frac{\partial h}{\partial x}=1-\frac{1}{F_0^2}\frac{u^2}{h^{2m}} \qquad (2)$$

where h = flow depth; u = flow velocity; x = distance from the dam axis; t = time; m = $^2/_3$ for Manning's formula or $m = ^1/_2$ for Chezy's formula; and F_0 = Froude numbei of initial uniform flow; h_0 = initial normal depth; and n is Manning's roughness coefficient. For Manning's formula $F_0 = h_0^m \sqrt{S_0}/n\sqrt{gh_0}$ and m = $^2/_3$. All variables in Eqs. (1) and (2) are normalized unless otherwise specified.

Although Eqs. (1) and (2) are the most accurate, they are the most demanding on human and computer resources. This has led to the development of several approximate flood routing models in order to minimize these demands. Two of which are presented below.

Approximate Flood Routing Models

Consider the general convection-diffusion equation given below

$$\frac{\partial h}{\partial t}+f\frac{\partial h}{\partial x}-\theta\frac{\partial^2 h}{\partial x^2}=0 \qquad (3)$$

where f is the wave speed and θ is the diffusion coefficient. Several models could be developed from Eq. (3) depending on the expression for f, and whether θ = 0 (kinematic model) or $\theta \neq 0$ (convection-diffusion model).

For $f=f(h)$ nonlinear model
f = *constant* linear model

Since the flow of water in a channel or river is a nonlinear phenomenon, f is set equal to $(\alpha + \beta h)$. The nonlinear kinematic model and the nonlinear convection-diffusion model are discussed below.

The Nonlinear Kinematic Model

When θ = 0 we obtain the nonlinear kinematic-wave model [5]

$$\frac{\partial h}{\partial t}+(\alpha+\beta h)\frac{\partial h}{\partial x}=0 \qquad (4)$$

where α and β are constants. It is interesting to note that the above equation is made up of a linear and a nonlinear part.

For gradually varied flow the left-hand side of Eq. (2) is zero, because their values are negligible compared to the right-hand side. According to Lighthill and Whitham [1955] Eq. (2) thus becomes

$$u = F_0 h^m \qquad (5)$$

To obtain α and β Eq. (4) is substituted into Eq. (1) and integrated once with respect to x to yield the flow velocity as

$$u=\left(F_0+\alpha(h-1)+0.5\beta(h^2-1)\right)/h \qquad (6)$$

Substituting Eq. (6) into Eq. (5) and taking the first and second derivatives with respect to h at h = 1, yields $\alpha + \beta = (m+1)F_0$ and $\beta = m(m+1)F_0$, respectively. Hence the constants are obtained as

$$\alpha=\frac{5}{9}F_0,\quad \beta=\frac{10}{9}F_0,\quad \alpha+\beta=\frac{5}{3}F_0 \qquad (7)$$

For the solution of Eq. (4) only the initial and upstream boundary conditions are required, but the downstream boundary condition is not required. This implies that the equation cannot model any backwater effects that may be present. Also because of the absence of diffusion term the model gives results with steep wave fronts.

The Nonlinear Convection-Diffusion Model

The kinematic model cannot be used for conditions where backwater effects may be present. In addition the nonlinearity effect is very strong in the kinematic model. Therefore the nonlinear convection-diffusion model [Onizuka and Odai, 1998] is developed by adding a diffusion term to Eq. (4) to yield

$$\frac{\partial h}{\partial t}+(\alpha+\beta h)\frac{\partial h}{\partial x}-\theta\frac{\partial^2 h}{\partial x^2}=0 \qquad (8)$$

Eq. (8) was extracted by, first substituting it into Eq. (1) and integrating once with respect to x, to yield the flow velocity as

$$u=\left(F_0+\alpha(h-1)+0.5\beta(h^2-1)-\theta\frac{\partial h}{\partial x}\right)/h \qquad (9)$$

Substituting Eqs. (8) and (9) into Eq. (2), simplifying, and collecting like-terms yields the coefficients of the Burgers equation [Odai, 1999a; Odai, 2002] as

$$\alpha = \frac{5}{9}F_0,\ \beta = \frac{10}{9}F_0,\ \ \theta = \frac{F_0}{2}(1-\frac{4}{9}F_0^{\ 2}),$$
$$\alpha + \beta = \frac{5}{3}F_0 \qquad (10)$$

According to Eq (8) the mechanism of open channel flow consists of: (i) nonlinear convection at the speed $(\alpha + \beta h)$; and (ii) attenuation or amplification depending on the sign of θ. The role of θ as the damping mechanism in diffusion and transport-related problems is well known. The presence of θ in the nonlinear convection-diffusion model is responsible for the attenuation effects on the wave fronts predicted by the model.

Numerical Experiments

A hypothetical example of a channel of length 50.0, initial normal depth 1.0, and initial uniform flow velocity F_0 is used in this study. To study how the rate of change of flow rate R_D (ratio of final normal flow rate to initial normal flow rate) and F_0 affect the ability of the approximate flood routing models to predict the flood stage and the arrival time, three examples are presented. In the analysis performed, the ratio of the final normal flow rate to the initial normal flow rate depicts the magnitude of the flood. Mathematically it is given by:

$$R_D = \frac{\text{final normal flow rate}}{\text{initial normal flow rate}} = \frac{Q_F}{Q_I} \qquad (11)$$

In Figures 1 through 3 the results of the nonlinear kinematic model (TW) and those of the nonlinear convection-diffusion model (TO) are compared with those of the St. Venant equations (TS). The results obtained by the St. Venant equations (TS) are used as the benchmark to judge the performance of the two approximate flood routing models. In the first two scenarios considered, the wave fronts predicted by the nonlinear kinematic model are very steep. This phenomenon is due to the strong nonlinear effect in the nonlinear kinematic model. This is attributed to the absence of diffusion effect in the model. Thus there is no attenuation of the wave predicted by the model.

Adding a diffusion term as shown in the nonlinear convection-diffusion model considerably reduces the nonlinear effect that was very strong in the nonlinear kinematic model. In Figures 1 and 2 the results by the nonlinear convection-diffusion model are between those of the St. Venant equations and those of the nonlinear kinematic model. The nonlinear convection-diffusion model gives a better description of open-channel flow because it accounts for both nonlinear propagation and diffusion effects as opposed to the nonlinear kinematic model, which does not account for attenuation effects.

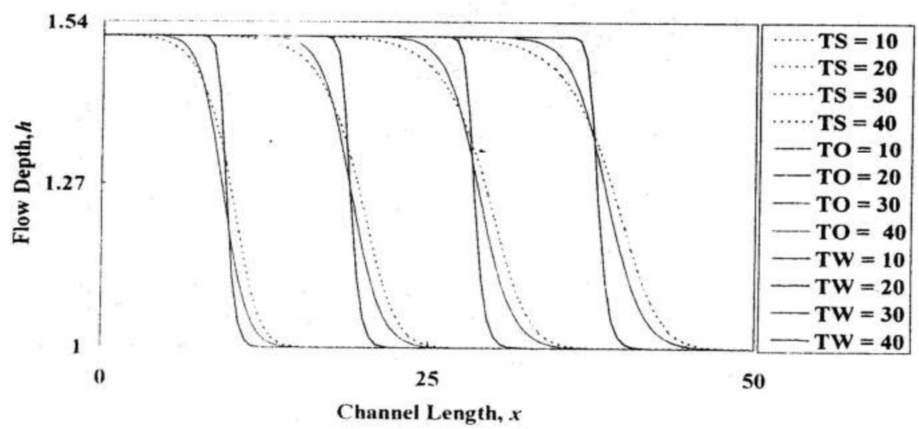
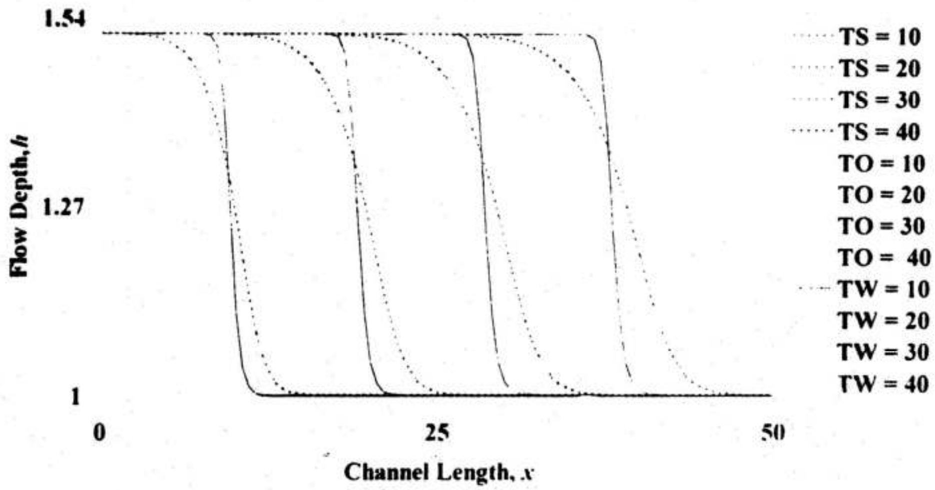


Figure 2 Transformation Process of Flood Wave of $F_0 = 0.5$ and $R_D = 2.0$



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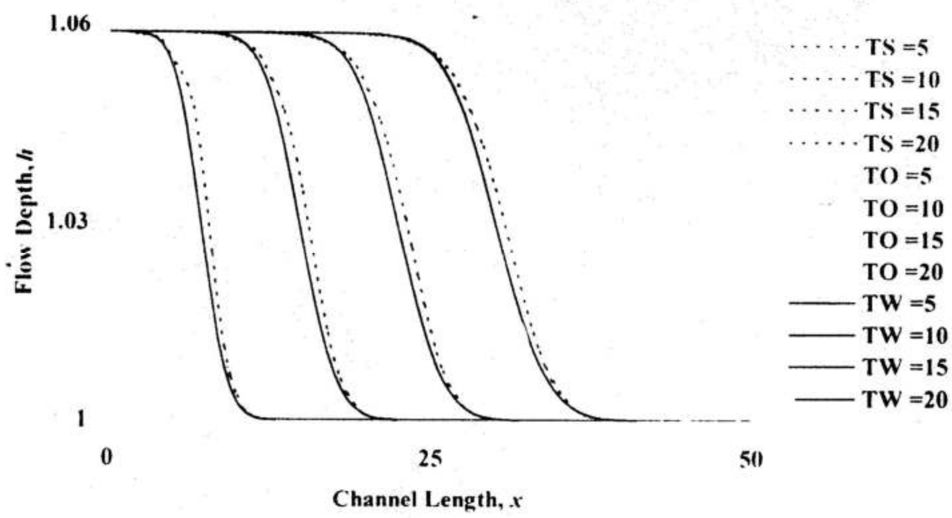

Figure 3. Transformation Process of Flood Wave of $F_0 = 0.9$ and $R_D = 1.1$

In Figs. 1 and 2 where there is 100% increase in flow, the effect of nonlinearity is more apparent as compared to the case in Figure 3, where there is only a 10% increase in flow. Thus in Fig. 3 the wave fronts predicted by the two approximate flood routing models coincide. This implies that for low nonlinear effects the two approximate flood routing models perform exactly the same. Comparing the three Figures shows that as the value of R_D departs from $R_D = 1.0$ the effect of nonlinearity increases and the nonlinear kinematic model should not be used.

Also, it is apparent that as F_0 (the flow velocity) increases the wave travels faster and the arrival time shortens. This is depicted in Figs. 1 through 3 by the successive shortening of the time taken by the wave fronts to travel the same distance downstream. This implies that shorter time is required to evacuate downstream dwellers.

On the other hand as R_D increases the change in flow depth increases. This gives an idea of the effect of R_D on the flow depth/stage, and hence the possible level of flood water inundation of the surrounding community. There may be sections of the watercourse dyke that need increase in height, and the result of the simulation can expose such vulnerable areas for earlier attention. The flood stage depicts the possible effect of the dam break or flood on the downstream dwellers.

Conclusions

Nonlinear approximate models have been used for estimating the arrival time and arrival flow depth of floods due to storms or dam breaks. The models are expressed in terms of flow depths; hence they are easy to apply, especially, when available data consists of observed upstream water depths or hydrographs on the reach. With these models, the engineer can predict accurately the expected time of arrival of the flood and the anticipated magnitude of the peak of the flood wave, with less difficulty. The results of the study have shown that when R_D is small the nonlinear kinematic-wave model may be used, however, as R_D increases the nonlinear convection-diffusion model should be used instead. The time of arrival of the wave front at a downstream point along the river channel can be predicted readily, hence evacuation of downstream dwellers can be planned more effectively. Also the depth of flow predicted gives an idea of which parts of the riverbank dykes need increase in height.

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